In the case we are considering, $\mu = \sqrt{-1}$, and therefore we infer that if P be any point on the hyperbola its polar with regard to the conjugate, touches the original hyperbola at P′, the other extremity of diameter CP, since $CP' = -CP = CP \times (\sqrt{-1})^2$.

As a final example, I take one involving focal properties. The projective property of the focus may be readily established geometrically, for we know that

1. A revolving right angle, makes with the circular asymptotes a harmonic pencil, and with no pair of *fixed* lines *but* the circular asymptotes.

2. If OL, OM be tangents to a conic, and the tangent at P meet LM in R, then OL, OP, OM, OR is harmonic.

3. If the tangent at P meet directrix in Z, then PSZ is a right angle. Combining these we see that the tangents to a conic from a focus, must pass through the circular points at infinity.

VII. A number of parabolas pass through a fixed point, and touch two lines. Show that the chord of contact envelopes a hyperbola. (*Aberdeen Senior Mathematical Examination* 1884).

Project orthogonally any hyperbola asymptotic to OP, OQ, (Fig. 32) into a circle, then OP, OQ become the circular asymptotes, and the parabolas remain parabolas, while O becomes their focus S. Hence the problem becomes—Given a point and a focus of a parabola, find the envelope of polar of the focus, or the directrix. And this is of course a circle, whose centre is the fixed point, and which passes through S. Returning to the original figure, we find that the envelope is a *hyperbola whose centre is* A, *which passes through* O, *and whose asymptotes are parallel to* OP, OQ.

On a number of concurrent spheres.

By R. E. Allardice, M.A.

This paper included an analytical proof of the following theorem:

If five spheres 1, 2, 3, 4, 5, pass through the same point, and if the four points in which the four spheres 1, 2, 3, 4, intersect in sets of three be coplanar; and if the same be true for the sets 1, 2, 3, 5; 1, 2, 4, 5; 1, 3, 4, 5; it will also be true for the remaining set 2, 3, 4, 5.

The same theorem is true for similar and similarly situated quadric surfaces.

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