The Mass Spectrum of Interstellar Clouds

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Abstract

The abundance of 21-cm absorption lines seen in surveys at high latitudes can be translated into a line of sight abundance of clouds vs. column density using an empirical relationship between temperature and optical depth. As VLA surveys of 21-cm absorption at low latitudes are now becoming available, it is possible to study the variation of this function with galactic radius. It is interesting to compare the abundance of these diffuse atomic clouds (with temperatures of 50 to 100 K and masses of 1 to 10 $M_\odot$) to the abundance of molecular clouds. To do the latter we must make assumptions about cloud cross-sections in order to convert the line of sight abundance of diffuse clouds into a number per unit volume, and to convert from cloud column density to mass. The spectrum of diffuse clouds matches fairly well the spectrum of molecular clouds, although observationally there is a gap of several orders of magnitude in cloud mass. Optical absorption studies also agree well with the 21-cm results for clouds of column density a few times $10^{20} M_\odot$.

I. Background

The 21-cm line is seen in absorption from cool atomic gas; the kinetic temperature in this medium is typically 50 to 100 K. In contrast 21-cm emission comes from atomic hydrogen at all temperatures; more than half of the atomic gas is warm ($\sim$6000 K) so that emission spectra consist of a blend of warm and cool material which is often impossible to separate. Because of this blending with warm gas, line profiles in 21-cm emission are blends of various inseparable features. Thus it is not possible to inventory the population of diffuse clouds using 21-cm emission surveys, as has been done for molecular clouds using CO surveys. On the other hand, absorption profiles are made up of distinct lines which can be fitted quite well by Gaussians. Recent surveys at low and high latitudes give us a good idea of the abundance of these lines (which we empirically identify as clouds) as a function of optical depth and position in the galaxy.

Physical conditions in these diffuse clouds are quite different from those in molecular clouds, even the “diffuse” clouds seen in molecular absorption. Densities are typically 20 to 100 cm$^{-3}$, and the hydrogen is mostly atomic (HI). The warm HI may be partially associated with these clouds as well, perhaps as a halo, but at least some of the warm gas is independent of clouds (Payne et al. 1983, Kulkarni and Heiles, 1988). Deep absorption at 21-cm is also seen from molecular clouds where the bulk of the hydrogen is molecular, because their lower temperatures
make the absorption coefficient per atom much greater, but these clouds are relatively rare. In the inner galaxy, however, the atomic and molecular clouds are highly correlated; there apparently most cool atomic gas is associated spatially with the molecular cloud regions, although it is not necessarily mixed with the molecular gas on a small scale.

II. $\Phi(N)$ – the Line of Sight Abundance vs. Column Density

Many lines of sight have been observed at high latitudes, so for the region within one or two kpc of the sun we have good statistics on the number of clouds per unit distance as a function of optical depth, $\phi(\tau)$, which is shown on figure 1. Also shown are estimates from a recent low latitude absorption survey for $\phi(\tau)$ in the inner galaxy (Garwood and Dickey 1989). Apparently $\phi(\tau)$ decreases with decreasing galactic radius (which is the opposite of what we expected before doing this survey). For the solar neighborhood $\phi(\tau)$ is fitted adequately over

Figure 1. Observational Results for $\phi(\tau)$, the abundance of absorption lines as a function of optical depth. The Garwood and Dickey survey is incomplete below $\tau=0.5$ because of line blending.

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the range $0.1 < \tau < 2$ by a simple power law: $\phi(\tau) \, d\tau = 0.35 \, \tau^{-1.6} \, d\tau \, \text{kpc}^{-1}$ (Crovisier 1981, Payne et al 1983).

The function $\phi(\tau)$ can be translated into $\Phi(N)$, the number of clouds per unit line of sight distance as a function of HI column density $N$, if the spin temperature, $T_{sp}$, of the gas is known, since the optical depth measures the ratio $N / T_{sp}$. Empirically there is a correlation between $T_{sp}$ and $\tau$, which may reflect the relative column density of warm and cool gas in clouds (Liszt 1983, Mebold et al 1982). Whatever its origin, the $T_{sp}$-$\tau$ relation allows us to translate 21-cm optical depth to HI column density, so that we can construct $\Phi(N)$ (Kulkarni and Heiles 1986). For very optically thick clouds the $T_{sp}$-$\tau$ relation must break down, as the HI will not get cooler than the molecular gas in dense clouds, which is typically in the range 5 to 20 K. It may be appropriate for clouds with optical depths above 2 to discard the $T_{sp}$-$\tau$ relation and simply assume that the HI has temperature about 20 K on average. These two approaches give roughly similar results for $\Phi(N)$, illustrated on figure 2 (Dickey and Garwood 1989). The actual range of $N$ over which these are measured

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is only about 0.5 to 2.5 in units of $10^{20}$ atoms cm$^{-2}$. Figure 2 also shows the result from surveys of optical absorption lines (Hobbs 1974), which is in good agreement with both the slope and the absolute normalization of the 21-cm result.

III. The Cloud Cross-Section Determines $\rho(m)$

Absorption observations at all wavelengths typically measure the properties of gas only along a narrow column in front of the background continuum source. Cloud sizes perpendicular to the line of sight are usually not measured directly. To compare with emission surveys we need to estimate the cloud cross-section, $\sigma$, which may be defined as the ratio of the cloud mass, $m$, to the mean column density, $<N>$, i.e.,

$$\sigma = \frac{m}{<N> \cdot m_H} = 1.3pc^2 \cdot \left(\frac{m}{M_\odot}\right) \cdot \left(\frac{<N>}{10^{20}\text{cm}^{-2}}\right)^{-1}$$  \hspace{1cm} (1)

where $<N>$ is the column density averaged over the projected area of the cloud,

$$<N> = \frac{1}{\Omega_{cl}} \int N(\theta, \phi) d\Omega$$  \hspace{1cm} (2)

where $\Omega_{cl}$ is the solid angle of the cloud.

