Canad. Math. Bull. Vol. **53** (4), 2010 pp. 690–705
doi:10.4153/CMB-2010-075-6

On the Maximal Operator Ideal Associated with a Tensor Norm Defined by Interpolation Spaces

M. E. Puerta and G. Loaiza

Abstract. The classical approach to studying operator ideals using tensor norms mainly focuses on those tensor norms and operator ideals defined by means of ℓ_p spaces. In a previous paper, an interpolation space, defined via the real method and using ℓ_p spaces, was used to define a tensor norm, and the associated minimal operator ideals were characterized. In this paper, the next natural step is taken, that is, the corresponding maximal operator ideals are characterized. As an application, necessary and sufficient conditions for the coincidence of the maximal and minimal ideals are given. Finally, the previous results are used in order to find some new metric properties of the mentioned tensor norm.

1 Introduction

In [17, 18] Matter introduced the notion of (p, σ)-absolutely continuous operators ($1 \leq p < \infty$, $0 < \sigma < 1$) between Banach spaces. For each choice of p and σ, these operators form an ideal. One of the most significant features of these ideals is their close relationship with typical calculation procedures in interpolation space theory. This is the case in [17], where interpolation spaces are defined by the real method and are used as essential tools in the characterization of some of the operators Matter introduced. Even though the ideals treated by Matter are all maximal, there is no attempt in [17, 18] to exploit the important relation between these ideals and tensor norms. This relation was used in [16] for the case $1 < p < \infty$, where the ideals of nuclear and integral operators corresponding to a tensor norm were characterized by means of factorizations through interpolation spaces. The case $p = 1$ was studied in [2]. These works put in evidence the role interpolation spaces may play in defining tensor norms that are more general than the classical ones. Each new tensor norm poses the problem of characterizing the associated (in the sense of [7]) nuclear and integral operator ideals. In [19], the authors introduced a tensor norm g_ξ^c by means of an interpolation space ξ and characterized the associated minimal operator ideals. The aim of the present paper is to give a characterization of the maximal operator ideal associated to this tensor norm, apply this and the characterization obtained in [19] to the study of the coincidence between components of the maximal and minimal operator ideals, and use these coincidence results to prove some metric properties of g_ξ^c and its dual.

Received by the editors December 13, 2007.
Published electronically July 26, 2010.
The writing of this paper was supported by EAFIT University.
AMS subject classification: **46M05**, 46M35, 46A32.
Keywords: maximal operator ideals, ultraproducts of spaces, interpolation spaces.

Notation is standard. We will always consider Banach spaces over the real field, since we shall use results in the theory of Banach lattices. The canonical inclusion map from Banach space E into the bidual E'' will be denoted by J_E. In general, if E is a subspace of F, the inclusion of E into F is denoted by $I_{E,F}$. The set of finite dimensional subspaces of a normed space E will be denoted by $FIN(E)$.

We recall the more relevant aspects on Banach lattices (we refer the reader to [1]). A Banach lattice E is order complete or Dedekind complete if every order bounded set in E has a least upper bound in E, and it is order continuous if every order convergent filter is norm convergent. Every dual Banach sequence lattice E' is order complete and all reflexive spaces are even order continuous. A linear map T between Banach lattices E and F is said to be positive if $T(x) \geq 0$ in F for every $x \in E, x \geq 0$. T is called order bounded if $T(A)$ is order bounded in F for every order bounded set A in E.

Let ω be the vector space of all scalar sequences and φ its subspace of the sequences with finitely many nonzero coordinates. A sequence space ξ is a linear subspace of ω containing φ provided with a topology finer than the topology of coordinatewise convergence. *A Banach sequence space* will be a sequence space ξ provided with a norm that makes it a Banach lattice and an ideal in ω, *i.e.,* such that if $|x| \leq |y|$ with $x \in \omega$ and $y \in \xi$, then $x \in \xi$ and $\|x\|_\xi \leq \|y\|_\xi$. A sectional subspace $S_k(\xi), k \in \mathbb{N}$, is the topological subspace of ξ of those sequences (α_i) such that $\alpha_i = 0$ for every $i \geq k$. Clearly $S_k(\xi)$ is 1-complemented in ξ. A Banach sequence space ξ will be called regular whenever the sequence $\{\mathbf{e}_i\}_{i=1}^\infty$, where $\mathbf{e}_i := (\delta_{ij})_j$ (Kronecker's delta) is a Schauder base in ξ.

Following [19], we establish some notation. Given a Banach space E and the compatible couple $\lambda = (\ell_{p_0}, \ell_{p_1})$ with $1 < p_0 \leq p_1 < \infty$ and its dual couple $\lambda' = (\ell_{p_0'}, \ell_{p_1'})$, we will denote by $\lambda_{q,\mathcal{J}}$, $\lambda_{q,\mathcal{K}}$ the interpolation spaces obtained using methods $\mathcal{J}$ and $\mathcal{K}$, respectively, from the couple λ (see [3]). Analogously, we denote by $\lambda'_{q,\mathcal{J}}$, $\lambda'_{q,\mathcal{K}}$ the interpolation spaces obtained from the couple λ'. With this notation, if $1 \leq q < \infty$ and $0 < \theta < 1$, dual spaces $(\lambda_{q,\mathcal{J}})' = ((\ell_{p_0}, \ell_{p_1})_{\theta,q,\mathcal{J}})' = (\ell_{p_0'}, \ell_{p_1'})_{\theta,q',\mathcal{K}} = \lambda'_{q',\mathcal{K}}$. We can see that $(\lambda_{q,\mathcal{J}})' \neq \lambda'_{q,\mathcal{J}}$. Recalling the duality for method $\mathcal{K}$, since $\lambda_{q,\mathcal{J}}$ is reflexive for $1 \leq q < \infty$, we get the isometry

$$(\lambda'_{q',\mathcal{K}})' = \lambda_{q,\mathcal{J}}.$$

We say that a sequence $(x_n)_{n=1}^\infty \in E^{\mathbb{N}}$ is strongly $\lambda_{q,\mathcal{J}}$-summing if $(\|x_n\|) \in \lambda_{q,\mathcal{J}}$, and we write $\pi_{\lambda_{q,\mathcal{J}}}((x_i)) := \|(\|x_i\|)_{i=1}^\infty\|_{\lambda_{q,\mathcal{J}}}$. It is said to be weakly $\lambda_{q,\mathcal{J}}$-summing if $\varepsilon_{\lambda_{q,\mathcal{J}}}((x_i)) := \sup_{\|x'\| \leq 1} \|(|\langle x_n, x'\rangle|)\|_{\lambda_{q,\mathcal{J}}}$. We denote by $\lambda_{q,\mathcal{J}}[E]$ (resp. $\lambda_{q,\mathcal{J}}(E)$) the space of all strongly (resp. weakly) $\lambda_{q,\mathcal{J}}$-summing in E with the norm $\pi_{\lambda_{q,\mathcal{J}}}(\,\cdot\,)$ (resp. $\varepsilon_{\lambda_{q,\mathcal{J}}}(\,\cdot\,)$).

Let (Ω, Σ, μ) be a measure space. We denote by $L_0(\mu)$ the space of equivalence classes, modulo equality μ-almost everywhere, of μ-measurable real-valued functions, endowed with the topology of local convergence in measure, and the space of all equivalence classes of μ-measurable X-valued functions is denoted by $L_0(\mu, X)$. By a Köthe function space $\mathcal{K}(\mu)$ on (Ω, Σ, μ), we shall mean an order dense ideal of $L_0(\mu)$, which is equipped with a norm $\|\cdot\|_{\mathcal{K}(\mu)}$ that makes it a Banach lattice (if $f \in L_0(\mu)$ and $g \in \mathcal{K}(\mu)$ $|f| \leq |g|$, then $f \in \mathcal{K}(\mu)$ with $\|f\|_{\mathcal{K}(\mu)} \leq \|g\|_{\mathcal{K}(\mu)}$).

https://doi.org/10.4153/CMB-2010-075-6 Published online by Cambridge University Press

Similarily, $\mathcal{K}(\mu, X) = \{f \in L_0(\mu, X) : \|f(\cdot)\|_X \in \mathcal{K}(\mu)\}$, endowed with the norm $\|f\|_{\mathcal{K}(\mu,X)} = \|\|f(\cdot)\|_X\|_{\mathcal{K}(\mu)}$.

For the theory of operator ideals and tensor norms, we refer to the books [21] and [7] of Pietsch and Defant and Floret respectively.

If E and F are Banach spaces and α is a tensor norm, then $E \otimes_\alpha F$ represents the space $E \otimes F$ endowed with the α-normed topology. The completion of $E \otimes_\alpha F$ is denoted by $E\hat{\otimes}_\alpha F$, and the norm of z in $E\hat{\otimes}_\alpha F$ by $\alpha(z; E \otimes F)$. If there is no risk of confusion, we write $\alpha(z)$ instead of $\alpha(z; E \otimes F)$.

2 The Tensor Norm $g_{\lambda_{q,\mathcal{J}}}$ and Associated $\lambda_{q,\mathcal{J}}$-Nuclear Operators

The more natural approach to defining a tensor norm analogous to Saphar's tensor norm is as follows. Let E and F be Banach spaces and $z \in E \otimes F$, we define

$$g_{\lambda_{q,\mathcal{J}}}(z; E \otimes F) := \inf \pi_{\lambda_{q,\mathcal{J}}}((x_n))\varepsilon_{\lambda'_{q',\mathcal{K}}}((y_n))$$

taking the infimum over all representations of z as $\sum_{n=1}^{m} x_n \otimes y_n$. We will write $g_{\lambda_{q,\mathcal{J}}}(z)$ instead of $g_{\lambda_{q,\mathcal{J}}}(z; E \otimes F)$ if there is no possibility of confusion.

