10

Double-Slit Experiments

10.1 Introduction

In this chapter we show how the quantized detector network (QDN) formalism describes the double-slit (DS) experiment. This is arguably the simplest experiment that demonstrates quantum affects such as wave–particle duality and quantum interference. It continues to be the focus of much debate and experiment (Mardari, 2005), because theoretical modeling of what is going on reflects current understanding of quantum physics and hence physical reality. We will apply QDN to two variants: the original DS experiment and the *monitored* DS experiment, where an attempt is made to determine the imagined path of the particle.

The DS experiment is widely acknowledged by physicists to be of importance to the understanding of quantum mechanics (QM). So much so that in 2002, the single electron version, first performed by Merli, Missiroli, and Pozzi (Merli et al., 1976), was voted by readers of *Physics World* to be “the most beautiful experiment in physics” (Rosa, 2012).

The DS experiment can be discussed in terms of three stages, shown in Figure 10.1. By the end of the preparation stage, Σ_0, a monochromatic beam of light or particles has been prepared by a source P, such as a laser. The beam emerges from point O and then passes through an information void V_1 to the first stage, Σ_1, which consists of a wall or barrier W. This wall has two openings denoted A and B that allow parts of the beam to pass through into another information void V_2 and onto the second and final stage Σ_2, which consists of a detecting screen S.

The screen S is in general some material that can absorb and record particle impacts. In reality, any screen will consist of a finite number of signal detectors, such as photosensitive molecules, but the typical QM modeling is done as if there were a continuum of sites on the screen, such as C, that could register particles.

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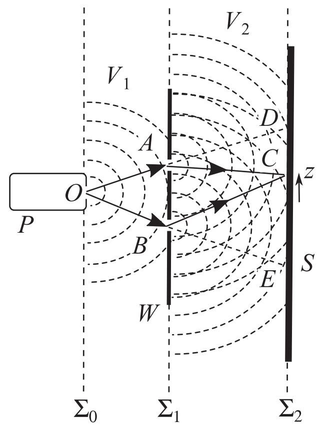

Figure 10.1. The DS experiment. C is a typical detector site in the detecting screen S.

10.2 Run Protocol

In addition to the above geometrical architecture, there are some important protocol features that need to be clarified.

Runs

Any DS experiment will consist of a large, possibly enormous, number of runs, or repetitions of a basic protocol.

Statistics

The conclusions of the experiment are based on a statistical analysis of the data averaged over all valid runs.

Preparation Stage

Each run consists of the observer establishing the start of that run, indicated by stage Σ_0 in Figure 10.1. This means that the observer will have reliable contextual information that some previously agreed-on procedure has been carried out at device P. If anything occurs to prevent confidence in that information, then that run is discarded.

Outcome Stage

At the end of each run, corresponding to stage Σ_2 in Figure 10.1, the observer looks at every accessible point/detector on the screen S and records whether each detector has a signal or not. The data from each run are entirely classical at this point, being in the form of a vast number of bits of yes/no values, each value coming from a given detector on the screen S.

Information Void

It is a critical feature of any quantum experiment, apart from the quantum Zeno-type experiments discussed in Chapter 15, that between stages Σ_0 and

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Σ_2, no attempt is made by the observer to extract any information from either information void V_1 or V_2, or at stage Σ_1.

Reset

At the end of each run, the detectors in screen S are reset to their ground states, ready for the next run, which takes place as if no other run had ever occurred.

10.3 Baseball in the Dark

It is significant that in the above description of protocol, Section 10.2, no reference is made to *particles*, *beams*, *interactions*, *waves*, or any such classical mechanics (CM) imagery. What has been described is only what the observer actually does in the laboratory, not what theorists imagine they are doing. Indeed, we not pushing this point too far to remind the reader that even the above "objective" account is contextual, being human-centric. From any other species' point of view, nothing of importance would be going on in the laboratory. A dog, for example, would be more concerned where the observer kept their sandwiches. Of course, when we discuss such experiments in practice, it is most convenient to objectify procedures in familiar terms, so we will usually talk about preparing a beam of particles at P and allowing that beam to pass through a double slit. We may even be caught out referring to photons impacting on a screen. But all of that is to be read as a convenience, not as a statement of belief that there are objective things known as photons.

When discussing quantum processes, we should be wary of invoking undue mental imagery. In this respect, a helpful analogy is to imagine the DS experiment as "baseball played in the dark." Suppose we were asked to describe a game of baseball or cricket played not in broad daylight but at night in pitch black conditions and with no sound. Now the game would look very different from what it would look like during the day. The pitcher or bowler would be the analogue of the preparation device P; the batter or batsman would be the analogue of one of the openings, A or B, in the wall W; and the fielders would be the detectors in the screen S. P would have some contextual information that they should throw the ball in a certain direction. Suppose they did that. If the ball reached the batter, it would be struck in some random direction. The odds of any of the fielders catching that ball at night (the analogue of a quantum experiment) would be quite different from the odds during daytime (the analogue of a classical experiment), because in daylight, fielders would have constant visual information as to the current position of the ball. At night, they would be faced with a real information void.

