
The book in question provides an ‘introduction’ to topology via surfaces in \( \mathbb{R}^3 \). The quotes are because it is written at a fairly high level (advanced undergraduate / beginning graduate student in US terms, which is the author's origin).

The basic ideas of point set topology such as continuity, connectedness, compactness, quotient spaces etc. are covered in the first 60 pages. After this is the meat of the text, with the classification of surfaces and a thorough introduction to the fundamental group presented in the remainder of the first half. The second half is devoted to covering spaces, CW complexes and a detailed development of singular homology, with an emphasis on particular examples and calculation via the Mayer-Vietoris sequence.

What makes this book different is the approach. The author adopts a ‘Moore method’ in which the reader does much of the work. The text is a series of motivated definitions and discussions, together with extensive exercises which develop the theory, and detailed examples to be worked which illustrate the uses of the various theorems as they are achieved.

I doubt that many second year British undergraduates would find this a useful introduction to topology, but a more advanced student would learn a great deal by working through this book. Students who worked the exercises in the second half would find themselves with an excellent grounding in the basics of algebraic topology, particularly useful for those wishing to specialise in differential topology, differential geometry, global analysis or related areas. A lecturer looking for examples to supplement a more traditional course will find all the old chestnuts and a great deal more herein.

MARK THORNBER
Durham Johnston Comprehensive School, Durham DH1 4SU


The theory of Riemann surfaces lies at the intersection of many of the principal areas of modern mathematics. Although its roots lie in complex analysis and the problem of finding a natural home for ‘multivalued functions’, the modern theory is a stunning fusion of analysis, topology, differential geometry and algebraic geometry; Jost cleverly exploits this to present a paradigm of the unity of contemporary mathematics. The richness of Riemann surfaces as a source of intuition and inspiration for more abstract generalisations means that the theory has constantly been reworked and reinvigorated. Jost’s account is no exception to this, and his most striking innovation lies in his use of harmonic maps, both as a novel tool to prove some of the key theorems and as a way of providing access to Teichmüller theory. The effect of this is to increase the ratio of Riemannian geometry and functional analysis to algebra, but I was constantly struck by the fresh perspectives revealed and the resequencing of standard theorems (often with non-standard proofs): one might say, ‘no sheaves, but a bumper harvest nonetheless!’.

Jost organises his material into five chapters. Chapters 1 and 2 deal with topological foundations (coverings and fundamental groups) and 2-dimensional differential geometry (including the topological classification of Riemann surfaces via fundamental polygons, Gauss-Bonnet and the ramifications of a hyperbolic metric). Chapter 3 erects the analytical framework (Sobolev spaces, PDE theory, variational ideas, etc.) necessary for a modern treatment of Dirichlet’s principle: to
paraphrase, the minimiser of the Dirichlet integral \( \frac{1}{2} \int |Dv|^2 \) exists and is unique as a weak solution of Poisson's equation which regularity theory then shows to be a classical solution. This is done in detail as a dry run for the definition of a harmonic map between two Riemann surfaces, \( u: \Sigma_1 \text{(compact)} \to \Sigma_2 \text{(compact, hyperbolic)} \).

Here, an energy integral \( E(u) \) is defined by analogy with the Dirichlet integral; conformal \( u \) are global minimisers, harmonic \( u \) are unique minimisers within each homotopy class (and are diffeomorphisms under natural hypotheses). Although Chapter 3 is technically the most intricate part of the book, the spin-offs are immediate and spectacular. The uniformisation theorem (for genus \( \geq 2 \)) is proved by a beautiful argument: the topological classification of Chapter 2 provides a homeomorphism \( u: \Sigma_1 \to S \text{(hyperbolic)} \); \( u \) is homotopic to a harmonic diffeomorphism which is then used to build a family of such, \( u^t, 0 < t < 1 \), in which \( u^0 \) is the desired conformal map. Chapter 4 provides an introduction to Teichmüller theory. Here, using harmonic maps enables one to side-step completely any use of quasiconformal mappings, and the (unique) harmonic map homotopic to the identity establishes directly the diffeomorphism between Teichmüller space and the vector space of holomorphic quadratic differentials. Chapter 5 is more classical in scope and deals with the algebraic geometry of Riemann surfaces, including divisors and the Riemann-Roch theorem, Abel's theorem, Jacobi inversion and (briefly) elliptic curves.

I was bowled over by this book. Jost writes in a stylish, engaging, 'let's get on with it' way and the fine detail is never allowed to obscure the big picture. It is no mean feat to provide a significantly fresh perspective on such a time-honoured area of mathematics and I warmly commend his achievement to other readers, especially those with some inkling of the underlying theory.

NICK LORD

Tonbridge School, Kent TN9 1JP


Model theory is the branch of mathematical logic that examines what it means for a first-order sentence (that is, a proposition, in some formal language, which only involves quantifiers over elements and not sets of elements) to be true in a particular structure (a set of objects together with functions, relations and constants). A theory is a set of sentences; a model of that theory is a structure in which all the sentences are true. By its very nature model theory is abstract, since it must use the concepts of language, structure and theory in the most general sense, and yet its entire motivation is concrete, in that it evolved precisely to study particular structures (such as the field of real numbers) and theories (such as Peano arithmetic or ordered Abelian groups). A strength of the author's approach is the fact that he is at pains, right from the first chapter, to stress the motivation behind the definitions by providing plenty of examples of each concept as it is introduced.

Traditionally there have been two principal themes in model theory. One begins with structures and looks at their related theories – all the sentences which are true in the structure. A question that can be asked is whether the theory is decidable: is there some algorithm for determining whether a statement is true or false in all models of the theory? Gödel showed that the theory of the ring of integers is undecidable, and Tarski that the theory of the field of reals is decidable. Marker proves Tarski's result using the method of quantifier elimination and places it in a wider context. The other theme begins with a theory and asks questions about its models: for instance, are all models of a particular cardinality isomorphic? The most