insurance is therefore of interest since the method is equally applicable in the non-life field.

An Introduction to Collective Risk Theory and Its Application to Stop-Loss Reinsurance by Dr. PAUL M. KAHN.

The purpose of this paper has been to present to the American actuary some of the methods and results of risk theory and to stimulate consideration of it on this side of the Atlantic. Dr. Paul Kahn provides a discussion of the distribution problem inherent in the collective theory of risk, a brief outline of the ruin problem and an application of the distribution theory to stop-loss reinsurance which, by its very nature, invites the application of collective risk theory. He concludes with some discussion of the usefulness of this work and of more useful tools which may be developed.

L. H. Longley-Cook

Austauschbare stochastische Variabeln und ihre Grenzwertsätze by HANS BÜHLMANN. University of California Press, Berkeley 1960.

In non-life insurance, the mathematical models frequently proceed from the simplifying assumption that the variables investigated are stochastically independent. This hypothesis generally does lead to a lucid presentation, however, practical experience has shown that under the assumption of independence, it often becomes difficult to reproduce the actual conditions satisfactorily. The investigation of dependent variables is therefore not merely of theoretical interest. The author of the present thesis deals with a special type of dependence, viz. exchangeability. This concept was introduced by de Finetti some time ago (1931, in a work in the "Mem. Reale Academia Naz. Lincei"), but has received little attention so far. A series of stochastic variables X_i is called exchangeable, if the distribution function for each finite selection $X_{i_1}, X_{i_2}, \ldots, X_{i_n}$ is identical to that of $X_1, X_2, \ldots, X_n$, i.e.

$$F_{X_{i_1}, X_{i_2}, X_{i_n}}(x_1, x_2, \ldots, x_n) = F_{X_1, X_2, \ldots, X_n}(x_1, x_2, \ldots, x_n)$$

The problems, which arise through the introduction of this new concept, are dealt with by the author in three chapters.

The basis of the theory is a proposition by de Finetti, which—as formulated by Loève—proves that every infinite series of exchangeable random variables can—in a certain well-defined sense—be regarded as conditionally independent and identically distributed. The analysis of the concepts 'exchangeable' and 'conditionally independent' and the proof of their equivalence form the contents of the first part.

In the second section, the author investigates the solution of the central limit theorem for exchangeable variables. Assuming finite variance of the variables X_i, it appears that the class of the possible limit distributions of standardized sums $S_n = \sum_{i=1}^{n} \frac{X_i}{B_n}$ coincides with the class of weighted normal distributions. Degenerate distributions occur in special cases. The question as to the necessary and sufficient conditions for the convergence permits a precise answer. In particular, the proposition holds good that for $B_n = \sqrt{n}$ the standardized partial sums of a sequence with mean 0 and variance σ_0^2 tend to the normal limit distribution if, and only if,

$$(\alpha) \sum_{i=1}^{n} \frac{X_i}{n} \xrightarrow{a.s.} 0 \qquad (\beta) \sum_{i=1}^{n} \frac{X_i^2}{n} \xrightarrow{P} \sigma_0^2$$

In this, as usual, $\xrightarrow{a.s.}$ signifies almost sure convergence, $\xrightarrow{P}$ convergence in probability.

In the last part of the paper the concept of exchangeability is further amplified to include stochastic processes. A process with exchangeable increments is said to occur when the variables

$$Y_{2^n, k} = X_{t_{2^n}, k} - X_{t_{2^n}, k-1}$$

(the index range of X is the interval $[a, b]$ and

$$t_{2^n, k} = a + [(b-a) k / 2^n], k = 0, 1, \ldots, 2^n)$$

for all n are terms of an infinite series of exchangeable variables. After discussing the fundamental characteristics of such processes, the author points out the connection with the class of infinitely divisible distribution laws. Of special interest are processes, the sample functions of which are either continuous or discrete (step-functions with constant height). In the first case, we get weighted normal distributions as probability laws of the processes. They are not weighted normal if, and only if, the increments are independent. Similar statements hold good for sample functions of the second type. Here weighted normal distributions have to be substituted by weighted Poisson distributions; the characterization of independence can be derived in an analogous way.

It is to be hoped that this interesting publication receives the attention it deserves and that it gives inducement for further investigations. Bühlmann's treatment of the subject is from a pure theoretical point of view, and a certain acquaintance with higher algebra and functional analysis is indispensable to the understanding. Only the future can show how far the results will prove useful for practical problems.

J. Kupper

The Objectives of an Insurance Company by Karl Borch. Skandinavisk Aktuarietidskrift 1962 (p. 162).

In continuation of some of his previous papers the author defines the objectives pursued by an insurance company by a mathematical function. This function to be maximized is usually referred to as utility function and defined by the risk situation of the company. Starting from the so-called Bernoulli hypothesis and assuming a rational behaviour two major decisions to be reached by a company are discussed. It is obvious that the decisions are reached so as to give the highest utility. The first decision concerns the determination of premiums to be offered to the public and the amounts to be spent for the promotion of sales, the second the companies reinsurance decisions.

A Contribution to the Theory of Reinsurance Markets by Karl Borch. Skandinavisk Aktuarietidskrift 1962 (p. 186).

In a previous paper the author has shown that there exists no market price in a reinsurance treaty which—when applied to all transactions—will lead