The above description of baseball in the dark is a classically based attempt to convey some of the attributes of observation; it cannot adequately account for all the nonclassical attributes of quantum processes. Any experimentalist who has done real DS experiments would possibly find the description of protocol

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in Section 10.2 simplistic and perhaps misleading. What we have described is a highly idealized version of rather complex procedures that involve a great deal of beam calibration, noise suppression, timing protocols, shielding, detector physics, and complex electronics. Indeed, major experiments such as the search for the Higgs particle at the Large Hadron Collider take data during a small fraction of the duration of the experiment, the rest of the time being spent in planning, funding, construction, and calibration of the apparatus. It is a common feature of experimental physics doctorates that constructing the apparatus takes two or more years in preparation, and then there is a frantic race to take enough data to justify submitting a thesis.

Perhaps the best way to think of these issues is to accept that theory does not describe reality directly but deals with *equivalence classes* of processes (Kraus, 1983). For example, what we mean by a "double-slit experiment" is a theoretical model of the equivalence class of activities in the laboratory that each has the essential features outlined in Section 10.2, disregarding many contexts such as whether the observer is male or female, wears a hat or not, and so on.

What is truly remarkable is that when we overlook these issues in much in the same way as friction is overlooked in Newtonian mechanics, then there emerges from the overall complexity of any experiment some simple theoretical rules as to what is going on. That is really how QM was discovered. Moreover, these rules have great applicability and in the case of QM, work particularly well and far better than CM in experiments such as the DS experiment. QDN should be seen in this light: it will give the essential architecture of an experiment but not a detailed description of the "friction" encountered in quantum experiments, unless that is called for.

10.4 Observed Phenomena

The DS experiment reveals a number of phenomena that continue to puzzle physicists, because those phenomena cannot be fully explained according to the principles and ideology of CM.

Line-of-Sight Violation

With reference to Figure 10.1, suppose opening B is blocked off but otherwise the experiment runs as described above. According to CM, particles passing through opening A should pass more or less undisturbed onto position D on the screen S. Of course, the opening at A would deflect some of the particles, since the wall W consists of atoms, so it is to be expected that at the edges of opening A, forces will act on any beam particles passing nearby. However, the fraction of all the particles passing through either opening that comes close to the edges of the slits is expected to be relatively small, so we expect to find a relatively large distribution of particles around position D on the screen S with relatively few to either side.

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What is observed is quite different. There is indeed a broad peak found centered on D, but it extends further over the screen than expected from naive CM expectations. Let us call the probability profile of this broad peak $P^A(z)$, where z is the position coordinate of points along the screen as shown in Figure 10.1. If now instead of B we blocked off opening A, then we would get a similar broad distribution $P^B(z)$, this time centered on E, on the line of sight from O to B.

Interference

What is astonishing is that if now we ran the experiment with both A and B open, then we would not find a simple distribution $P^A(z) + P^B(z)$ as expected from CM principles. Instead, we would find a distribution $P^A(z) + P^B(z) + I^{AB}(z)$, where I^{AB} is known as an interference term. It is this interference term that causes all the fuss.

Self-interference

It was originally believed that a theoretical particle–wave conflict could be avoided if the explanation of the interference term I^{AB} was that there were interactions of the particles coming from opening A that were somehow interacting with particles coming from opening B, thereby disrupting the basic addition of P^A to P^B on the detecting screen.

But the mystery was only deepened when DS experiments were done such that the rate of particles falling on the screen was extremely low (Taylor, 1909). So low in fact that there would be one (or even fewer than one)[1] on average landing on the screen S per run. It was found that the interference term occurred in the statistical analysis when a large number of runs was performed, even when only one particle could possibly have passed through at a time. The point is that the aforementioned picture of clouds of particles interfering with each other cannot be valid here when only one particle passes through at a time. The paradox is that it is then hard if not impossible to understand why the interference term should occur in the analysis of many separate, uncorrelated runs. When QM is interpreted in terms of particle–waves interfering with each other, the low-intensity interference pattern is generally referred to as *self-interference*, a term commonly attributed to Dirac in his famous book on QM (Dirac, 1958).

Not all theorists subscribe to this view (Mardari, 2005). From the QDN perspective, the term *self-interference* is a dangerous one to take seriously, as it invokes a confusing picture of a classical particle that behaves unlike a classical particle. We do *not* have to believe in particles: the only things that we can be sure of are signals in detectors.

[1] This is deep. An average of fewer than one particle per run would mean, of course, that during some runs, *no* particles were recorded as having landed on the screen. An average of fewer than one particle per run vindicates the view that what matters is what the observer does, not what we think they do. Observers push buttons and look at screens. They do *not* know for sure what is happening beyond that description.

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Path Indeterminacy

The interference term I_{AB} requires both openings A and B to be present. If any attempt is made to block off one or another opening, then I^{AB} disappears. Now according to CM, every time a detector registers a signal at S, that is evidence for a particle having traveled from source P to that detector. According to CM principles, that particle had to have traveled along a continuous path from O through either A or B. If true, it should be possible to establish which of the two openings it was. But any attempt to do this appears to destroy I^{AB}.

It is as if the observer has two mutually exclusive choices: either have no knowledge about which path was taken, and then P^A, P^B, and the interference term occur, or know which path was taken, such as through A, but then the interference term and P^B disappear and only P^A is observed. In QM, the principle that there is this exclusive choice is generally referred to as *complementarity*.

It may be possible to trade off information, with the observer having only a probability estimate for each of these alternatives. In that case, the overall distribution on the screen would depend on that probability estimate in some way. However, predicting that distribution would undoubtedly require the most careful analysis of context. Experiments along such lines have been attempted, such as that of Afshar (Afshar, 2005). The results of such experiments remain controversial (Kastner, 2005).