It is possible that for some interpolation space $\lambda_{q,\mathcal{J}}$, the functional $g_{\lambda_{q,\mathcal{J}}}$ does not satisfy the triangle inequality, but it is always a reasonable *quasi norm* on $E \otimes F$, see [6, 9]. We denote by $E \otimes_{g_{\lambda_{q,\mathcal{J}}}} F$ the quasi normed space and by $E\hat{\otimes}_{g_{\lambda_{q,\mathcal{J}}}} F$ the corresponding quasi Banach space.

To obtain a tensor norm $g^c_{\lambda_{q,\mathcal{J}}}$ in [19], we took the Minkowski functional, denoted $g^c_{\lambda_{q,\mathcal{J}}}(z; E \otimes F)$, of the absolutely convex hull of the unit closed ball $B_{g_{\lambda_{q,\mathcal{J}}}} := \{z \in E \otimes F / g_{\lambda_{q,\mathcal{J}}}(z) \leq 1\}$ of the quasi norm $g_{\lambda_{q,\mathcal{J}}}$ in $E \otimes F$, such that

$$g^c_{\lambda_{q,\mathcal{J}}}(z; E \otimes F) := \inf \sum_{i=1}^{n} \pi_{\lambda_{q,\mathcal{J}}}((x_{ij}))\varepsilon_{\lambda'_{q',\mathcal{K}}}((y_{ij})),$$

taking the infimum over all representations of z as $\sum_{i=1}^{n} \sum_{j=1}^{m} x_{ij} \otimes y_{ij}$. Again, we will write $g_{\lambda_{q,\mathcal{J}}}$ instead of $g_{\lambda_{q,\mathcal{J}}}(z; E \otimes F)$ if there is no possibility of confusion.

It is easy to see that $g^c_{\lambda_{q,\mathcal{J}}}$ *is a tensor norm* on the class of all Banach spaces, using [7, Criterion 12.2] and bearing in mind that if $1 \leq q \leq \infty$, then

$$\big\|(\mathbf{e}_i)\big\|_{\lambda_{q,\mathcal{J}}} = \varpi_q^{-1} y \big\|(\mathbf{e}_i)\big\|_{\lambda_{q,\mathcal{K}}} = \varpi_q,$$

where

$$\varpi_q = \Big(\int_0^\infty \Big(\frac{\min\{1,t\}}{t^\theta}\Big)^q \frac{dt}{t}\Big)^{\frac{1}{q}}$$

for every $i \in \mathbb{N}$, in relation to method $\mathcal{K}$ and that for all $z \in E \otimes F$, $g^c_{\lambda_{q,\mathcal{J}}}(z; E \otimes F) \leq g_{\lambda_{q,\mathcal{J}}}(z; E \otimes F)$. We denote by $E\hat{\otimes}_{g^c_{\lambda_{q,\mathcal{J}}}} F$ the corresponding Banach space.

Proceeding as in [6, 23], it is easy to see that if $z \in E\hat{\otimes}_{g_{\lambda_{q,\mathcal{J}}}} F$, there are $(x_i)_{i=1}^\infty \in \lambda_{q,\mathcal{J}}[E]$ and $(y_i)_{i=1}^\infty \in \lambda'_{q',\mathcal{K}}(F)$ such that $\pi_{\lambda_{q,\mathcal{J}}}((x_i))\varepsilon_{\lambda'_{q',\mathcal{K}}}((y_i)) < \infty$ and $z =$

https://doi.org/10.4153/CMB-2010-075-6 Published online by Cambridge University Press

$\sum_{i=1}^{\infty} x_i \otimes y_i$. Moreover, the quasi norm of z in $E\hat{\otimes}_{g_{\lambda_{q,\mathcal{J}}}} F$ (again denoted by $g_{\lambda_{q,\mathcal{J}}}(z)$) is given by

$$g_{\lambda_{q,\mathcal{J}}}(z) = \inf \pi_{\lambda_{q,\mathcal{J}}}((x_i))\varepsilon_{\lambda'_{q',\mathcal{K}}}((y_i))$$

taking the infimum over all such representations of z as $\sum_{n=1}^{m} x_n \otimes y_n$. Similarly, if $z \in E\hat{\otimes}_{g^c_{\lambda_{q,\mathcal{J}}}} F$ then z can be represented as $z = \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} x_{ij} \otimes y_{ij}$, where $(x_{ij})_{i=1}^{\infty} \in \lambda_{q,\mathcal{J}}[E]$ for each $j \in \mathbb{N}$, $(y_{ij})_{i=1}^{\infty} \in \lambda'_{q',\mathcal{K}}(F)$ for each $j \in \mathbb{N}$ and

$$\sum_{j=1}^{\infty} \pi_{\lambda_{q,\mathcal{J}}}((x_{ij}))\varepsilon_{\lambda'_{q',\mathcal{K}}}((y_{ij})) < \infty.$$

Moreover, the norm of z in $E\hat{\otimes}_{g^c_{\lambda_{q,\mathcal{J}}}} F$ is

$$g^c_{\lambda_{q,\mathcal{J}}}(z) = \inf \sum_{j=1}^{\infty} \pi_{\lambda_{q,\mathcal{J}}}((x_{ij}))\varepsilon_{\lambda'_{q',\mathcal{K}}}((y_{ij})),$$

taking the infimum over all representations of z as $\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} x_{ij} \otimes y_{ij}$.

The topology defined by the quasi norm $g_{\lambda_{q,\mathcal{J}}}$ on $E \otimes F$ is normable with norm equivalent to $g^c_{\lambda_{q,\mathcal{J}}}$. In fact, being $\lambda_{q,\mathcal{J}}$ a reflexive interpolation space for $1 \le q < \infty$ and following the arguments of [6, Proposition 16], we consider the bilinear onto map $R\colon \lambda_{q,\mathcal{J}}[E] \times \lambda'_{q',\mathcal{K}}(F) \to E\hat{\otimes}_{g_{\lambda_{q,\mathcal{J}}}} F$, such that $R((x_i),(y_i)) = \sum_{i=1}^{\infty} x_i \otimes y_i$. R is continuous with quasi norm less than or equal to one. Then there exists a unique linear and continuous map $\lambda_{q,\mathcal{J}}[E] \otimes_\pi \lambda'_{q',\mathcal{K}}(F) \to E\hat{\otimes}_{g_{\lambda_{q,\mathcal{J}}}} F$, see [25]. This map can be extended to a continuous linear and onto map $\lambda_{q,\mathcal{J}}[E]\hat{\otimes}_\pi \lambda'_{q'}, \mathcal{K}(F) \to E\hat{\otimes}_{g_{\lambda_{q,\mathcal{J}}}} F$ which is open by the open mapping theorem. Then $E\hat{\otimes}_{g_{\lambda_{q,\mathcal{J}}}} F$ is isomorphic to a quotient of a Banach space and so it is a Banach space itself. In this way, there is *a norm* $w_{\lambda_{q,\mathcal{J}}}(\,\cdot\,; E \otimes F)$ equivalent to the quasi norm $g_{\lambda_{q,\mathcal{J}}}(\,\cdot\,; E \otimes F)$; furthermore, it is easy to see that $w_{\lambda_{q,\mathcal{J}}}(\,\cdot\,; E \otimes F)$, $g_{\lambda_{q,\mathcal{J}}}(\,\cdot\,; E \otimes F)$ and $g^c_{\lambda_{q,\mathcal{J}}}(\,\cdot\,; E \otimes F)$ are equivalent with $g^c_{\lambda_{q,\mathcal{J}}}(\,\cdot\,; E \otimes F) \le w_{\lambda_{q,\mathcal{J}}}(\,\cdot\,; E \otimes F)$. Given the last equivalence, $g_{\lambda_{q,\mathcal{J}}}$ seems appropriate for our purposes, but we need $g^c_{\lambda_{q,\mathcal{J}}}$ for our main results.

To introduce $\lambda_{q,\mathcal{J}}$-nuclear operators, bearing in mind that every representation of $z \in E'\hat{\otimes}_{g^c_{\lambda_{q,\mathcal{J}}}} F$ as $\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} x_{ij} \otimes y_{ij}$, defines a map $T_z \in \mathcal{L}(E,F)$ such that for $x \in E$,

$$T_z(x) := \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} \langle x'_{ij}, x\rangle y_{ij}.$$

Furthermore, T_z is well defined and independent of the chosen representation for z. Let $\Phi_{EF}\colon E'\hat{\otimes}_{g^c_{\lambda_{q,\mathcal{J}}}} F \to \mathcal{L}(E,F)$ be defined by $\Phi_{EF}(z) := T_z$.

Definition 2.1 An operator between Banach spaces $T\colon E \to F$ is said to be $\lambda_{q,\mathcal{J}}$-nuclear if $T = \Phi_{EF}(z)$ for some $z \in E'\hat{\otimes}_{g^c_{\lambda_{q,\mathcal{J}}}} F$.

