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CONVERGENCE RATE FOR STATISTICS OF POINT PROCESSES

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This thesis investigates the intractable statistical behaviour of interactions of random events in large space. The intractable behaviour can often be closely matched by a simple model with little loss.

We use Stein's method to establish the rates of normal approximation in terms of the total variation distance for a large class of sums of score functions of samples arising from random events on the integer lattice $\mathbb{Z}^d$ or driven by a Poisson point process on $\mathbb{R}^d$. As in the study under the weaker Kolmogorov distance, the score functions are assumed to satisfy stabilising and moment conditions. At the cost of an additional nonsingularity condition, we show that the rates are in line with those under the Kolmogorov distance for both cases. We also consider score functions having polynomial stabilising radii. We demonstrate the use of the theorems in six applications: local maxima, record with missing data, k-nearest neighbours, Voronoi tessellation, timber volume and maximal layers. Some of this research has been published in [2].

For the data of repeated observations of point processes, the counterpart of the sum of independent and identically distributed random variables is the superposition of independent and identically distributed point processes. The asymptotic behaviour of the superposition of independent and identically distributed point processes has been of considerable interest since the 1960's. One of the main differences between the central limit theorem and the Poisson law of small numbers is that the former possesses the large sample property, that is, the error of normal approximation to the sum of n independent and identically distributed random variables converges to 0 as $n \to \infty$. Since the 1980's, considerable effort has been devoted to recovering the large sample property for the law of small numbers in discrete random variable approximation. One

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of the most frequently used ideas is to introduce more parameters into the reference distribution. We establish a large sample property for the superposition of independent and identically distributed finite point processes. Some of this research has been published in [1].

References

[1] T. Cong and A. Xia, 'Normal approximation in total variation for statistics in geometric probability', Preprint, 2020, arXiv:2011.07781.

[2] T. Cong, A. Xia and F. Zhang, 'A large sample property in approximating the superposition of i.i.d. finite point processes', *Stochastic Process. Appl.* **130** (2020), 4493–4511.

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