Wave–Particle Duality

We shall see in the next section that physicists can get a good theoretical handle on the experiment by applying the particle–wave concept inherent to Schrödinger wave mechanics. The DS experiment touches on particle-like attributes because the detectors in the screen S respond in a discrete *yes/no* way characteristic of particle impacts, while on the other hand the distribution $P^A + P^B + I^{AB}$ is characteristic of wave dynamical processes. The mystery only arises when theorists suppose that there are objects with particle and wave properties simultaneously and fail to recognize empirical context as a critical factor in the experiment. In actuality, each aspect (particle or wave) is significant within its own particular empirical context, and there are no real paradoxes in the laboratory. The so-called wave–particle paradox occurs only because of the way humans generally choose to interpret their experiments.

10.5 A Wave-Mechanics Description

In this section we discuss the DS scenario using standard non-relativistic QM for a wave–particle of nonzero mass m propagating in three spatial dimensions. The standard theory assumes that apart from the production process in the source P, the interaction with the wall W, and the screen S, the quantum state representing the dynamics propagates in the information void regions V_1 and V_2 according to the Schrödinger–Dirac equation

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$$i\hbar \frac{d}{dt}|\Psi, t\rangle = \widehat{H}|\Psi, t\rangle, \tag{10.1}$$

where $\widehat{H}$ is the free particle Hamiltonian given by

$$\widehat{H} = \frac{\widehat{\boldsymbol{p}} \cdot \widehat{\boldsymbol{p}}}{2m}. \tag{10.2}$$

Relative to a standard improperly normalized particle position basis $\{|\boldsymbol{x}\rangle, \boldsymbol{x} \in \mathbb{E}^3\}$, the state vector $|\Psi, t\rangle$ is given by

$$|\Psi, t\rangle = \int d^3\boldsymbol{x} \Psi(\boldsymbol{x}, t)|\boldsymbol{x}\rangle. \tag{10.3}$$

The wave-function part $\Psi(\boldsymbol{x}, t)$ of the solution to Eq. (10.1) is readily found by standard methods to be given by

$$\Psi(\boldsymbol{x}, t) = \int d^3\boldsymbol{y} F(\boldsymbol{x}, t; \boldsymbol{y}, t_0)\Psi(\boldsymbol{y}, t_0), \qquad t > t_0, \tag{10.4}$$

where the propagator $F(\boldsymbol{x}, t; \boldsymbol{y}, t_0) \equiv \langle \boldsymbol{x}|e^{-i\widehat{H}(t-t_0)/\hbar}|\boldsymbol{y}\rangle$ is given by (Feynman and Hibbs, 1965)

$$F(\boldsymbol{x}, t; \boldsymbol{y}, t_0) = \left[\frac{-im}{2\pi\hbar(t - t_0)}\right]^{3/2} \exp\left\{\frac{im(\boldsymbol{x} - \boldsymbol{y})^2}{2\hbar(t - t_0)}\right\}, \qquad t > t_0. \tag{10.5}$$

This propagator will be valid for events $(\boldsymbol{x}, t)$ and $(\boldsymbol{y}, t_0)$ in the same information void region. We shall consider what happens in region V_1 and then in region V_2.

In the following, we take standard Cartesian coordinates $\boldsymbol{x} \equiv (x, y, z)$ with origin at O in Figure 10.1, x-axis along the beam direction, y-axis transverse to the beam, and z-axis in the direction from B to A. For simplicity, we shall suppress any transverse effects and assume that openings A and B are almost point-like, with coordinates $\boldsymbol{x}_A = (d, 0, a)$ and $\boldsymbol{y}_B = (d, 0, -a)$, respectively, where d is the distance of the wall W from the opening O and a is positive. We shall consider what happens at a point C on the detecting screen S, with coordinates $\boldsymbol{x}_C = (d + D, 0, c)$, where D is the distance of the screen S from the wall W.

For the rest of this section we use the notation $x_1 \equiv (\boldsymbol{x}_1, t_1)$, $y_2 \equiv (\boldsymbol{y}_2, t_2)$, and so on.

Region V_1

We imagine that at the first stage Σ_0, a normalized pulse is emitted from O, characterized by wave function $\Phi(x_0)$. At a given event with coordinates x_1 on the wall W, the wave function $\Psi(x_1)$ after propagation through V_1 is given by

$$\Psi(x_1) = \int d^3\boldsymbol{x}_0 F(x_1; x_0)\Phi(x_0), \qquad t_1 > t_0. \tag{10.6}$$

Conservation of probability requires that

$$\int d^3\boldsymbol{x}_1 |\Psi(x_1)|^2 = \int d^3\boldsymbol{x}_0 |\Phi(x_0)|^2, \tag{10.7}$$

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which is satisfied by virtue of the relation

$$\int d^3\boldsymbol{x}_1 F^*(x_1; y_0) F(x_1; x_0) = \delta^3(\boldsymbol{x}_0 - \boldsymbol{y}_0). \tag{10.8}$$

Region V_2

Equation (10.6) gives the wave function impacting on the wall W on the side facing the first information void V_1. We need an expression for the wave function on the other side of W that acts as an initial wave function propagating into information void V_2 and hitting the screen S. Stage Σ_1 can be thought of in this respect as a preparation stage.

