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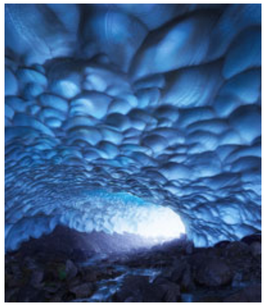
Sculpting with flow



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Flowing air and water are persistent sculptors, gradually working stone, clay, sand and ice into landforms and landscapes. The evolution of shape results from a complex fluid–solid coupling that tends to produce stereotyped forms, and this morphology offers important clues to the history of a landscape and its development. Claudin *et al.* (*J. Fluid Mech.*, vol. 832, 2017, R2) shed light on how we might read the rippled and scalloped patterns written into dissolving or melting solid surfaces by a flowing fluid. By better understanding the genesis of these patterns, we may explain why they appear in different natural settings, such as the walls of mineral caves dissolving in flowing water, ice caves in wind, and melting icebergs.

Key words: geophysical and geological flows, morphological instability, pattern formation

1. Flowing fluids and shapeable solids

It is an exciting time for two important research areas in fluid mechanics. On one hand, we have fluid–structure interactions, the classic example of which is aeroelastic phenomena such as the flapping of a flag. On the other, we have moving-boundary problems that focus on the interface between two phases of matter, for example a freezing and advancing ice-water front or its recession by melting. Where these two disciplines overlap are diverse problems unified by the interaction of flowing fluids with solid but shapeable boundaries. Many processes may drive changes in shape, such as erosion, dissolution, melting, corrosion and so on, and flows are inevitably present.

Upon more reflection, one gets the impression that cases for which a flow encounters a truly immovable solid are in the minority, even if these are the problems that fill up our fluid mechanics textbooks. Even mountains gradually slump and flow due to erosion. The field of geomorphology, which seeks to explain landforms and their development, provides many examples of boundary–flow interactions that yield stereotyped shapes and patterns. Given their beauty, it is perhaps no wonder that landscapes have inspired an entire class of paintings and that our art of sculpting borrows the natural media of stone, clay and ice.

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Perhaps we fluid dynamicists are drawn to such shape–flow coupled problems because of their contrast with the textbook settings where flows must satisfy no-slip and no-penetration. In a process like erosion or dissolution, the fluid invades the solid by eating it away. The compliant fluid indeed gives way to the solid, but only on short time scales. Gradually and incrementally, but persistently, and always eventually, the flow leaves its imprint. What makes these problems physically and mathematically interesting is that the solid–fluid dynamics are interdependent, often leading to characteristic shapes or shape dynamics. In other words, complex feedback processes are at work, and better understanding translates to better control in chemical or pharmaceutical applications, for example.

2. Dissolution sculptures

The work of Claudin, Durán & Andreotti (2017) is a great example that illustrates some of the common threads in shape–flow problems. The team studies the flow of a fluid over a solid and flat but dissolvable bed, a setting whose simplicity conveys a sense of generality. The authors formulate a model that tracks the average and fluctuating components of a turbulent flow as well as the concentration of dissolved solute. Their stability analysis indicates that the fluid invades the solid not uniformly but such as to give rise to a wavy interface, and this evolution is shown schematically in figure 2(*a*).

Before digging deeper, we note that the authors are asking some form of the question 'What's in a shape?' This is the key question for all studies in geomorphology that seek to understand the changing face of the Earth as well as other celestial surfaces. Shape is the first clue, and sometimes the only clue available, when we encounter a new topography, and we would like to read into shape to infer environmental conditions past and present. An example of such historical inference is the case for flowing water on Mars as suggested by erosion channels and characteristically shaped river islands.

This particular work follows others (Blumberg & Curl 1974; Meakin & Jamtveit 2009) aiming to understand the ripply and scalloped patterns seen in different situations such as the walls of ice caves carved by wind, the walls of mineral caves carved by flowing water, and the sides of melting icebergs (figure 1). This nicely illustrates the surprising convergence of form that is a theme of shape–flow interaction problems. The analogy between dissolution and melting is precise, as the concentration of the dissolved solid is akin to temperature. That gets us to the nature of the two-way interaction, which is given by a boundary evolution law, rather than just a boundary condition that the fluid must satisfy. In this case, the gradient of the solute concentration (or temperature) in the fluid near the interface dictates how fast material will exit the solid into the fluid, and thus determines the recession velocity of the boundary (Huang, Moore & Ristroph 2015).

About those ripples: for those of us steeped in fluid dynamics, the appearance of a wavy interface as the result of an instability is familiar from many hyphenated phenomena, Rayleigh–Plateau, Kelvin–Helmholtz, Saffman–Taylor and so on. But the authors do not simply declare victory at the appearance of an instability of the flat bed. Rather they have taken the next step of imposing even higher amplitude roughness and asking whether this too is amplified. This approach has led them to conclude that there is a finite amplitude to the roughness: shallower patterns are amplified and more deeply etched patterns are dissolved away (figure 2*b*). These findings lead to predictions for the wavelength and amplitude or depth of the pattern that would be

FIGURE 1. Natural patterning. (*a*) Water-driven mineral scallops (reproduced with permission from Johannes Lundberg). (*b*) and (*c*) Water-driven iceberg scallops, exposed by turnover (reproduced with permission from Phillip Colla).