Given any pair of Banach spaces E and F, the space of the $\lambda_{q,\mathcal{J}}$-nuclear operators $T\colon E \to F$ endowed with the topology of the norm

$$\mathbf{N}^c_{\lambda_{q,\mathcal{J}}}(T) := \inf\{g^c_{\lambda_{q,\mathcal{J}}}(z)/\Phi_{EF}(z) = T\},$$

or with the equivalent quasi-norm $\mathbf{N}_{\lambda_{q,\mathcal{J}}}(T) := \inf\{g_{\lambda_{q,\mathcal{J}}}(z)/\Phi_{EF}(z) = T\}$, is denoted by $\mathcal{N}_{\lambda_{q,\mathcal{J}}}(E,F)$. Also $(\mathcal{N}_{\lambda_{q,\mathcal{J}}}(E,F), \mathbf{N}^c_{\lambda_{q,\mathcal{J}}})$ denotes a component of the minimal Banach operator ideal $(\mathcal{N}_{\lambda_{q,\mathcal{J}}}, \mathbf{N}^c_{\lambda_{q,\mathcal{J}}})$ associated to the tensor norm $g^c_{\lambda_{q,\mathcal{J}}}$. As in [19], we obtain the following result.

Theorem 2.2 *Let E, F be any pair of Banach spaces and an operator $T \in \mathcal{L}(E,F)$. Then the following are equivalent:*

(i) *T is $\lambda_{q,\mathcal{J}}$-nuclear.*

(ii) *T factors continuously in the following way:*

$$\begin{tikzcd} E \arrow[r, "T"] \arrow[d, "A"'] & F \\ \ell_\infty \arrow[r, "B"'] & \lambda_{q,\mathcal{J}} \arrow[u, "C"'] \end{tikzcd}$$

where B is a diagonal multiplication operator defined by a positive sequence $(b_i) \in \lambda_{q,\mathcal{J}}$.

Furthermore, $\mathbf{N}_{\lambda_{q,\mathcal{J}}}(T) = \inf\{\|C\|\|B\|\|A\|\}$, infimum taken over all such factors.

(iii) *T factors continuously in the following way:*

$$\begin{tikzcd} E \arrow[r, "T"] \arrow[d, "A"'] & F \\ \ell_\infty[\ell_\infty] \arrow[r, "B"'] & \ell_1[\lambda_{q,\mathcal{J}}] \arrow[u, "C"'] \end{tikzcd}$$

where B is a diagonal multiplication operator defined by a positive sequence $(b_i) \in \ell_1[\lambda_{q,\mathcal{J}}]$.

Furthermore. $\mathbf{N}^c_{\lambda_{q,\mathcal{J}}}(T) = \inf\{\|C\|\|B\|\|A\|\}$, infimum taken over all such factors.

There are other important operator ideals associated with $g_{\lambda_{q,\mathcal{J}}}$ and $g^c_{\lambda_{q,\mathcal{J}}}$.

Definition 2.3 Let $T \in \mathcal{L}(E,F)$. We say that T is $\lambda_{q,\mathcal{J}}$-*absolutely summing* if a real number $C > 0$ exists such that for all sequences (x_i) in E, with $\varepsilon_{\lambda_{q,\mathcal{J}}}((x_i)) < \infty$, the following is satisfied:

$$\|(T(x_i))\|_{\lambda_{q,\mathcal{J}}} \leq C\varepsilon_{\lambda_{q,\mathcal{J}}}((x_i)). \tag{2.1}$$

For $\mathcal{P}_{\lambda_{q,\mathcal{J}}}(E,F)$, we denote the Banach ideal of the $\lambda_{q,\mathcal{J}}$-absolutely summing operators $T\colon E \to F$ endowed with the topology of the norm $\mathbf{P}_{\lambda_{q,\mathcal{J}}}(T) := \inf C$, taking the infimum over all C that satisfies (2.1).

Theorem 2.4 *Let E and F be Banach spaces. Then $(E \otimes_{g^c_{\lambda_{q,\mathcal{J}}}} F)' = \mathcal{P}_{\lambda'_{q'},\mathcal{K}}(F, E')$ isometrically.*

https://doi.org/10.4153/CMB-2010-075-6 Published online by Cambridge University Press

3 $\lambda_{q,\mathcal{J}}$-Integral Operators

In this paper, we give a characterization of the maximal operator ideal by considering the structure of finite dimensional subspaces of the interpolation spaces involved. The behavior of the interpolation sequences spaces under ultraproducts is also crucial, see [14, 15].

For ultraproducts of Banach spaces, we refer the reader to [11]. We only set the notation we will use. Let D be a nonempty index set and $\mathcal{U}$ a non-trivial ultrafilter in D. Given a family $\{X_d, d \in D\}$ of Banach spaces, $(X_d)_{\mathcal{U}}$ denotes the corresponding ultraproduct Banach space. If every X_d, $d \in D$, coincides with a fixed Banach space X, the corresponding ultraproduct is called an ultrapower of X and is denoted by $(X)_{\mathcal{U}}$. Recall that if every $X_d, d \in D$ is a Banach lattice, $(X_d)_{\mathcal{U}}$ has a canonical order that makes it a Banach lattice. If we have another family of Banach spaces $\{Y_d, d \in D\}$ and a family of operators $\{T_d \in \mathcal{L}(X_d, Y_d), d \in D\}$ such that $\sup_{d\in D} \|T_d\| < \infty$, then $(T_d)_{\mathcal{U}} \in \mathcal{L}((X_d)_{\mathcal{U}}, (Y_d)_{\mathcal{U}})$ denotes the canonical ultraproduct operator.

We now give a local definition that was inspired by Gordon and Lewis' definition of local unconditional structure.

Definition 3.1 Given a sequence space ξ, we say that a Banach space X has an $S_k(\xi)$-local unconditional structure if there exists a real constant $c > 0$ such that for every finite dimensional subspace F of X, there is a section $S_n(\xi)$ of ξ and linear operators $u\colon F \to S_n(\xi)$ and $v\colon S_n(\xi) \to X$ such that $\|u\|\|v\| \le c$ and $vu = I_{F,X}$.

The constant c that appears in above definition is called a $S_k(\xi)$-local unconditional structure constant of X, and in this case we say that X has c-$S_k(\xi)$-local unconditional structure. If a Banach space X has c-$S_k(\xi)$-local unconditional structure for every $c > C$, we say that it has C^+-$S_k(\xi)$-local unconditional structure.

The following definition was introduced by Pelczyǹski and Rosenthal in 1975 [20].

Definition 3.2 A Banach space X has the *uniform projection property* if there is a $b > 0$ such that, for each natural number n, there is a natural number $m(n)$ such that, for every n-dimensional subspace $M \subset X$, there exists a k-dimensional and b-complemented subspace Z of X containing M with $k \le m(n)$.

The constant b of the above definition is called a uniform projection property constant of X, and in this case we say that X has the b-uniform projection property. If X has the b-uniform projection property for every $b > B$, we say that X has the B^+-uniform projection property.

Now, the uniform projection property is satisfied by L_p spaces for $1 < p < \infty$ [20], and since $\lambda_{q,\mathcal{J}}$ is an interpolation space defined by means of ℓ_{p_1} and ℓ_{p_0}, with $1 < p_0 \le p_1 < \infty$, with continuous inclusions $\ell_{p_0} \subset \lambda_{q,\mathcal{J}} \subset \ell_{p_1}$, the uniform projection property is satisfied by $\lambda_{q,\mathcal{J}}$

Furthermore, if $1 \le p \le \infty$, then the Bochner space $L_p(\mu, E)$ and $\ell_p(E)$ has the b-uniform projection property if E does, see [11]. We highlight that the uniform projection property is stable under ultrapowers, see [11]. Moreover, from [22], since $\lambda_{q,\mathcal{J}}$ is reflexive, then every ultrapower of $\ell_1[\lambda_{q,\mathcal{J}}]$ (of $\lambda_{q,\mathcal{J}}$) has 1^+-$S_r(\ell_1)[S_k(\lambda_{q,\mathcal{J}})]$-local unconditional structure (resp. 1^+-$S_k(\lambda_{q,\mathcal{J}})$-local unconditional structure).

https://doi.org/10.4153/CMB-2010-075-6 Published online by Cambridge University Press

According to the general theory of tensor norms and operator ideals, the normed ideal of $\lambda_{q,\mathcal{J}}$-integral operators $(\mathcal{I}_{\lambda_{q,\mathcal{J}}}, \mathbf{I}_{\lambda_{q,\mathcal{J}}})$ is the maximal operator ideal associated with the tensor norm $g^c_{\lambda_{q,\mathcal{J}}}$ in the sense of Defant and Floret [7], or in an equivalent way, the maximal normed operator ideal associated with the normed ideal of $\lambda_{q,\mathcal{J}}$-nuclear operators in the sense of Pietsch [21]. From [7], for every pair of Banach spaces E and F, an operator $T\colon E \to F$ is $\lambda_{q,\mathcal{J}}$-integral if and only if $J_F T \in (E \otimes_{(g^c_{\lambda_{q,\mathcal{J}}})'} F')'$.

For every pair of Banach spaces E, F we define the finitely generated tensor norm $g'_{\lambda_{q,\mathcal{J}}}$ such that if $M \in FIN(E)$ and $N \in FIN(F)$, for every $z \in M \otimes N$,

$$g'_{\lambda_{q,\mathcal{J}}}(z; M \otimes N) := \sup\left\{ |\langle z, w\rangle| / g_{\lambda_{q,\mathcal{J}}}(w; M' \otimes N') \le 1 \right\}.$$

Clearly, $g'_{\lambda_{q,\mathcal{J}}} = (g^c_{\lambda_{q,\mathcal{J}}})'$, since the unit ball in $M' \otimes_{g^c_{\lambda_{q,\mathcal{J}}}} N'$ is the convex hull of the unit ball of $M' \otimes_{g_{\lambda_{q,\mathcal{J}}}} N'$. But we remark that $E' \otimes_{g^c_{\lambda_{q,\mathcal{J}}}} F'$ (and no $E' \otimes_{g_{\lambda_{q,\mathcal{J}}}} F'$) is an isometric subspace of $(E \otimes_{g'_{\lambda_{q,\mathcal{J}}}} F)'$ because $g^c_{\lambda_{q,\mathcal{J}}}$ is finitely generated, see [7, 15.3].

