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Mathematical modelling of crime and security: Special Issue of EJAM

This special issue of the European journal of applied mathematics features research articles that involve the application of mathematical methodologies to the modelling of a broad range of problems related to crime and security. Some specific topics in this issue include recent developments in mathematical models of residential burglary, a dynamical model for the spatial spread of riots initiated by some triggering event, the analysis and development of game-theoretic models of crime and conflict, the study of statistically based models of insurgent activity and terrorism using real-world data sets, models for the optimal strategy of police deployment under realistic constraints, and a model of cyber crime as related to the study of spiking behaviour in social network cyberspace communications. Overall, the mathematical and computational methodologies employed in these studies are as diverse as the specific applications themselves and the scales (spatial or otherwise) to which they are applied. These methodologies range from statistical and stochastic methods based on maximum likelihood methods, Bayesian equilibria, regression analysis, self-excited Hawkes point processes, agent-based random walk models on networks, to more traditional applied mathematical methods such as dynamical systems and stability theory, the theory of Nash equilibria, rigorous methods in partial differential equations and travelling wave theory, and asymptotic methods that exploit disparate space and time scales.

Although not nearly as well-developed as in the applied and biological sciences, over the past seven years there has been a more enhanced focus and increased interest in the development and analysis of mathematical models in the social sciences, and more specifically to problems related to crime and security. In particular, a previous special issue of EJAM [EJAM, Vol. 21 (2010)] was one of the first dedicated special issues of an applied mathematics journal to the study of crime. It is notable that, at the time of writing, four of the 10 most read articles published in EJAM, and four of the most cited articles (according to Google Scholar Metrics) from EJAM, appeared in this issue of the journal. With a broader, but related focus, the special issue of Discrete and Continuous Dynamical Systems Vol. 19, No. 5, (2014) was devoted to the mathematical study of social systems, including crime modelling and swarming behaviour in complex systems. Publications applying formal mathematical models have appeared in high impact criminology journals, including Criminology (e.g. Brantingham *et al.*, 2012) and the Journal of Research in Crime and Delinquency (e.g. Pitcher and Johnson, 2011), demonstrating an appetite for the application of applied mathematics in other disciplines. Numerous conferences have also been arranged including a workshop on “Computational Criminology” in 2012 held at Simon Fraser University in Canada. This particular meeting was supported by the Pacific Institute of Mathematical Sciences, and brought together diverse groups of researchers with expertise in social sciences, computational and statistical methodologies, and mathematical analyses, for the identification and dissemination of results for key problems in the modelling, prediction, and control of crime. In 2009, the Engineering and Physical Sciences Research Council (EPSRC) launched a 10 M call for projects to address real-world problems using tools from complexity science, signalling a broader acknowledgement of the contribution that applied mathematics can play in understanding social systems. Two of the articles (Baudains *et al.* and Lloyd *et al.*) published as part of this special issue were funded under that call.

The emerging role of mathematical modelling in the study of crime and security is multi-faceted. On the one hand, the development and analysis of specific theoretical models can lead to new insights in crime behaviour, such as the conditions for the propagation of riots or social unrest, conditions for the emergence of spatially localized patterns of elevated crime activity, and theoretical predictions for optimal strategies to prevent or mitigate crime with limited police resources. Alternatively, mathematical modelling efforts can be "data-driven", with the focus on the implementation of new statistical and computational methodologies to analyse essential features of real-world data, which can then inspire the formulation of new mathematical models incorporating these key features. Often either real-world data or results from agent-based models can be used to refine and validate more theoretically based mathematical models, and to optimally choose or fit realistic choices of parameter values in these models. Over the past five years, there has been a new and significant interplay between real-world data and mathematical modelling in the study of problems related to criminology.

We now provide a brief description of the specific contributions to this special issue. Many of the key references to this emerging field of mathematical criminology can be found in the bibliographies of the wide spectrum of papers that comprise this special issue.