To this end we introduce *shape functions* $G^A(x_1)$ and $G^B(x_1)$ that characterize the openings A and B. Feynman and Hibbs (1965) discuss this calculation where a Gaussian shape function is assumed. The prepared wave function on the V_2 side of the wall is then given by $\left\{G^A(x_1) + G^B(x_1)\right\}\Psi(x_1)$.

Propagation through V_2 follows the same pattern as through V_1. The final stage wave function $\Psi(x_1)$ for an event on S at stage Σ_2 is given by

$$\Psi(x_2) = \int d^3\boldsymbol{x}_1 F(x_2; x_1) \left\{G^A(x_1) + G^B(x_1)\right\}\Psi(x_1). \tag{10.9}$$

What matters here is the squared modulus $|\Psi(x_2)|^2$, which according to the Born interpretation (Born, 1926) is the probability density relevant to outcome detection on the screen S. From (10.9) we find

$$|\Psi(x_2)|^2 = P^A(x_2) + P^B(x_2) + I^{AB}(x_2), \tag{10.10}$$

where

$$\begin{aligned} P^A(x_2) &= \int d^3\boldsymbol{x}_1 d^3\boldsymbol{y}_1 F(x_2; x_1) F^*(x_2; y_1) G^A(x_1) G^{*A}(y_1) \Psi(x_1)\Psi^*(y_1), \\ P^B(x_2) &= \int d^3\boldsymbol{x}_1 d^3\boldsymbol{y}_1 F(x_2; x_1) F^*(x_2; y_1) G^B(x_1) G^{*B}(y_1) \Psi(x_1)\Psi^*(y_1), \\ I^{AB}(x_2) &= \int d^3\boldsymbol{x}_1 d^3\boldsymbol{y}_1 F(x_2; x_1) F^*(x_2; y_1) \left\{ \begin{array}{c} G^A(x_1) G^{*B}(y_1) + \\ G^B(x_1) G^{*A}(y_1) \end{array} \right\} \Psi(x_1)\Psi^*(y_1). \end{aligned} \tag{10.11}$$

In these integrals, we may use (10.6) to work out the outcome probabilities from a knowledge of the detectors in the screen S and the characteristics of the preparation device P; that is, we should be able to specify the initial wave function $\Phi(x_0)$ reasonably well.

Blocking off opening B corresponds to setting G^B to zero, and then we see from (10.11) that the interference term and P^B vanish. A similar remark applies the blocking off of opening A.

10.6 The QDN Account of the Double-Slit Experiment

We are now in position to describe the DS experiment via QDN. Applying our bitification process, we introduce qubits at all those sites where significant

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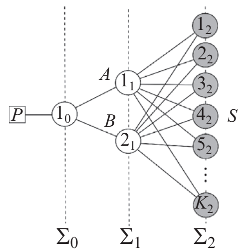

Figure 10.2. The DS experiment.

information could in principle be extracted. This excludes the information void regions V_1 and V_2, and all parts of the wall W apart from the two openings A and B. The QDN architecture is given by Figure 10.2. We model the detecting screen as a (possibly vast) number K of detectors and now discuss each run, stage by stage.

Stage $\boldsymbol{\Sigma_0}$

The initial normalized labstate $\boldsymbol{\Psi}_0$ prepared by apparatus P by stage Σ_0 is denoted in QDN by

$$\boldsymbol{\Psi}_0 = \mathbf{1}_0, \tag{10.12}$$

where $\mathbf{1}_0$ is the signal state element of the one-qubit quantum computational basis representation (CBR) $\{\mathbf{0}_0, \mathbf{1}_0\}$. Essentially, $\boldsymbol{\Psi}_0$ carries the information that the run should proceed, analogous to the proposition "*go for burn*," validated by Mission Control prior to moon rocket engine ignition, *after* all safety checks had been completed (Apollo Program Office, 1969). We shall refer to such a one-qubit register as a *preparation switch*.

Note that we are ignoring the internal spin state of the electromagnetic radiation, as polarization is not a factor in this version of the experiment. However, such effects can be included easily if required.

Stage $\boldsymbol{\Sigma_1}$

The QDN description of $\boldsymbol{\Psi}_1$, the labstate at stage Σ_1, has a dual role. On the one hand it represents what the observer could find (statistically) *if* they looked at either or both openings A and B at that stage. Actually, doing this would be part of the complex calibration processes involved in setting up the experiment in the first place. On the other hand, $\boldsymbol{\Psi}_1$ should be regarded as the initial labstate for propagation to the next stage, Σ_2, landing on the screen S. If this latter alternative is chosen, then the observer should make no attempt to extract information from $\boldsymbol{\Psi}_1$.

The rules of QM lead us to assert that

$$\boldsymbol{\Psi}_1 = \mathbb{U}_{1,0}\boldsymbol{\Psi}_0 = (\alpha^1\widehat{\mathbb{A}}_1^1 + \alpha^2\widehat{\mathbb{A}}_1^2)\mathbf{0}_1, \tag{10.13}$$

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where α^1 and α^2 are complex numbers satisfying the normalization condition $|\alpha^1|^2 + |\alpha^2|^2 = 1$.