In this case, we define $\mathbf{I}_{\lambda_{q,\mathcal{J}}}(T)$ to be the norm of $J_F T$ considered as an element of the topological dual of the Banach space $E \otimes_{g'_\lambda} F'$. Remark that $\mathbf{I}_{\lambda_{q,\mathcal{J}}}(T) = \mathbf{I}_{\lambda_{q,\mathcal{J}}}(J_F T)$ as a consequence of F' being canonically complemented in F'''.

First we give a non trivial example of $\lambda_{q,\mathcal{J}}$-integral operators.

Theorem 3.3 *Let (Ω, Σ, μ) be a measure space and let $\lambda_{q,\mathcal{J}}$ be an interpolation space. Then every order bounded operator $S\colon L_\infty(\mu) \to \lambda_{q,\mathcal{J}}$ and every order bounded operator $S\colon L_\infty(\mu) \to \ell_1[\lambda_{q,\mathcal{J}}]$ are $\lambda_{q,\mathcal{J}}$-integral with $\mathbf{I}_{\lambda_{q,\mathcal{J}}}(S) = \|S\|$.*

Proof We will only give the proof if $S\colon L_\infty(\mu) \to \lambda_{q,\mathcal{J}}$ is an order bounded operator, since the proof in the other case is similar.

The predual space of $\lambda_{q,\mathcal{J}}$ is $\lambda'_{q',\mathcal{K}}$, which is a regular space for $1 \le q < \infty$, because $\lambda'_{q',\mathcal{K}}$ is an interpolation space between regular spaces $\ell_{p'_0}$ and $\ell_{p'_1}$ with $1 < p_0 \le p_1 < \infty$, where $\ell_{p'_1}$ is dense in $\lambda'_{q',\mathcal{K}}$ and $\lambda'_{q',\mathcal{K}}$ is dense in $\ell_{p'_0}$. Then, the linear span $\mathcal{T}$ of the set $\{\mathbf{e}_i,\ i \in \mathbb{N}\}$ is dense in $\lambda'_{q',\mathcal{K}}$ and by the representation theorem of maximal operator ideals (see [7, 17.5]) and the density lemma ([7, Theorem 13.4]) we only have to see that $S \in (L_\infty(\mu) \otimes_{g'_{\lambda_{q,\mathcal{J}}}} \mathcal{T})'$.

Given $z \in L_\infty(\mu) \otimes_{g'_{\lambda_{q,\mathcal{J}}}} \mathcal{T}$ and $\varepsilon > 0$, let X and Y be finite dimensional subspaces of $L_\infty(\mu)$ and $\mathcal{T}$ respectively such that $z \in X \otimes Y$ and

$$g'_{\lambda_{q,\mathcal{J}}}(z; X \otimes Y) \le g'_{\lambda_{q,\mathcal{J}}}(z; L_\infty(\mu) \otimes \mathcal{T}) + \varepsilon.$$

Let $\{\mathbf{g}_s\}_{s=1}^m$ be a basis for Y and let $k \in \mathbb{N}$ be such that $\forall 1 \le s \le m \mathbf{g}_s = \sum_{i=1}^k c_{si}\mathbf{e}_i$. Then $\forall f \in X, \forall 1 \le s \le m$

$$\langle S, f \otimes \mathbf{g}_s\rangle = \langle f, S'(\mathbf{g}_s)\rangle = \left\langle f, \Big(\sum_{i=1}^k c_{si}\Big) S'(\mathbf{e}_i)\right\rangle = \left\langle f \otimes \sum_{j=1}^k c_{sj}\mathbf{e}_j, \sum_{i=1}^k S'(\mathbf{e}_i) \otimes \mathbf{e}_i \right\rangle.$$

Then if U denotes the tensor $U := \sum_{i=1}^k S'(\mathbf{e}_i) \otimes \mathbf{e}_i \in L_\infty(\mu)' \otimes \lambda_{q,\mathcal{J}}$, by bilinearity we get $\forall z \in X \otimes Y \langle z, S\rangle = \langle U, z\rangle$.

https://doi.org/10.4153/CMB-2010-075-6 Published online by Cambridge University Press

Given $\nu > 0$, for every $1 \leq i \leq k$ there is $f_i \in L_\infty(\mu)$ such that $\|f_i\| \leq 1$ and $\|S'(\mathbf{e}_i)\| \leq |\langle S'(\mathbf{e}_i), f_i\rangle| + \nu$. Then $f := \sup_{1\leq i\leq k} f_i$ lies in the closed unit ball of $L_\infty(\mu)$. On the other hand, $\lambda_{q,\mathcal{J}}$ is a dual lattice and hence it is order complete. By the Riesz–Kantorovich theorem (see [1, Theorem 1.13], for instance), the modulus $|S|$ of the operator S exists in $\mathcal{L}(L_\infty(\mu), \lambda_{q,\mathcal{J}})$. By the lattice properties of $\lambda_{q,\mathcal{J}}$, we have

$$\begin{aligned}
\pi_{\lambda_{q,\mathcal{J}}}((S'(e_i)) &= \pi_{\lambda_{q,\mathcal{J}}}\Big(\sum_{i=1}^k \|S'(\mathbf{e}_i)\|\mathbf{e}_i\Big)\\
&\leq \pi_{\lambda_{q,\mathcal{J}}}\Big(\sum_{i=1}^k |\langle S'(\mathbf{e}_i), f_i\rangle|\mathbf{e}_i\Big) + \nu\pi_{\lambda_{q,\mathcal{J}}}\Big(\sum_{i=1}^k \mathbf{e}_i\Big)\\
&\leq \pi_{\lambda_{q,\mathcal{J}}}\Big(\sum_{i=1}^k |\langle S(f_i), \mathbf{e}_i\rangle|\mathbf{e}_i\Big) + \nu\pi_{\lambda_{q,\mathcal{J}}}\Big(\sum_{i=1}^k \mathbf{e}_i\Big)\\
&\leq \pi_{\lambda_{q,\mathcal{J}}}\Big(\sum_{i=1}^k \langle |S(f_i)|, \mathbf{e}_i\rangle\Big) + \nu\pi_{\lambda_{q,\mathcal{J}}}\Big(\sum_{i=1}^k \mathbf{e}_i\Big)\\
&\leq \pi_{\lambda_{q,\mathcal{J}}}\Big(\sum_{i=1}^k \langle |S|(|f_i|), \mathbf{e}_i\rangle\mathbf{e}_i\Big) + \nu\pi_{\lambda_{q,\mathcal{J}}}\Big(\sum_{i=1}^k \mathbf{e}_i\Big)\\
&\leq \pi_{\lambda_{q,\mathcal{J}}}\Big(\sum_{i=1}^k \langle |S|(|f|), \mathbf{e}_i\rangle\mathbf{e}_i\Big) + \nu\pi_{\lambda_{q,\mathcal{J}}}\Big(\sum_{i=1}^k \mathbf{e}_i\Big)\\
&= \pi_{\lambda_{q,\mathcal{J}}}(|S|(|f|)) + \nu\pi_{\lambda_{q,\mathcal{J}}}\Big(\sum_{i=1}^k \mathbf{e}_i\Big) \leq \||S|\| + \nu\pi_{\lambda_{q,\mathcal{J}}}\Big(\sum_{i=1}^k \mathbf{e}_i\Big).
\end{aligned}$$

Moreover, $\varepsilon_{\lambda'_{q',\mathcal{K}}}((\mathbf{e}_i)_{i=1}^k) \leq 1$. Hence, denoting by I_X and I_Y the corresponding inclusion maps into $L_\infty(\mu)$ and $\lambda_{q,\mathcal{J}}$ respectively, we have

$$\begin{aligned}
|\langle S, z\rangle| &= |\langle U, z\rangle| = |\langle U, ((I_X)' \otimes (I_Y)')(z)\rangle|\\
&\leq g^c_{\lambda_{q,\mathcal{J}}}(U; X\otimes Y) g'_{\lambda_{q,\mathcal{J}}}(((I_X)' \otimes (I_Y)')(z); X'\otimes Y')\\
&\leq g_{\lambda_{q,\mathcal{J}}}(U; X\otimes Y) g'_{\lambda_{q,\mathcal{J}}}(((I_X)' \otimes (I_Y)')(z); X'\otimes Y')\\
&\leq (g_{\lambda_{q,\mathcal{J}}}(U; L_\infty \otimes (\lambda_{q,\mathcal{J}})) + \varepsilon) g'_{\lambda_{q,\mathcal{J}}}(z; L_\infty(\mu) \otimes \lambda'_{q',\mathcal{K}})\\
&\leq g'_{\lambda_{q,\mathcal{J}}}(z; L_\infty(\mu) \otimes \lambda'_{q',\mathcal{K}})(\pi_{\lambda_{q,\mathcal{J}}}((S'(\mathbf{e}_i))\varepsilon_{\lambda'_{q',\mathcal{K}}}((\mathbf{e}_i)) + \varepsilon)\\
&\leq g'_{\lambda_{q,\mathcal{J}}}(z; L_\infty(\mu) \otimes \lambda'_{q',\mathcal{K}})\Big(\||S|\| + \nu\pi_{\lambda_{q,\mathcal{J}}}\Big(\sum_{i=1}^k \mathbf{e}_i\Big) + \varepsilon\Big),
\end{aligned}$$

and, ν being arbitrary, $|\langle S, z\rangle| \leq g'_{\lambda_{q,\mathcal{J}}}(z; L_\infty(\mu) \otimes \lambda'_{q',\mathcal{K}})(\||S|\| + \varepsilon)$. Finally, since ε is arbitrary, we get $|\langle S, z\rangle| \leq g'_{\lambda_{q,\mathcal{J}}}(z; L_\infty(\mu) \otimes \lambda'_{q',\mathcal{K}}\||S|\|$. But from [1, Theorem 1.10], $|S|(\chi_\Omega) = \sup\{|S(f)|, |f| \leq \chi_\Omega\}$ and as $\lambda_{q,\mathcal{J}}$ is order continuous,

$$\||S|\| = \||S|(\chi_\Omega)\| = \sup\{\||S(f)|\|, \|f\| \leq 1\} = \|S\|.$$

Then S is $\lambda_{q,\mathcal{J}}$−integral with $\mathbf{I}_{\lambda_{q,\mathcal{J}}}(S) \leq \|S\|$. But as $(\mathcal{I}_{\lambda_{q,\mathcal{J}}}, \mathbf{I}_{\lambda_{q,\mathcal{J}}})$ is a Banach operators ideal, $\|S\| \leq \mathbf{I}_{\lambda_{q,\mathcal{J}}}(S)$, hence $\mathbf{I}_{\lambda_{q,\mathcal{J}}}(S) = \|S\|$. ∎