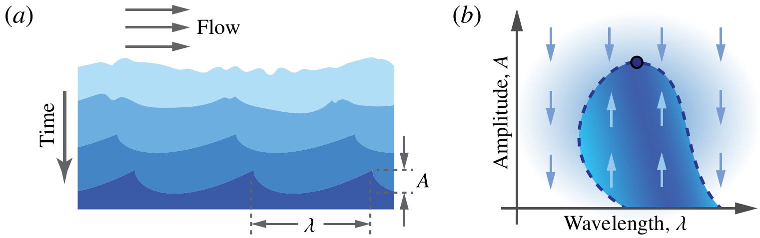

FIGURE 2. Birth of a scalloped pattern. (*a*) Flow over a melting ice sheet or dissolving mineral bed. The initial bed may contain many ripples of various heights, and the shape evolution selects a specific wavelength λ and amplitude A. (*b*) The observed pattern may represent the most prominent amplitude (circle) on the boundary between growth and shrinking.

maintained as the surface recedes. A comparison with previous observations of ice scallops shows good correspondence.

This study makes big strides towards understanding the steady state or terminal patterns seen in nature. Geomorphological problems often involve this emergence and persistence of stereotyped forms, which may be interpreted as stable attractors of the shape dynamics. In studying the growth of an icicle or stalactite (Short *et al.* 2005), for example, we believe that there is a fundamental form that would arise under ideal conditions, with natural sculptures being close but imperfect realizations. We seek these ideal forms by taking the problem out of nature and into the laboratory or, as this study does, packaging it into a model or simulation.

3. Outlook and future directions

The work of Claudin *et al.* (2017) invites future study of this instability, and a controlled laboratory experiment may be informative. We also note that the authors' approach does not explicitly evolve the shape nor exactly identify its ideal terminal form, but rather asks whether the amplitude of a sinusoidal waveform would tend to grow or shrink. The fully nonlinear and coupled problem awaits, as do three-dimensional formulations descriptive of the scalloping seen on mineral and ice surfaces. But this raises a good point: often we may consider a decoupled problem in which the flow field alone is first solved, then the boundary is incrementally moved according to the time-averaged effect of this flow. This quasistatic scheme works when the boundary speed is very slow compared to the flow speed. This means we

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should not toss out those fluid mechanics textbooks full of immovable-solid problems, as their solutions must be stitched together in time to get the boundary evolution.

I might also speculate that the mechanism identified here accounts for some but not all ripply patterns seen in nature. For example, the pocked surfaces of meteorites may be of different origin, as these are carved by ablative heating and melting during entry through the atmosphere (Feldman 1959). These may be more akin to the formation of water waves, with wind shear destabilizing the molten surface. Regardless, this again highlights the interesting similarities between disparate processes, a related example being the conical shapes assumed by both meteorites and bodies subject to fluidic erosion (Ristroph *et al.* 2012). Further investigation is needed to say if such similarities are only superficial or something more.

More generally, future research might compare receding (e.g. erosion or melting) and advancing (deposition or freezing) processes and address the issue of reversibility. I am not aware of any scallops formed by advancing fronts. And I am excited to know what new themes and unifying principles will emerge in the future. Certainly, as this and other studies have shown, the 'common wisdom' that processes like dissolution and erosion are smoothers of shape is not to be taken literally. Here, a flat surface spontaneously develops curvature, and previous studies show that erosion carves sharp points, corners and edges from smooth initial geometries (Ristroph *et al.* 2012). Progress will likely come from the detailed study of such archetypal problems, where clean and controlled experiments may make close contact with simulations (Mitchell & Spagnolie 2017) and mathematical models (Moore 2017).

If more urgency is needed, we may appeal to a darker motivation for studying this class of problems: the looming consequences of our changing climate. Our oceans and water supplies are increasingly acidified, and with it comes increased dissolution and corrosion. Global temperatures are increasing, and with that comes increased melting of the polar ice. The sea levels are rising, and hence erosion of our coasts. So as we appreciate the inherent beauty in all these processes, we should respect their persistent power, the consequences of which may grow more dire in the near future.

References

BLUMBERG, P. N. & CURL, R. L. 1974 Experimental and theoretical studies of dissolution roughness. *J. Fluid Mech.* **65** (4), 735–751.

CLAUDIN, P., DURÁN, O. & ANDREOTTI, B. 2017 Dissolution instability and roughening transition. *J. Fluid Mech.* **832**, R2.

FELDMAN, S. 1959 On the instability theory of the melted surface of an ablating body when entering the atmosphere. *J. Fluid Mech.* **6** (1), 131–155.

HUANG, J. M., MOORE, M. N. J. & RISTROPH, L. 2015 Shape dynamics and scaling laws for a body dissolving in fluid flow. *J. Fluid Mech.* **765**, R3.

MEAKIN, P. & JAMTVEIT, B. 2009 Geological pattern formation by growth and dissolution in aqueous systems. *Proc. R. Soc. Lond.* A **466** (2115), 659–694.

MITCHELL, W. H. & SPAGNOLIE, S. E. 2017 A generalized traction integral equation for Stokes flow, with applications to near-wall particle mobility and viscous erosion. *J. Comput. Phys.* **333**, 462–482.

MOORE, M. N. J. 2017 Riemann–Hilbert problems for the shapes formed by bodies dissolving, melting, and eroding in fluid flows. *Commun. Pure Appl. Maths* **70**, 1810–1831.

RISTROPH, L., MOORE, M. N. J., CHILDRESS, S., SHELLEY, M. J. & ZHANG, J. 2012 Sculpting of an erodible body by flowing water. *Proc. Natl Acad. Sci. USA* **109** (48), 19606–19609.

SHORT, M. B., BAYGENTS, J. C., BECK, J. W., STONE, D. A., TOOMEY, R. S. III & GOLDSTEIN, R. E. 2005 Stalactite growth as a free-boundary problem: a geometric law and its platonic ideal. *Phys. Rev. Lett.* **94** (1), 018501.