Here $\mathbb{U}_{2,1}$ is a semi-unitary operator taking normalized labstates from $\mathcal{Q}_0$ to $\mathcal{Q}_1 \equiv Q_1^1 Q_1^2$. We shall comment on the nature of this operator presently. At this point, we note that signality is preserved in the transition from stage Σ_0 to stage Σ_1, since the initial labstate $\boldsymbol{\Psi}_0 \equiv \widehat{\mathbb{A}}_0^1 \mathbf{0}_0$ has signality one, and by inspection of (10.13), $\boldsymbol{\Psi}_1$ also has signality one. We rule out (by hand) terms proportional to the signality-zero state, $\mathbf{0}_1$, or to the signality-two state $\widehat{\mathbb{A}}_1^1 \widehat{\mathbb{A}}_1^2 \mathbf{0}_1$, as these represent dynamics different from the one that we wish to explore here. When charged particles such as electrons are involved in the DS experiment, charge conservation rules out changes of signality. For bosons such as photons, it is quite possible to encounter signality nonconservation, such as in the case of parametric down-conversion (Burnham and Weinberg, 1970; Klyshko et al., 1970).

Stage $\boldsymbol{\Sigma_2}$

The transition from stage Σ_1 to stage Σ_2 is handled as follows. We assert that there is a semi-unitary operator $\mathbb{U}_{2,1}$ such that for each term $\widehat{\mathbb{A}}_1^a \mathbf{0}_1$, $a = 1, 2$, on the right-hand side of equation (10.13),

$$\mathbb{U}_{2,1} \widehat{\mathbb{A}}_1^a \mathbf{0}_1 = \sum_{j=1}^{K} U_{2,1}^{j,a} \widehat{\mathbb{A}}_2^j \mathbf{0}_2, \quad a = 1, 2, \tag{10.14}$$

assuming there are K detector sites on the detecting screen S. This process preserves signality. We shall comment on the nature of $\mathbb{U}_{2,1}$ presently. Conservation of probability requires the complex coefficients $\left\{U_{2,1}^{j,a}\right\}$ to satisfy the semi-unitarity rule

$$\sum_{j=1}^{K} U_{2,1}^{j,b*} U_{2,1}^{j,a} = \delta^{ab}, \tag{10.15}$$

where $U_{2,1}^{j,b*}$ is the complex conjugate of $U_{2,1}^{j,b}$. We note that Eq. (10.15) is the QDN analogue of Eq. (10.8).

The linearity rules of QM now give the relationship between the initial labstate $\boldsymbol{\Psi}_0$ and the final labstate $\boldsymbol{\Psi}_2$ to be

$$\boldsymbol{\Psi}_2 = \mathbb{U}_{2,1} \mathbb{U}_{1,0} \boldsymbol{\Psi}_0 = \sum_{a=1}^{2} \sum_{j=1}^{K} \alpha^a U_{2,1}^{j,a} \widehat{\mathbb{A}}_2^j \mathbf{0}_2. \tag{10.16}$$

It can be readily checked, using the normalization condition and the above semi-unitarity rules that $\overline{\boldsymbol{\Psi}_2} \boldsymbol{\Psi}_2 = 1$.

We can now readily calculate all outcome probabilities, by choosing any of the 3^N maximal or partial questions. For example, the conditional probability $\Pr(k_2|\boldsymbol{\Psi}_0)$ that the kth detector at stage Σ_2 would be in its signal state is given by $\Pr(k_2|\boldsymbol{\Psi}_0) = \overline{\boldsymbol{\Psi}_2} \widehat{\mathbb{P}}_2^k \boldsymbol{\Psi}_2$, which readily evaluates to the value

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$$\Pr(k_2|\mathbf{\Psi}_0) = \sum_{a,b=1}^{2} \alpha^{a*}\alpha^b U_{2,1}^{k,a*} U_{2,1}^{k,b}, \quad k = 1, 2, \ldots, K. \tag{10.17}$$

Exercise 10.1 Prove (10.17) and show that total probability is conserved, that is,

$$\sum_{k=1}^{K} \Pr(k_2|\mathbf{\Psi}_0) = 1. \tag{10.18}$$

10.7 Contextual Subspaces

Suppose $\mathbb{U}_{n+1,n}$ is the semi-unitary evolution operator from quantum register $\mathcal{Q}_n$ at stage Σ_n to quantum register $\mathcal{Q}_{n+1}$ at stage Σ_{n+1}. Suppose further that we have a complete specification of the action of $\mathbb{U}_{n+1,n}$, in the form of the rules for CBR element evolution given by

$$\mathbb{U}_{n+1,n}\boldsymbol{i}_n = \sum_{j=0}^{2^{r_{n+1}}-1} U_{n+1,n}^{j,i}\boldsymbol{j}_{n+1}, \qquad r_{n+1} = rank\ \mathcal{Q}_{n+1}. \tag{10.19}$$

Then using completeness, we find the dyadic representation

$$\mathbb{U}_{n+1,n} = \sum_{i=0}^{2^{r_n}-1} \sum_{j=0}^{2^{r_{n+1}}-1} \boldsymbol{j}_{n+1} U_{n+1,n}^{j,i} \overline{\boldsymbol{i}^n}. \tag{10.20}$$

Then we have the rule

$$\overline{\mathbb{U}}_{n+1,n}\mathbb{U}_{n+1,n} = \mathbb{I}_n, \tag{10.21}$$

where $\overline{\mathbb{U}}_{n+1,n}$ is the retraction of $\mathbb{U}_{n+1,n}$ and $\mathbb{I}_n$ is the identity operator over $\mathcal{Q}_n$.