Corollary 3.4 *Let (Ω, Σ, μ) be a measure space and $n, k \in \mathbb{N}$. Then every operator $T\colon L_\infty(\mu) \to S_k(\lambda_{q,\mathcal{J}})$ and every operator $T\colon L_\infty(\mu) \to S_n(\ell_1)[S_k(\lambda_{q,\mathcal{J}})]$ satisfy $\mathbf{I}_{\lambda_{q,\mathcal{J}}}(T) = \|T\|$.*

Proof The result follows easily from Theorem 3.3, since every operator $T\colon L_\infty(\mu) \to S_k(\lambda_{q,\mathcal{J}})$ ($T\colon L_\infty(\mu) \to S_n(\ell_1)[S_k(\lambda_{q,\mathcal{J}})]$ in the other case) is order bounded and $S_k(\lambda_{q,\mathcal{J}})$ (resp. $S_n(\ell_1)[S_k(\lambda_{q,\mathcal{J}})]$) is reflexive, hence order continuous. ∎

For our next theorem we need a very deep technical result of Lindenstrauss and Tzafriri [13] that gives us a kind of "uniform approximation" of finite dimensional subspaces by finite dimensional sublattices in Banach lattices.

Lemma 3.5 ([13]) *Let $\varepsilon > 0$ and $n \in \mathbb{N}$ be fixed. There is a natural number $h(n, \varepsilon)$ such that for every Banach lattice X and every subspace $F \subset X$ of dimension* $\dim(F) =$ *n, there are $h(n, \varepsilon)$ disjoints elements $\{z_i, 1 \leq i \leq h(n, \varepsilon)\}$ and an operator A from F into the linear span G of $\{z_i, 1 \leq i \leq h(n, \varepsilon)\}$ such that*

$$\forall x \in F \|A(x) - x\| \leq \varepsilon \|x\|.$$

Theorem 3.6 *Let $\lambda_{q,\mathcal{J}}$ be a interpolation space, regular for $1 \leq q < \infty$, G an abstract M-space, and X a Banach space with c-$S_k(\lambda_{q,\mathcal{J}})$ or c-$S_k(\ell_1)[S_n(\lambda_{q,\mathcal{J}})]$-local uniform structure . Then every operator $T\colon G \longrightarrow X$ is $\lambda_{q,\mathcal{J}}$-integral and* $\mathbf{I}_{\lambda_{q,\mathcal{J}}}(T) \leq c\|T\|$.

Proof We will prove the case where X has c-$S_k(\lambda_{q,\mathcal{J}})$-local unconditional structure since the other case is similar. By the representation theorem of maximal operator ideals (see [7, 17.5]), we only need to show that $J_X T \in (G \otimes_{g'_{\lambda_{q,\mathcal{J}}}} X')'$.

Given $z \in G \otimes X'$ and $\varepsilon > 0$, let $P \subset G$ and $Q \subset X'$ be finite dimensional subspaces and let $z = \sum_{i=1}^n f_i \otimes x'_i$ be a *fixed* representation of z with $f_i \in P$ and $x'_i \in Q$, $i = 1, 2, \dots, n$ such that

$$g'_{\lambda_{q,\mathcal{J}}}(z; G \otimes X') \leq g'_{\lambda_{q,\mathcal{J}}}(z; P \otimes Q) \leq g'_{\lambda_{q,\mathcal{J}}}(z; G \otimes X') + \varepsilon.$$

From Lemma 3.5, we have a finite dimensional sublattice P_1 of G and an operator $A\colon P \to P_1$ so that $\forall f \in P, \|A(f) - f\| \leq \varepsilon \|f\|$. Then, if id_G denotes the identity map on G, we have

$$\begin{aligned} |\langle J_X T, z\rangle| = \Big|\sum_{i=1}^n \langle T(f_i), x'_i\rangle\Big| &\leq \Big|\sum_{i=1}^n \langle T(id_G - A)(f_i), x'_i\rangle\Big| + \Big|\sum_{i=1}^n \langle TA(f_i), x'_i\rangle\Big| \\ &\leq \varepsilon \|T\| \sum_{i=1}^n \|f_i\| \|x'_i\| + \Big|\sum_{i=1}^n \langle TA(f_i), x'_i\rangle\Big|. \end{aligned}$$

Let $X_1 := T(P_1)$. As X has $S_k(\lambda_{q,\mathcal{J}})$-local unconditional structure, there are $k \in \mathbb{N}$, $u\colon X_1 \to S_k(\lambda_{q,\mathcal{J}})$ and $v\colon S_k(\lambda_{q,\mathcal{J}}) \to X$ such that $I_{X_1,X} = vu$ and $\|u\|\|v\| \leq c$.

Let $X_2 := vu(X_1)$, which is a finite dimensional subspace of X containing X_1 and $I_{X_1,X_2} = vu$. Put $K_2\colon X''' \longrightarrow X_2' = X'''/X_2^\circ$ be the canonical quotient map. Then

$$\begin{aligned}\sum_{i=1}^n \langle T(A(f_i)), x_i'\rangle &= \sum_{i=1}^n \langle I_{X_1,X_2} T(A(f_i)), K_2(x_i')\rangle = \sum_{i=1}^n \langle vuT(A(f_i)), K_2(x_i')\rangle \\ &= \sum_{i=1}^n \langle uT(A(f_i)), v'K_2(x_i')\rangle = \langle uT, \sum_{i=1}^n A(f_i) \otimes v'K_2(x_i')\rangle\end{aligned}$$

with $\sum_{i=1}^n A(f_i) \otimes v'K_2(x_i') \in P_1 \otimes (S_k(\lambda_{q,\mathfrak{J}}))'$ and $uT\colon P_1 \to S_k(\lambda_{q,\mathfrak{J}})$. Since P_1 is a reflexive abstract M-space, it is lattice isometric to some $L_\infty(\mu)$ space, hence, by Corollary 3.4, this map is $\lambda_{q,\mathfrak{J}}$-integral with $\mathbf{I}_{\lambda_{q,\mathfrak{J}}}(uT) \le \|u\|\|T\|$. Then

$$\begin{aligned}\Big|\sum_{i=1}^n \langle T(A(f_i)), x_i'\rangle\Big| &= \Big|\Big\langle uT, \sum_{i=1}^n A(f_i) \otimes v'K_2(x_i')\Big\rangle\Big| \\ &\le \mathbf{I}_{\lambda_{q,\mathfrak{J}}}(uT) g'_{\lambda_{q,\mathfrak{J}}}\Big(\sum_{i=1}^n A(f_i) \otimes v'K_2(x_i'\Big); P_1 \otimes S_k(\lambda_{q,\mathfrak{J}})) \\ &\le \|u\|\|T\| g'_{\lambda_{q,\mathfrak{J}}}((A \otimes v'K_2)(z); P_1 \otimes S_k(\lambda_{q,\mathfrak{J}})) \\ &\le \|u\|\|T\|\|A\|\|v'\|\|K_2\| g'_{\lambda_{q,\mathfrak{J}}}(z; P \otimes Q) \\ &\le (1+\varepsilon)c\|T\| g'_{\lambda_{q,\mathfrak{J}}}(z; P \otimes Q) \\ &\le (1+\varepsilon)c\|T\|(g'_{\lambda_{q,\mathfrak{J}}}(z; G \otimes X') + \varepsilon),\end{aligned}$$

and since ε is arbitrary, we obtain $|\langle J_X T, z\rangle| \le c\,\|T\| g'_{\lambda_{q,\mathfrak{J}}}(z; G \otimes X')$. ■

Concerning the characterization theorem of $\lambda_{q,\mathfrak{J}}$-integral operators, we have the following.

Theorem 3.7 *Let $\lambda_{q,\mathfrak{J}}$ be a regular interpolation space for $1 \le q < \infty$ and let E and F be Banach spaces. The following statements are equivalent:*

(i) $T \in \mathfrak{I}_{\lambda_{q,\mathfrak{J}}}(E, F)$.