The first two articles in this special issue concern the modelling of crime through the use of game theory. In McBride, Kendall, D'Orsogna, and Short (this issue), the authors examine further game-theoretic properties of a model of crime known as the SBD Adversarial Game [Short *et al.*, Phys. Rev. E. 82, (2010), 066114, Short *et al.*, EJAM 24, (2013), pp. 131–159]. In this evolutionary game model, the three classic strategies known from public goods games of "Punishers", "Cooperators", and "Defectors", are modified by the inclusion of a fourth strategy of "Informants", which leads to the emergence of cooperation in specific simulations of the game. In an earlier paper, Short *et al.* EJAM 24, (2013) conjectured that the presence of criminal informants leads to diminishing crime, and they formulated an optimal control problem to determine the most effective strategy to recruit such informants under given constraints. In Mcbride *et al.*, this conjecture is revisited for a wider parameter set in the game model. They found that under the best response dynamics, the role informants play does depend on parameter choices, and that their mere presence does not necessarily drive the system to low crime states as they did in SBD's original work. Specific criteria, in terms of the model parameters, for the emergence of low crime states in the presence of informants are determined. In Espejo, L'Huillier, and Weber (this issue), the authors use a game-theoretical approach to model the interaction between police forces and delinquents in public places. In their model, the police must simultaneously deal with different types of offenders, who may either be in

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organized gangs or act independently of each other. Their modelling framework is an extension of the well-known two-player Stackelberg game, which consists of a leader and a follower, with the leader, representing the police, committing to a specific strategy and the follower, representing the criminal element, then optimizing their reward based on the action chosen by the leader. In the model of Espejo *et al.*, their novel contribution is the analysis of two games that occur simultaneously: the interaction between the police and a coordinated gang in a classic Stackelberg game, and the interaction among criminals that act independently when deciding where to commit crime, modelled by a Stackelberg–Nash game with multiple followers. The theory of Nash equilibria is used to obtain optimal distributions for offenders, which are then employed, for simulated data sets, to suggest optimal decisions regarding police surveillance that minimize the criminals' utility.

The next two articles in this special issue focus on developing mathematical models and statistical methods to understand data related to insurgent or terrorist attacks. In Johnson, Johnson, and Restrepo (this issue), the authors use generative mathematical models to interpret certain statistical patterns observed in insurgent, terrorist, and cyber-attacks across different geographic settings including cyberspace. Both the mathematical modelling and the empirical statistical analysis draw on approaches popularized within the physics community. In particular, in the first part of their study, the authors reveal important mathematical insights underlying a previously identified universal power law exponent that fits various real-world datasets of attack severity associated with insurgent and terrorist activities. The second part of the paper focusses on the timing and frequency of successive attacks, identifying an empirical relation between the initial attack frequency, and the escalation rate of follow-up attacks. This is based on the formulation and study of a coagulation/fragmentation process, which models the interaction of distinct human social groups. From this analysis, the authors conjecture that, irrespective of the underlying circumstances and locations, groups of humans tend to "do" insurgency and terrorism in a surprisingly generic and universal way. In Tench, Fry, and Gill (this issue), the authors use the Hawkes processes to model improvized explosive device (IED) attacks during the time period known as "The Troubles" in Northern Ireland. These point processes are used to analyse a unique dataset of IED attacks that took place across various spatial zones, including six counties and Belfast, and across several distinct phases of the conflict. To account for edge effects, where events in one phase influence those in another, a novel approach involving a moving time frame is implemented. The models are assessed for goodness of fit and estimated parameters are interpreted in terms of the theoretical mechanisms (of conflict) at play. By incorporating a second dataset concerning British Security Force interventions, a multidimensional Hawkes process model is used to test the effectiveness of counter-terrorism operations in Northern Ireland. The results from this latter analysis are consistent with previous research showing the retaliatory nature of terrorist groups.

The next article, by Lacey and Tsardakas (this issue), also deals with a real-world dataset and examines the timing of crime events. The authors first present an analysis of a dataset that consists of both major and minor crimes in England over a 30 month period, noting some interesting relationships between the two such as the observation that local maxima of serious crime generally coincide with that of more minor crime. Neglecting spatial effects, the authors then propose a novel three-component ODE-based model that allows for two distinct criminal types associated with either major or minor crime, together with a dynamic attractiveness field. Additionally, the authors examine a stochastic variant of this ODE model that provides a more realistic model for the "generation" of new criminals. The authors then devise a least-squares scheme to estimate model parameters from their data. Numerical solutions from both the deterministic and stochastic models are presented and compared with actual crime data for the Greater Manchester area. Agreement between simulations and actual data is encouraging. A preliminary statistical analysis of the data also supports the model's potential to describe crime.