At this point, we are confronted with what appears to be a serious problem; we do not have all the information that allows us to construct the full evolution operator $\mathbb{U}_{2,1}$ in the *DS* experiment. The number of elements in the initial preferred basis B_1 for $\mathcal{Q}_1$ is four, but of these, only two have signality one, that is, $\widehat{\mathbb{A}}_1^1\mathbf{0}_1$ and $\widehat{\mathbb{A}}_1^2\mathbf{0}_1$. The only specific information we have is given by the relations (10.14) for those two elements of the preferred basis.

Fortunately, this problem is easily circumvented by the observation that for a DS experiment, an observer will not in general be interested in the complete quantum registers $\mathcal{Q}_n$ and $\mathcal{Q}_{n+1}$ but only in the subspaces spanned by the signality-one elements of their respective preferred bases. This is really the meaning of the term *self-interference*.

This leads us to define the notion of *contextual basis* and *contextual subspace*.

Definition 10.2 In a given experiment, a *contextual basis* B_n^c is a subset of the preferred basis B_n for the quantum register $\mathcal{Q}_n$ at stage Σ_n, the elements of B_n^c being dictated by the context of the experiment.

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Definition 10.3 A *contextual subspace* is a subspace $\mathcal{Q}_n^c$ of a quantum register $\mathcal{Q}_n$, the preferred basis for $\mathcal{Q}_n^c$ being a given contextual subset B_n^c of the preferred basis B_n for $\mathcal{Q}_n$

In the case of the DS experiment, the contextual bases B_1^c, B_2^c are given by all the respective signality-one states, so we have

$$B_1^c \equiv \left\{ \widehat{\mathbb{A}}_1^1 \mathbf{0}_1, \widehat{\mathbb{A}}_1^2 \mathbf{0}_1 \right\}, \quad B_2^c \equiv \left\{ \widehat{\mathbb{A}}_2^i \mathbf{0}_2 : i = 1, 2, \ldots, K \right\}. \tag{10.22}$$

These define the contextual subspaces $\mathcal{Q}_1^c$ and $\mathcal{Q}_2^c$. From (10.14), we can construct the contextual evolution operator $\mathbb{U}_{2,1}^c$ and its retraction $\overline{\mathbb{U}}_{2,1}^c$. These are given in the CBR by

$$\mathbb{U}_{2,1}^c \equiv \sum_{a=1}^{2} \sum_{j=1}^{K} U_{2,1}^{j,a} \mathbf{2}_2^{j-1} \overline{\mathbf{2}_1^{a-1}}, \quad \overline{\mathbb{U}}_{2,1}^c \equiv \sum_{a=1}^{2} \sum_{j=1}^{K} U_{2,1}^{j,a*} \mathbf{2}_2^{a-1} \overline{\mathbf{2}_1^{j-1}} \tag{10.23}$$

and satisfy the required relation $\overline{\mathbb{U}}_{2,1}^c \mathbb{U}_{2,1}^c = \mathbb{I}_1^c$, where $\mathbb{I}_1^c$ is the stage-Σ_1 *contextual identity*

$$\mathbb{I}_1^c = \mathbf{1}_1 \overline{\mathbf{1}_1} + \mathbf{2}_1 \overline{\mathbf{2}_1}. \tag{10.24}$$

Remark 10.4 The real world of experience and the world of the theorist's imagination are each far too complex to understand fully. Experiments attempt to limit the complexity of the former by focussing on a limited number of detectors. Contextual subspaces implement that strategy as closely as possible, limiting the amount of complexity that the theorist needs to face.

Usually it will be clear by context when we are dealing with contextual subspaces, so we shall usually drop the superscript c in our notation.

10.8 The Sillitto–Wykes Variant

The DS experiment was identified by Feynman, Leighton, and Sands (FLS) as *the* fundamental experiment to understand. They wrote that "it contains the only mystery" (Feynman et al., 1966). From the beginning of QM, experimentalists sought to probe this mystery deeper, such as greatly reducing the light intensity (Taylor, 1909) in the case of electromagnetic waves. Technology finally made possible experiments where one electron came through the device at a time (Merli et al., 1976).

Another variant was performed by Sillitto and Wykes, who arranged for the two openings, A and B, to be opened and closed so that only one was open at a time (Sillitto and Wykes, 1972). Nevertheless, an interference pattern was observed. This seems at first sight impossible to understand if we think in terms of particles.

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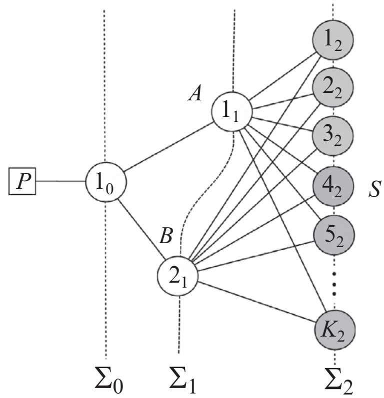

Figure 10.3. The QDN explanation of the Sillitto-Wykes experiment result is that stage Σ_1 need not coincide with a hyperplane of simultaneity in the laboratory, or indeed, in any other frame of reference.