(ii) *$J_F T$ factors continuously in the following way:*

$$\begin{tikzcd} E \arrow[r, "J_F T"] \arrow[d, "A"'] & F'' \\ L_\infty(\mu) \arrow[r, "B"'] & X \arrow[u, "C"'] \end{tikzcd}$$

where X is an ultrapower of $\ell_1[\lambda_{q,\mathfrak{J}}]$ and B is a lattice homomorphism. Furthermore, $\mathbf{I}_{\lambda_{q,\mathfrak{J}}}(T)$ is equivalent to $\inf\{\|D\|\|B\|\|A\|\}$, *taking it over all such factors.*

https://doi.org/10.4153/CMB-2010-075-6 Published online by Cambridge University Press

Proof (i) $\Longrightarrow$ (ii) Let $D := \{(P, Q) : P \in FIN(E), Q \in FIN(F')\}$, where $FIN(Y)$ is the set of finite dimensional subspace of a Banach space Y, endowed with the natural inclusion order

$$(P_1, Q_1) \leq (P_2, Q_2) \Longleftrightarrow P_1 \subset P_2, Q_1 \subset Q_2.$$

For every $(P_0, Q_0) \in D$, $R(P_0, Q_0) := \{(P, Q) \in D : (P_0, Q_0) \subset (P, Q)\}$ and $\mathcal{R} = \{R(P, Q), (P, Q) \in D\}$. $\mathcal{R}$ is filter basis in D, and according to Zorn's lemma, let $\mathcal{D}$ be an ultrafilter on D containing $\mathcal{R}$. If $d \in D$, P_d and Q_d denote the finite dimensional subspaces of E and F' respectively so that $d = (P_d, Q_d)$. For every $d \in D$, if

$$z \in P_d \otimes Q_d,\ J_F T_{|P_d \otimes Q_d} \in (P_d \otimes_{g'_{\lambda_{q,\mathcal{J}}}} Q_d)' = P'_d \otimes_{g_{\lambda_{q,\mathcal{J}}}} Q'_d = \mathcal{N}_{\lambda_{q,\mathcal{J}}}(P_d, Q'_d).$$

Then from Theorem 2.2 of characterization of $\lambda_{q,\mathcal{J}}$-nuclear operators, $J_F T_{|P_d \otimes Q_d}$ factors as

$$\begin{tikzcd}
P_d \arrow[r, "J_F T_{|P_d \otimes Q_d}"] \arrow[d, "A_d"'] & Q'_d \\
\ell_\infty[\ell_\infty] \arrow[r, "B_d"'] & \ell_1[\lambda_{q,\mathcal{J}}] \arrow[u, "C_d"']
\end{tikzcd}$$

where B_d is a positive diagonal operator and

$$\|A_d\| \|B_d\| \|C_d\| \leq \mathbf{N}^c_{\lambda_{q,\mathcal{J}}}(T_{|P_d \otimes Q_d}) + \varepsilon = \mathbf{I}_{\lambda_{q,\mathcal{J}}}(T_{|P_d \otimes Q_d}) + \varepsilon.$$

Then

$$\|A_d\| \|B_d\| \|C_d\| \leq \mathbf{I}_{\lambda_{q,\mathcal{J}}}(T_{|P_d \otimes Q_d}) + \varepsilon \leq \mathbf{I}_{\lambda_{q,\mathcal{J}}}(T) + \varepsilon$$

Without loss of generality, we can suppose that $\|A_d\| = \|C_d\| = 1$. We define $W_E \colon E \to (P_d)_{\mathcal{D}}$ such that $W_E(x) = (x_d)_{\mathcal{D}}$, so that $x_d = x$ if $x \in P_d$, and $x_d = 0$ if $x \notin P_d$. In the same way, we define $W_{F'} : F' \to (Q_d)_{\mathcal{D}}$ such that $W_{F'}(a) = (a_d)_{\mathcal{D}}$, so that $a_d = a$ if $a \in Q_d$, and $a_d = 0$ if $a \notin Q_d$. Then we have the following commutative diagram:

$$\begin{tikzcd}
E \arrow[rr, "J_F T"] \arrow[d, "W_E"'] & & F'' \\
(P_d)_{\mathcal{D}} \arrow[r, "(J_F T_{|P_d \otimes Q_d})_{\mathcal{D}}"'] \arrow[d, "(A_d)_{\mathcal{D}}"'] & (Q'_d)_{\mathcal{D}} \arrow[r, "I"'] & ((Q_d)_{\mathcal{D}})' \arrow[u, "W'_{F'}"'] \\
(\ell_\infty[\ell_\infty])_{\mathcal{D}} \arrow[r, "(B_d)_{\mathcal{D}}"'] & (\ell_1[\lambda_{q,\mathcal{J}}])_{\mathcal{D}} \arrow[u, "(C_d)_{\mathcal{D}}"'] &
\end{tikzcd}$$

where I is the canonical inclusion map. As in [13] $((\ell_1[\lambda_{q,\mathcal{J}}])_{\mathcal{D}})''$ is a 1-complemented subspace of some ultrapower $((\ell_1[\lambda_{q,\mathcal{J}}])_{\mathcal{D}})_{\mathcal{U}}$, which, from [24], is another

https://doi.org/10.4153/CMB-2010-075-6 Published online by Cambridge University Press

ultrapower $(\ell_1[\lambda_{q,\mathcal{J}}])_{\mathcal{U}_1}$ with projection Q. The result follows with $A = (A_d)_{\mathcal{D}}$, $B = ((B_d)_{\mathcal{D}})''$, which is a lattice homomorphism, $C = P_{F''''}(W'_{F'}I(C_d)_{\mathcal{D}})''Q$, where $P_{F''''}$ is the projection of F'''' in F'', and $X = (\ell_1[\lambda])_{\mathcal{U}_1}$, having in mind that as $(\ell_\infty[\ell_\infty])_{\mathcal{D}}$ is an abstract M-space, there is a measure space such that $L_\infty(\mu) = ((\ell_\infty[\ell_\infty])_{\mathcal{D}})''$, where equality means that the spaces are lattice isometric.

(ii) $\Longrightarrow$ (i) As $(\mathcal{I}_{\lambda_{q,\mathcal{J}}}, \mathbf{I}_{\lambda_{q,\mathcal{J}}})$ is an operator ideal, it follows easily from Theorem 3.6 and considering that every ultrapower of $\ell_1[\lambda_{q,\mathcal{J}}]$ has 1^+-$S_r(\ell_1)[S_k(\lambda_{q,\mathcal{J}})]$-local unconditional structure. ■

The following new formulation of the preceding characterization theorem is needed in our context.

Theorem 3.8 *Let $\lambda_{q,\mathcal{J}}$ be an interpolation space. For every pair of Banach spaces E and F, the following statements are equivalent:*

(i) $T \in \mathcal{I}_{\lambda_{q,\mathcal{J}}}(E, F)$.

(ii) *There exists a σ-finite measure space $(\mathcal{O}, \mathcal{S}, \nu)$ and a Köthe function space $\mathcal{K}(\nu)$ that is complemented in a space with $S_k(\ell_1)[S_n(\lambda_{q,\mathcal{J}})]$-local unconditional structure, such that $J_F T$ factors continuously in the following way:*

$$\begin{array}{ccc} E & \xrightarrow{\quad J_F T \quad} & F'' \\ {\scriptstyle A}\downarrow & & \uparrow{\scriptstyle C} \\ L_\infty(\nu) & \xrightarrow[\quad B \quad]{} & \mathcal{K}(\nu) \end{array}$$

where B is a multiplication operator for a positive function of $\mathcal{K}(\nu)$. Furthermore $\mathbf{I}_{\lambda_{q,\mathcal{J}}}(T) = \inf\{\|C\|\|B\|\|A\|\}$, taking the infimum over all such factors.

Proof Starting from Theorem 3.7, as $\ell_1[\lambda_{q,\mathcal{J}}]$ has finite cotype, see [4, 5], and additionally, $\ell_1[\lambda_{q,\mathcal{J}}]$ is order continuous ([10, 4.6]), and for [12, Theorem 1.a.9] $\ell_1[\lambda_{q,\mathcal{J}}]$ can be decomposed into an unconditional direct sum of a family of mutually disjoint ideals $\{X_h, h \in H\}$ having a positive weak unit, and then from [12, 1.b.14], as every X_h is order isometric to a Köthe space of functions defined on a probability space $(\mathcal{O}_h, \mathcal{S}_h, \nu_h)$, then $(\ell_1[\lambda_{q,\mathcal{J}}])_{\mathcal{U}}$ is order isometric to a Köthe function space $\mathcal{K}(\nu^1)$ over a measure space $(\mathcal{O}^1, \mathcal{S}^1, \nu^1)$, hence we can substitute $(\ell_1[\lambda_{q,\mathcal{J}}])_{\mathcal{U}}$ for $\mathcal{K}(\nu^1)$ in Theorem 3.7. If we denote $z := B(\chi_\Omega)$ with $z = \sum_{i=1}^\infty y_{h_i}$, with $y_{h_i} \in X_{h_i}$ for every $i \in \mathbb{N}$, then $B(L_\infty(\mu))$ is contained in the unconditional direct sum of $\{X_{h_i}, i \in \mathbb{N}\}$, which is order isometric to a space of Köthe function space $\mathcal{K}(\nu)$ over a σ-finite measure space $(\mathcal{O}, \mathcal{S}, \nu)$, which is 1-complemented in $\mathcal{K}(\nu^1)$.