For a homogeneous sphere the internal density is simply

$$n = 38.6 \frac{<N>^{\frac{1}{3}}}{\sqrt{m}} \text{ cm}^{-3}$$  \hspace{1cm} (3)

(with $<N>$ in units of $10^{20}$ cm$^{-2}$ and $m$ in $M_\odot$), but diffuse clouds are well known to have highly elongated shapes (eg. Heiles 1967), so it is better to model their shapes with ellipsoids with fairly high axis ratios. The ratio of $<N>$ to that for a sphere with the same mass goes only as axis ratio to the $1/3$ power, so even for axis ratios of 100 or so the actual internal density is within a factor 5 of the value given by equation 3.

Although the interstellar medium is by no means at constant pressure (eg. Jenkins et al. 1983), it is probably a reasonable approximation to say that the gas pressure in diffuse clouds is generally roughly 3000 cm$^{-3}$ K. Using this and the spin temperature as discussed above we can fix the density, and solve for the mass as a function of $<N>$. The same geometrical assumptions give us the cloud cross-section,

$$\rho = \frac{\phi}{\sigma}$$  \hspace{1cm} (4)

where $\phi$ is the line of sight density as above, and $\rho$ is the number of clouds per unit volume. Now we can convert from $\Phi(N)$ to $\rho(m)$, the mass spectrum of interstellar clouds, which is shown on figure 3 (from Dickey and Garwood 1989). This function we can compare directly with the result for molecular clouds (Scoville and Sanders 1987, Terebey et al. 1987, Elmegreen 1987), which is also shown on figure 3. The molecular cloud spectrum is known only for clouds with masses in the range $10^5$ to $10^6 M_\odot$ for which the CO surveys are complete, and for which virial mass estimates are reliable. In between the giant molecular clouds and the diffuse clouds (with masses of a few solar masses) there is a gap of several orders of magnitude which is not well sampled; these intermediate mass clouds may correspond to the high latitude molecular clouds.
seen in recent surveys (e.g. Magnani et al. 1988). It is also interesting that the slope of the mass spectrum of diffuse clouds is near the critical value, \( \rho(m) \propto m^{-2} \), for which equal mass contributions are made by equal mass intervals, and both the maximum and minimum mass are critical to the total mass density of cloud material:

\[
\rho = \int m \cdot \rho(m) dm
\] 5.

We can check the normalization of the atomic portion of this spectrum, since we know that in the solar neighborhood the cool HI makes up about half of the total atomic hydrogen (e.g. Colgan et al. 1988), whose surface density is 5 to 6 \( M_\odot \text{pc}^{-2} \) at the solar circle (Burton 1987 and references therein). The diffuse cloud spectrum shown on figure 3 gives this value if the limits of integration in equation 5 are taken to be about 0.1 and 1000 \( M_\odot \).

It is important to distinguish between the approach taken here and other derivations of the "cloud spectrum" based on measurements of cloud sizes. Not only are cloud edges difficult to determine observationally due to spatial blending by the telescope beam in emission surveys, but more importantly the relation between cloud size and mass is very steep. For the two alternatives of constant temperature clouds or the \( T_{sp} - \tau \) relation, the scaling relation \( < N > \) goes as radius to the
$2^{nd}$ or $3^{rd}$ power, whereas for molecular clouds which are self-gravitating and therefore centrally condensed the central column density is an even steeper function of the radius. Thus it is unwise to construct a cloud mass spectrum starting from a cloud size spectrum derived indirectly from observations.

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References:


Discussion:

MUNCH (Comment): Your success in establishing that the estimate of the first moment in the mass distribution of IS "clouds" (their mean number per kpc) is the same when derived from 21cm absorption data as from optical IS lines multiplicity, is most gratifying. But we should recall that the same number is obtained from extinction data (stellar color excesses) and from brightness fluctuations of the Milky Way. This overall agreement shows that the concept of "clouds" as one dimensional structures of zero measure has a physical meaning. We owe to the intuition of Ambartsumian this abstraction of an immensely complicated physical situation. Chandrasekhar and I worked out in detail the mathematics of the stochastic processes involved, as Chandra mentioned during the Banquet of this meeting. After my "divergence" to Caltech I studied further the questions on the basis of "infinitely divisible distributions" and applied the results to various sub-sets of measured stellar color excesses. My efforts to extend the analysis to joint distributions of optical depths and velocities of IS absorption lines met only partial success. With the recent "explosion" of observational data, however, new theoretical tools for interpretation need to be elaborated. I fully agree with John Dickey's remark that to ask for the "shape" of an interstellar cloud is a meaningless proposition. In this context I should further add that the "pictures" of molecular clouds under higher and higher angular resolution, as shown by J. Bally, are a beautiful visualization of the concept of "infinite divisibility", that is to say of the topological invariance of their 2D distribution under varying angular resolution.

SCWARZ: In deriving the optical depth distribution as function of distance from the center, do you take "shadowing" (you cannot see through optically thick clouds) effects into account?

DICKEY: Yes, we take account of line blanketing which leads to incompleteness in the cloud spectrum $\phi(r)$. There are very few velocities where the total optical depth saturates so as to difficult to measure (optical depths add linearly, in any case).