We can readily explain the result of the Sillitto–Wykes experiment in QDN by pointing out that time is handled in QDN in terms of *stages*. In Figure 10.2, we note that the openings A, B are associated with *stage* Σ_1, not a specific laboratory time. We have stressed previously that stages are analogues of space-like hypersurfaces in conventional relativity, but with an important proviso: shielding can play a role. In the case of the Sillitto–Wykes experiment, a more appropriate diagram is Figure 10.3. In that representation, stage Σ_1 is not necessarily physically space-like everywhere. What matters is that the optical paths (in the case of photons) or trajectory channels (in the case of electrons) from P through openings A and B are such that wave trains from the two slits subsequently "intersect and interfere" at the screen S. Whether A and B are open "at the same labtime" turns out to be irrelevant. Indeed, that is as it should be, because it is not possible anyway to determine when any "particle" passes through either hole without destroying the interference pattern on S.

10.9 The Monitored Double-Slit Experiment

A fundamental question raised by the DS experiment concerns the particle interpretation of quanta. Speaking classically, if a particle such as an electron is released from source P and lands on screen S, then "it stands to reason" that that particle must have passed through either opening A or opening B. That is, after all, what is meant by a "particle."

But this is just an appeal to intuition. We have emphasized that our interpretation of QM is that it is really a theory of *entitlement*: QM tells us what we are entitled to say in a given context, and no more and no less. Therefore, if we have not monitored through which slit an electron has gone, we are not entitled to say it had to have gone through one opening for sure. We do not even have to

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think of that as having happened in any way; physicists are not in the business of believing in unverified propositions.

It is that lack of entitlement that really upsets people, because it destroys the comfortable belief that real things are going on, even when we are not observing them. Physicists, being people, felt compelled to find out whether the path of electron could be determined when an interference pattern on screen S was found. The *monitored DS experiment* was devised in order to investigate this issue.

The Feynman–Leighton–Sands Discussion

We shall first discuss the treatment given by FLS in Feynman et al. (1966). The apparatus is given in Figure 10.4.

The apparatus is the same as for the original DS experiment except for three items. L is a source of light and D^A and D^B are photon detectors. The idea is that if an electron passes through opening A, then there is a possibility that it will interact with light from the source L, causing a signal in D^A. A signal in D^A may therefore be an indicator that the electron has passed through opening A and not B. A similar remark applies if the electron passes through opening B.

The analysis goes as follows. First, consider the case when the light source is absent. If opening B is closed off and an electron passes through opening A and lands on the screen at C, then the amplitude is ϕ_1. Conversely, if opening A is closed off and an electron passes through opening B and lands on C, then the amplitude is ϕ_2. If A and B are both open, then the amplitude at C is $\phi_1 + \phi_2$. This is just the QM description of the original DS experiment.

When the light source L is present and both slits are open, then the situation needs some careful analysis. Suppose an electron has been detected at C *and*

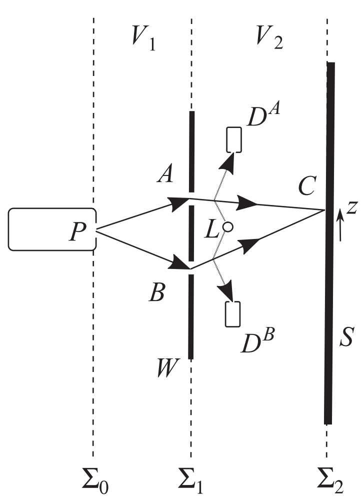

Figure 10.4. The monitored DS experiment.

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a photon detected at D^A. Then two alternative paths contribute to the overall amplitude $\mathcal{A}(C|P)$.

Path A

The electron passes through slit A and interacts with the light, with an amplitude at C written as $a\phi_1$, where a is a factor representing the electron–electromagnetic field interaction.

Path B

The electron passes through slit B and interacts with the light, with an amplitude at C written as $b\phi_2$, where b is a factor representing the electron–electromagnetic field interaction.

The coefficients a and b are expected to be different, from the geometry of the situation. The overall amplitude at C, according to Feynman's sum of paths prescription, is therefore the sum of the two contributions, and so given by $\mathcal{A}(C, D^A|P) = a\phi_1 + b\phi_2$. Assuming suitable normalization, the probability $\Pr(C, D^A|P)$ for an electron to land on C and a photon to be registered at D^A, conditional on the electron being fired from P, is therefore given by

$$\Pr(C, D^A|P) = |\mathcal{A}(C, D^A|P)|^2 = |a\phi_1 + b\phi_2|^2. \tag{10.25}$$

If, on the other hand, an electron has been detected at C *and* a photon detected at D^B, then a similar argument (using symmetry) gives $\Pr(C, D^B|P) = |a\phi_2 + b\phi_1|^2$.

The probability $\Pr(C, D^A \text{ or } D^B|P)$ of an electron landing at C and a photon being detected at either D^A or D^B is the sum of the two probabilities, not the squared modulus of the sum of the two amplitudes (a point stressed by FLS), and is therefore given by

$$\Pr(C, D^A \text{ or } D^B|P) = |a\phi_1 + b\phi_2|^2 + |a\phi_2 + b\phi_1|^2, \tag{10.26}$$

which is Eq. (3.10) in Feynman et al. (1966).

The QDN Discussion

The relevant QDN version of Figure 10.4 is Figure 10.5. In the latter figure, we have two extra nodes, labeled A_2 and B_2, added to stage Σ_2 detectors in the screen S. These new nodes represent the detectors D^A and D^B shown in Figure 10.4.

The QDN analysis in this case goes by the three stages discussed in the original scenario. Nothing is different by stage Σ_1, so Eq. (10.13) is still valid. However, the jump to stage Σ_2 has to treated more carefully. We deal with each opening separately.