Now, since $\mathcal{K}(\nu)$ is order complete, there exists $g := \sup_{\|f\|_{L_\infty(\mu)}} B(f)$ in $\mathcal{K}(\nu)$. Then the operators $B_1 \colon L_\infty(\mu) \to L_\infty(\nu)$ and $B_2 \colon L_\infty(\nu) \to \mathcal{K}(\nu)$, such that

$$B_1(f)(\omega) := B(f)(\omega)/g(\omega),$$

for all $f \in L_\infty(\mu)$, $\omega \in \mathcal{O}$ with $g(\omega) \neq 0$ and $B_1(f)(\omega) = 0$ otherwise, and $B_2(h)(\omega) := g(\omega)h(\omega)$ for all $h \in L_\infty(\nu)$, $\omega \in \mathcal{O}$, satisfy that $B = B_2 B_1$ and B_2 is a multiplication operator for a positive element $g \in \mathcal{K}(\nu)$. ■

https://doi.org/10.4153/CMB-2010-075-6 Published online by Cambridge University Press

4 On Equality Between $\lambda_{q,\mathcal{J}}$-Nuclear and $\lambda_{q,\mathcal{J}}$-Integral Operators

Finally, using the preceding characterization theorems, we give some properties of $\lambda_{q,\mathcal{J}}$-nuclear and $\lambda_{q,\mathcal{J}}$-integral operators. Let us now establish a necessary condition for equality between components of $\lambda_{q,\mathcal{J}}$-nuclear and $\lambda_{q,\mathcal{J}}$-integral operator ideals. First, we introduce a new operator ideal, which is contained in the ideal of the $\lambda_{q,\mathcal{J}}$-integral operators.

Definition 4.1 Given E and F Banach spaces, let $\lambda_{q,\mathcal{J}}$ be an interpolation space. We say that $T \in \mathcal{L}(E,F)$ is *strictly $\lambda_{q,\mathcal{J}}$-integral* if there exists a σ-finite measure space $(\mathcal{O}, \mathcal{S}, \nu)$ and a Köthe function space $\mathcal{K}(\nu)$ which is complemented in a space with $S_k(\ell_1)[S_n(\lambda_{q,\mathcal{J}})]$-local unconditional structure, such that T factors continuously in the following way:

$$\begin{tikzcd} E \arrow[r, "T"] \arrow[d, "A"'] & F \\ L_\infty(\nu) \arrow[r, "B"'] & \mathcal{K}(\nu) \arrow[u, "C"'] \end{tikzcd}$$

where B is a multiplication operator for a positive function of $\mathcal{K}(\nu)$, endowed with the topology of the norm $\mathbf{SI}_{\lambda_{q,\mathcal{J}}}(T) = \mathbf{I}_{\lambda_{q,\mathcal{J}}}(T)$.

Obviously, if F is a dual space, or it is complemented in its bidual space, then $\mathcal{SI}_{\lambda_{q,\mathcal{J}}}(E,F) = \mathcal{I}_{\lambda_{q,\mathcal{J}}}(E,F)$.

Theorem 4.2 *Let $\ell_{\lambda_{q,\mathcal{J}}}$ be a interpolation space, and let E and F be Banach spaces, such that E' satisfies the Radon–Nikodým property, then $\mathcal{N}_{\lambda_{q,\mathcal{J}}}(E,F) = \mathcal{SI}_{\lambda_{q,\mathcal{J}}}(E,F)$.*

Proof Let $T \in \mathcal{SI}_{\lambda^c}(E,F)$, where E' has the Radon–Nikodým property.

(a) First, we suppose that B is a multiplication operator for a function $g \in \mathcal{K}(\nu)$ with finite measure support D. We denote by ν_D the restriction of ν to D.

As $(\chi_D A)\colon E \to L_\infty(\nu_D)$, then $(\chi_D A)'\colon (L_\infty(\nu_D))' \to E'$ and the restriction of $(\chi_D A)' \restriction_{L_1(\nu_D)}\colon L_1(\nu_D) \to E'$, thus, for every $x \in E$ and $f \in L_1(\nu_D)$,

$$\langle x, (\chi_D A)'(f)\rangle = \langle \chi_D A(x), f\rangle = \int_D \chi_D A(x) f d(\nu_D).$$

As E' has the Radon–Nikodým property, by [8, III(5)], we have that $(\chi_D A)'$ has a Riesz representation, therefore a function $\phi \in L_\infty(\nu_D, E')$ exists such that for every $f \in L_1(\nu_D)$

$$(\chi_D A)'(f) = \int_D f\phi d(\nu_D).$$

Then, for every $x \in E$, we have that $\chi_D A(x)(t) = \langle \phi(t), x\rangle$, ν_D-almost everywhere in D, and then $B(\chi_D A)(x) =< g\phi(\,\cdot\,), x >$, ν_D-almost everywhere in D. Let $g\phi$ be this last operator, and we can consider it as a $\mathcal{K}(\nu_D, E')$ element. As the simple functions are dense in $\mathcal{K}(\nu_D, E')$, $g\phi$ can be approximated by a sequence of simple functions $((S_k)_{k=1}^\infty)$.

https://doi.org/10.4153/CMB-2010-075-6 Published online by Cambridge University Press

We suppose $S_k = \sum_{j=1}^{m_k} x'_{kj}\chi_{A_{kj}}$, where $\{A_{ki} : i = 1, \dots, m\}$ is a family of ν-measure set of Ω pairwise disjoint. For each $k \in \mathbb{N}$, we can interpret S_k as a map $S_k\colon E \to \mathcal{K}(\nu)$ such that $S_k(x) = \sum_{j=1}^{m_k} < x'_{kj}, x > \chi_{A_{kj}}$ with norm less than or equal to the norm of S_k in $\mathcal{K}(\nu, E')$.

Clearly for all $k \in \mathbb{N}$, S_k is $\lambda_{q,\mathcal{J}}$-nuclear since it has finite range, but we need to evaluate its $\lambda_{q,\mathcal{J}}$-nuclear norm coinciding with its $\lambda_{q,\mathcal{J}}$-integral norm. Let $S_k^1\colon E \to L_\infty(\nu)$ be such that

$$S_k^1(x) = \sum_{j=1}^{m_k} \frac{\langle x'_{kj}, x\rangle}{\|x'_{kj}\|}\chi_{A_{kj}},$$

and let $S_k^2\colon L_\infty(\nu) \to \mathcal{K}(\nu)$ be such that $S_k^2(f) = \sum_{j=1}^{m_k} \|x'_{kj}\| f\chi_{A_{kj}}$.

Then $\|S_k^1\| \le 1$, $\|S_k^2\| \le \|S_k\|_{\mathcal{K}(\nu,E')}$ and $S_k = S_k^2 S_k^1$. But as $\mathcal{K}(\nu)$ is a complemented subspace of space with $S_k(\ell_1)(S_n(\lambda_{q,\mathcal{J}}))$-local unconditional structure, from Theorem 3.6, there is $K > 0$ such that $\mathbf{I}_{\lambda_{q,\mathcal{J}}}(S_k^2) \le K\|S_k^2\| \le K\|S_k\|_{\mathcal{K}(\nu,E')}$, hence $\mathbf{N}^c_{\lambda_{q,\mathcal{J}}}(S_k^2) \le K\|S_k^2\| \le \|S_k\|_{\mathcal{K}(\nu,E')}$, hence $\mathbf{N}^c_{\lambda_{q,\mathcal{J}}}(S_k) \le K\|S_k\|_{\mathcal{K}(\nu,E')}$.

Then, as $(S_k)_{k=1}^\infty$ converges in the $\mathcal{K}(\nu_D, E')$ space, it is a Cauchy sequence in $\mathcal{N}_{\lambda_{q,\mathcal{J}}}(E, \mathcal{K}(\nu_D))$, and since this is complete, $(S_k)_{k=1}^\infty$ converges to $g\phi$, that is to say, $g\phi \in \mathcal{N}_{\lambda_{q,\mathcal{J}}}(E, \mathcal{K}(\nu_D))$. Therefore, $g\phi = B\chi_D A$ is $\lambda_{q,\mathcal{J}}$-nuclear and so T is also $\lambda_{q,\mathcal{J}}$-nuclear.

(b) Now, if g is any element of $\mathcal{K}(\nu)$, it can be approximated in norm by means of a sequence $(t_n)_{n=1}^\infty$ of simple functions with finite measure support, and therefore by (a), the sequence $T_n = CB_{t_n}A$ is a Cauchy sequence in $\mathcal{N}_{\lambda_{q,\mathcal{J}}}(E, F)$ converging to T in $\mathcal{L}(E, F)$, and then $T \in \mathcal{N}_{\lambda_{q,\mathcal{J}}}(E, F)$. ■

As consequence of the former result and of the factorization Theorems 3.8 and 2.2, we obtain the following metric properties of $g^c_{\lambda_{q,\mathcal{J}}}$ and $(g^c_{\lambda_{q,\mathcal{J}}})'$.

Theorem 4.3 *$(g^c_{\lambda_{q,\mathcal{J}}})'$ is a totally accessible tensor norm.*

Proof Since $(g^c_{\lambda_{q,\mathcal{J}}})'$ is finitely generated, it is sufficient to prove that the map

$$F \otimes_{(g^c_{\lambda_{q,\mathcal{J}}})'} E \hookrightarrow \mathcal{P}_{\lambda'_{q',\mathcal{K}}}(E', F'')$$

is isometric.

Let

$$z = \sum_{i=1}^{n}\sum_{j=1}^{l_i} y_{ij} \otimes x_{ij} \in F \otimes_{(g^c_{\lambda_{q,\mathcal{J}}})'} E,$$

and let $H_z \in \mathcal{P}_{\lambda'_{q',\mathcal{K}}}(E', F'')$ be the canonical map associated with z, that is to say,

$$H_z(x') = \sum_{i=1}^{n}\sum_{j=1}^{l_i} \langle x_{ij}, x'\rangle y_{ij}$$

for all $x' \in E'$, with $H_z \in \mathcal{L}(E', F) \subset \mathcal{L}(E', F'')$.