In Tayebi, Glässer, Ester, and Brantingham (this issue), a random walk algorithm called CRIMETRACER is developed to predict the locations of future crimes due to the actions of individual offenders, on the basis of existing data on previous crime locations and offender profiles held by the police. In contrast to focussing on regions of crime hotspots, which is what most studies do, the aim of the CRIMETRACER algorithm is to predict crime locations in regions away from hotspot locations, which for certain real-world data sets can still represent nearly half of all urban crimes. To study such coldspot regions, the authors propose a probabilistic model of spatial behaviour of known offenders within their activity space, as informed by Crime Pattern Theory, as discussed in Brantingham *et al.* [Counterterrorism and Open Source Intelligence (U. K. Wiil, editor), (2011), Springer pp. 73–102]. This theory proposes that offenders, rather than venturing into unknown territory, frequently commit crimes by taking advantage of opportunities they encountered at or near to those places they typically visit as part of their routine (legitimate) activities. From a modelling perspective, the CRIMETRACER algorithm demonstrates how to incorporate individual-level offender heterogeneity into a spatio-temporal model of crime. Through the data mining of operational police records for crimes committed over a five-year period in Metro Vancouver, the CRIMETRACER algorithm is shown to be highly successful in predicting the locations of crimes in coldspot regions.

The next two articles in this special issue are related to the agent-based modelling (ABM) framework of residential burglary of Short *et al.* [Math. Models. Meth. Appl. Sci., 18, (2008) pp. 1249–1267], which models the influence on crime patterns of time-stable features of the urban environment and self-excitation due to offender activity, and its corresponding mean-field PDE reaction–diffusion limit. In Lloyd, Santitissadeekorn, and Short (this issue), a procedure is formulated and implemented for assimilating burglary data into the 1-D version of the PDE model of Short *et al.* of urban crime. In this data assimilation procedure, the authors aim is to construct the continuum attractiveness and criminal density fields described by the PDE model in such a way that they incorporate discrete "attack data" which consists of a set of criminal attack times and locations generated by the ABM. For the attractiveness field, a key simplifying step in the analysis is achieved by noting that the self-excitation in the model should respond to actual events and not to the averaged intensity with which events are expected. As such, the authors remove the excitation process from the evolution of the PDE and re-insert excitation as discrete events via the use of delta functions in the initial conditions. This leads to an attractiveness field that is completely uncoupled from the criminal density field. It is then supposed that the burglaries occur as a result of an inhomogeneous Poisson process, which leads to the construction of a likelihood function that can be maximized to estimate parameter values in the original PDE model that most closely fits the data.

The authors then favourably compare results from the original ABM and the assimilated PDE model, and provide a successful forecasting analysis of both stationary and non-stationary hotspots. In Camacho, Lee, and Smith (this issue), the authors extend the ABM of residential burglary of Short *et al.* [Math. Models. Meth. Appl. Sci., 18, (2008) pp. 1249–1267] to study the effect of various policing strategies for the prevention of urban crime and the elimination of crime hotspots. The main contribution of this paper is to incorporate into the ABM some of the more realistic constraints faced by police departments in preventing crime with limited resources. The authors first systematically discuss some previous modelling efforts for incorporating the effect of police into the ABM formulations, including peripheral interdiction which attempts to prevent the spread of hotspots, the cops-on-the-dots strategy of Jones *et al.* [Math. Models Meth. Appl. Sci., 20, (2010), pp. 1397–1423] of focussing considerable resources on known hotspot locations, and of random patrolling. From this ABM framework, a detailed comparison on the effect of crime hotspots is made for various realistic real-world options of patrol strategies including beat patrols, intermittent removal from patrolling, and various mixed patrolling strategies. The effect of a neighbouring deterrent effect on the criminals is also considered.