In the following, $a = 1$ corresponds to opening A and $a = 2$ corresponds to opening B. We expect the following dynamics, which is more general than that considered in Feynman et al. (1966):

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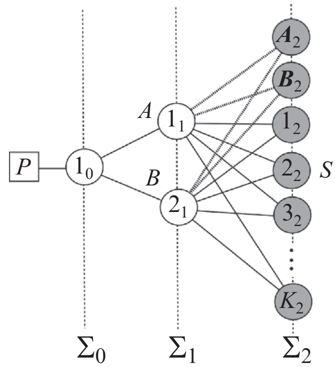

Figure 10.5. The QDN stage diagram for the monitored DS experiment.

$$\mathbb{U}_{2,1}\widehat{\mathbb{A}}_1^a\mathbf{0}_1 = \sum_{j=1}^{K}(\underbrace{U^{j,a}\widehat{\mathbb{A}}_2^j}_{(a)} + \underbrace{V^{j,a}\widehat{\mathbb{A}}_2^j\widehat{\mathbb{A}}_2^A}_{(b)} + \underbrace{W^{j,a}\widehat{\mathbb{A}}_2^j\widehat{\mathbb{A}}_2^B}_{(c)} + \underbrace{X^{j,a}\widehat{\mathbb{A}}_2^j\widehat{\mathbb{A}}_2^A\widehat{\mathbb{A}}_2^B}_{(d)})\mathbf{0}_2. \quad (10.27)$$

The four terms on the right-hand side represent the following alternatives:

(a) $U^{j,a}$ is the signality-one amplitude that an electron has passed through opening a and has landed on the screen at site j and no signal is detected in either D^A or D^B.

(b) $V^{j,a}$ is the signality-two amplitude that an electron has passed through opening a and has landed on the screen at site j and a signal is detected in D^A and not in D^B.

(c) $W^{j,a}$ is the signality-two amplitude that an electron has passed through opening a and has landed on the screen at site j and a signal is detected in D^B and not in D^A.

(d) $X^{j,a}$ is the signality-three amplitude that an electron has passed through opening a and has landed on the screen at site j and a signal is detected in D^A and a signal is detected in D^B. This possibility is not considered in the calculation of FLS.

The labstate $\mathbf{\Psi}_2$ is given by

$$\mathbf{\Psi}_2 = \sum_{a=1}^{2}\sum_{j=1}^{K}\alpha^a(U^{j,a}\widehat{\mathbb{A}}_2^j + V^{j,a}\widehat{\mathbb{A}}_2^j\widehat{\mathbb{A}}_2^A + W^{j,a}\widehat{\mathbb{A}}_2^j\widehat{\mathbb{A}}_2^B + X^{j,a}\widehat{\mathbb{A}}_2^j\widehat{\mathbb{A}}_2^A\widehat{\mathbb{A}}_2^B)\mathbf{0}_2. \quad (10.28)$$

From this we can readily determine all the various probabilities of interest by asking the appropriate questions. For example, if we ask for the probability that the electron has landed on the kth detector site, and there is a signal in D^A, and there is no signal in D^B, we need to calculate the expectation value $\overline{\mathbf{\Psi}}_2\widehat{\mathbb{P}}_2^k\widehat{\mathbb{P}}_2^A\mathbb{P}_2^B\mathbf{\Psi}_2$. To evaluate this, we first note that the signal algebra gives

$$\widehat{\mathbb{P}}_2^k\widehat{\mathbb{P}}_2^A\mathbb{P}_2^B\mathbf{\Psi}_2 = \sum_{a=1}^{2}\alpha^a V^{k,a}\widehat{\mathbb{A}}_2^k\widehat{\mathbb{A}}_2^A\mathbf{0}_2, \quad (10.29)$$

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and then we readily find

$$\overline{\boldsymbol{\Psi}_2}\widehat{\mathbb{P}}_2^k\widehat{\mathbb{P}}_2^A\mathbb{P}_2^B\boldsymbol{\Psi}_2 = \left|\sum_{a=1}^{2}\alpha^a V^{k,a}\right|^2 = \left|\alpha^1 V^{k,1} + \alpha^2 V^{k,2}\right|^2. \tag{10.30}$$

Similarly, the probability $\overline{\boldsymbol{\Psi}_2}\widehat{\mathbb{P}}_2^k\mathbb{P}_2^A\widehat{\mathbb{P}}_2^B\boldsymbol{\Psi}_2$, that the electron landed on the kth screen site and there was no signal in D^A and there was a signal in D^B, is given by

$$\overline{\boldsymbol{\Psi}_2}\widehat{\mathbb{P}}_2^k\mathbb{P}_2^A\widehat{\mathbb{P}}_2^B\boldsymbol{\Psi}_2 = |\alpha^1 W^{k,1} + \alpha^2 W^{k,2}|^2. \tag{10.31}$$

We recover the FLS result in their notation if we use symmetry, setting $\alpha^1 = \alpha^2$ and

$$V^{k,1} = a\phi_1, \quad V^{k,2} = b\phi_2, \quad W^{k,1} = a\phi_2, \quad W^{k,2} = b\phi_1. \tag{10.32}$$

The QDN calculation allows greater flexibility in the architecture of the experiment than that assumed in the FLS calculation, but the price is that more assumptions have to be made about the various amplitudes.

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