Applying [7, Theorem 15.5] for $\alpha = (g^c_{\lambda_{q,\mathcal{J}}})'$, the theorem 2.4, and the equality $(g^c_{\lambda_{q,\mathcal{J}}})'' = g^c_{\lambda_{q,\mathcal{J}}}$, since $g^c_{\lambda_{q,\mathcal{J}}}$ is finitely generated, we have that inclusion

$$F \otimes_{\overleftarrow{(g^c_{\lambda_{q,\mathcal{J}}})'}} E \hookrightarrow (F' \otimes_{g^c_{\lambda_{q,\mathcal{J}}}} E')' \to \mathcal{P}_{\lambda'_{q',\mathcal{K}}}(E', F'')$$

is an isometry, and therefore by [7, Proposition 12.4], we obtain

$$\mathbf{P}_{\lambda'_{q',\mathcal{K}}}(H_z) = \overleftarrow{(g^c_{\lambda_{q,\mathcal{J}}})'}(z; F \otimes E) \leq (g^c_{\lambda_{q,\mathcal{J}}})'(z; F \otimes E).$$

Now, given N, a finite dimensional subspace of F such that $z \in N \otimes_{(g^c_{\lambda_{q,\mathcal{J}}})'} E$, there exists

$$V \in (N \otimes_{(g^c_{\lambda_{q,\mathcal{J}}})'} E)' = \mathcal{I}(N, E')$$

such that $\mathbf{I}_{\lambda_{q,\mathcal{J}}}(V) \leq 1$ and $(g^c_{\lambda_{q,\mathcal{J}}})'(z; N \otimes E) = \langle z, V \rangle$. Clearly enough

$$V \in \mathcal{SI}_{\lambda_{q,\mathcal{J}}}(N, E') = \mathcal{I}_{\lambda_{q,\mathcal{J}}}(N, E')$$

because E' is a dual space, and N', being finite dimensional, has the Radon–Nikodým property. Therefore, by Theorem 4.2, $V \in \mathcal{N}_{\lambda_{q,\mathcal{J}}}(N, E')$ and by Theorem 2.2, given $\epsilon > 0$, there is a factorization V in the way

$$\begin{tikzcd}
N \arrow[r, "T"] \arrow[d, "A"'] & E' \\
\ell_\infty[\ell_\infty] \arrow[r, "B"'] & \ell_1[\lambda_{q,\mathcal{J}}] \arrow[u, "C"']
\end{tikzcd}$$

such that $\|C\|\|B\|\|A\| \leq \mathbf{N}^c_{\lambda_{q,\mathcal{J}}}(V) + \epsilon = \mathbf{I}^c_{\lambda_{q,\mathcal{J}}}(V) + \epsilon \leq 1 + \epsilon$.

As $\ell_\infty[\ell_\infty]$ has the extension metric property, (see [21, Proposition 1, C.3.2]), A can be extended to a continuous map $\overline{A} \in \mathcal{L}(F, \ell_\infty[\ell_\infty])$ such that $\|\overline{A}\| = \|A\|$. By Theorem 2.2 again, $W := CB\overline{A}$ is in $\mathcal{N}_{\lambda_{q,\mathcal{J}}}(F, E')$, so there is a representation $w =: \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} y'_{ij} \otimes x'_{ij} \in F' \widehat{\otimes}_{g^c_{\lambda_{q,\mathcal{J}}}} E'$ of W verifying

$$\sum_{i=1}^{\infty} \pi_{\lambda_{q,\mathcal{J}}}((y'_{ij}))\varepsilon_{\lambda'_{q',\mathcal{K}}}((x'_{ij})) \leq \mathbf{N}^c_{\lambda_{q,\mathcal{J}}}(W) + \epsilon \leq \|C\| \, \|B\| \, \|\overline{A}\| + \epsilon \leq 1 + 2\epsilon.$$

Then, $(g^c_{\lambda_{q,\mathcal{J}}})'(z; F \otimes E) \leq (g^c_{\lambda_{q,\mathcal{J}}})'(z; N \otimes E) = \langle z, V \rangle = \langle z, W \rangle$. It follows that

$$(g^c_{\lambda_{q,\mathcal{J}}})'(z; F \otimes E) \leq g^c_{\lambda_{q,\mathcal{J}}}(w)\mathbf{P}_{\lambda'_{q',\mathcal{K}}}(H_z) \leq (1 + 2\epsilon)\mathbf{P}_{\lambda'_{q',\mathcal{K}}}(H_z),$$

whence $(g^c_{\lambda_{q,\mathcal{J}}})'(z; F \otimes E) \leq \mathbf{P}_{\lambda'_{q',\mathcal{K}}}(H_z)$, and the equality is obvious. ∎

Finally, as a consequence of the former theorem and of [7, Proposition 15.6], we have the following.

Corollary 4.4 *$g^c_{\lambda_{q,\mathcal{J}}}$ is an accessible tensor norm.*

References

[1] C. D. Aliprantis and O. Burkinshaw, *Positive operators.* Pure and Applied Mathematics, 119, Academic Press, Inc., Orlando, FL, 1985.

[2] G. Arango, J. A. López Molina, and M. J. Rivera, *Characterization of $g_{\infty,\sigma}$-integral operators*. Math. Nachr. **278**(2005), no. 9, 995–1014. doi:10.1002/mana.200310286

[3] B. Beauzamy, *Espaces d'interpolation reéls: topologie et géométrie.* Lecture Notes in Mathematics, 666, Springer, Berlin, 1978.

[4] J. Bergh and J. Lofstrom, *Interpolation spaces. An introduction*. Grundlehren der Mathematischen Wissenschaften, 223, Springer-Verlag, Berlin-New York, 1976.

[5] J. Creekmore, *Type and cotype in Lorentz L_{pq} spaces*. Nederl. Akad. Wetensch. Indag. Math. **43**(1981), no. 2, 145–152.

[6] N. De Grande-De Kimpe, *Λ-mappings between locally convex spaces.* Indag. Math. **33**(1971), 261–274.

[7] A. Defant and K. Floret, *Tensor norms and operator ideals*. North-Holland Mathematics Studies, 176, North-Holland Publishing Co., Amsterdam, 1993.

[8] J. Diestel and J. J. Uhl Jr., *Vector measures*. Mathematical Surveys, 15, American Mathematical Society, Providence, RI, 1977.

[9] J. Harksen, *Tensornormtopologien*. Dissertation, Christian-Albrechts-Universität zu Kiel, 1979.

[10] R. Haydon, M. Levy, and Y. Raynaud, *Randomly normed spaces*. Travaux en Cours, 41, Hermann, Paris, 1991.

[11] S. Heinrich, *Ultraproducts in Banach spaces theory*. J. Reine Angew. Math. **313**(1980), 72–104. doi:10.1515/crll.1980.313.72

[12] J. Lindenstrauss and L. Tzafriri, *Classical Banach spaces I. Sequence spaces.* Ergebnisse der Mathematik und ihrer Grenzgebiete, 92, Springer-Verlag, Berlin-New York, 1977, and *Classical Banach spaces II. Function spaces.* Ergebnisse der Mathematik und ihrer Grenzgebiete, 97, Springer-Verlag, Berlin-New York, 1979.

[13] ———, *The uniform approximation property in Orlicz spaces.* Israel J. Math. **23**(1976), no. 2, 142–155.

[14] J. A. López Molina, M. E. Puerta, and M. J. Rivera, *On the structure of ultraproducts of real interpolation spaces*. Extracta Math. **16**(2001), no. 3, 367–382. doi:10.1007/BF02756794

[15] ———, *Ultraproducts of real interpolation spaces between L^p-spaces*. Bull. Braz. Math. Soc. (N.S.) **37**(2006), no. 2, 191–216. doi:10.1007/s00574-006-0010-5

[16] J. A. López Molina and E. A. Sánchez Pérez, *On operator ideals related to (p,σ)-absolutely continuous operators*. Studia Math. **138**(2000), no. 1, 25–40.

[17] U. Matter, *Factoring through interpolation spaces and super-reflexive Banach spaces*. Rev. Roumaine Math. Pures Appl. **34**(1989), no. 2, 147–156.

[18] ———, *Absolutely continuous operators and super-reflexivility*. Dissertation, Universität Zürich, 1985.

[19] M. E. Puerta and G. Loaiza, *Sobre un ideal minimal de operadores definido a través de espacios de interpolación*. Ingeniería y Ciencia **3**(2007), no. 6, 63–89.

[20] A. Pelczyński and H. P. Rosenthal, *Localization techniques in L^p spaces.* Studia Math. **52**(1974/75), 263–289.

[21] A. Pietsch, *Operator ideals*. North-Holland Mathematical Library, 20, North-Holland Publishing Co., Amsterdam-New York, 1980.

[22] M. J. Rivera, *On the classes of $\mathcal{L}^\lambda$, quasi-$\mathcal{L}^E$ and $\mathcal{L}^{\lambda,g}$ spaces.* Proc. Amer. Math. Soc. **133**(2005), no. 7, 2035–2044. doi:10.1090/S0002-9939-05-07761-0

[23] P. Saphar, *Produits tensoriels d'espaces de Banach et classes d'applications linéaires.* Studia Math. **38**(1970), 71–100.

[24] B. Sims, *"Ultra"-techniques in Banach space theory.* Queen's Papers in Pure and Applied Mathematics, 60, Queen's University, Kingston, ON, 1982.

[25] S. Tomášek, *Projectively generated topologies on tensor products.* Comment. Math. Univ. Carolinae **11**(1979), no. 4, 745–768.

Universidad EAFIT, Departamento de Ciencias Bsicas, Medelln, Colombia
e-mail: mpuerta@eafit.edu.co
gloaiza@eafit.edu.co