In Zipkin, Schoenberg, Coronges, and Bertozzi (this issue), the authors consider the problem of analysing temporal statistics of activity on a social network, in this case one involving email traffic. They are interested ultimately in the missing data problem – namely if information about a transmission is withheld (e.g. the sender or receiver) can one infer that information from the surrounding information? The authors use self-exciting point processes as a model for email traffic statistics and provide an overview of those models and how they might be estimated from data. They go on to study the missing data problem both in simluated datasets and a real-world dataset, IkeNet dataset of e-mails within a relatively small network of students over approximately a year. For the final task of missing data reconstruction, the authors provide a variety of methods, one of which is taken from related work on gang violence, and compare results between these methods for IkeNet and for several artificially constructed datasets.

In both historical and modern conflicts, geography typically plays a critical role in how interactions occur over time. However, the spatial distribution of adversaries has often been neglected in the development of mathematical models of such conflicts. In Baudins, Fry, Davies, Wilson, and Bishop (this issue), the authors extend the classical linear model of conflict escalation, known as the Richardson model, to include the effect of the spatial distribution of adversaries. More specifically, they propose an entropy-maximizing spatial interaction method for disaggregating the impact of space, employing a general notion of ‘threat’ between two adversaries. The resulting mathematical formulation is a large ODE system which incorporates the impact of space via a distance metric. By analysing this system using techniques from dynamical systems and bifurcation theory, the authors show, for certain model parameters sets, that as the level of conflict intensifies there can be a bifurcation from the state in which the conflict is distributed across many spatial locations to a new state where the conflict becomes much stronger at only a few locations that are in close proximity to each other. This paper also provides a comprehensive review of the literature of mathematical models of conflicts and develops the properties of the model, encapsulated by the ODE system, through increasingly complex cases.

The final two papers in this special issue involve a detailed analysis of some PDE models related to spatial-temporal dynamics of crime. In Berestycki and Rodriguez (this issue), the authors propose an exploratory mathematical model to understand the spatio-temporal dynamics of social outbursts, such as protests or riots, which are driven by the availability of information. The resulting model is a two-component reaction–diffusion system modelling the level of rioting or social upheaval and social tension in an environment where the spread of information is a key parameter. The non-linearity in this system models various factors such as social proximity, geography, and the rate of transition of information. The social tension is assumed to have a local diffusion part, but also has non-local effects. The main contribution of this paper is to analyse, for the local model, the existence of travelling waves of rioting that can exist in various parameter regimes, and to explore how these waves are affected by periodicity, or by abrupt changes in the availability of information, which can lead to pinning effects. Global in time existence results are also obtained for the Cauchy problem, which includes the non-local term. In Tse and Ward (this issue), the authors use methods of asymptotic analysis and bifurcation theory to analyse the existence, stability, and dynamics of localized regions of elevated crime, referred to as hotspots, for the original PDE model of residential burglary of Short *et al.* [Math. Models. Meth. Appl. Sci., 18, (2008), pp. 1249–1267]. By using path-following methods, hotspot equilibria are constructed on a finite one-dimensional domain and their bifurcation properties analysed as the diffusivity of criminals is varied. It is shown, both analytically and numerically, that new hotspots of criminal activity can be nucleated in low crime regions with inconspicuous crime activity gradient when the spatial extent of these regions exceeds a critical threshold. These nucleations are shown to occur near a saddle-node bifurcation point characterizing hotspot equilibria. For the time-dependent problem, a differential algebraic (DAE) system characterizing the slow dynamics of a collection of hotspots over long time-scales is derived, which shows that adjacent hotspots are mutually repulsive and cannot undergo merging.

As the articles included in this special issue show, the application of applied mathematics to problems of crime and security is growing. Since the 2010 issue of EJAM on crime and criminality, it is apparent that there has been both a substantive development of existing mathematical models of crime, a greater exploitation of real-world data, and a focus on new problems to which the techniques of applied mathematics have been brought to bare. Those illustrated in the current issue include offender spatial decision making, gang violence, insurgency, and cyber attacks. With the publication of this special issue, our dual aim was to provide a snapshot of the broad spectrum of problems in crime and security that are being studied and the variety of mathematical tools applied, as well as to encourage more researchers to become immersed in this exciting, and largely new, area for cross-disiplinary research.

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