In order to enhance the independent viability of high-orbit satellites, an X-ray pulsar-based navigation (XNAV)/celestial navigation system (CNS) integrated navigation method is proposed. An improved kinematic and static filter is derived to fulfil data fusion that can obtain an optimal estimation for global use. In the filter, unscented transformation is used to reduce linearization error, and the technique of separate-bias is used to reduce the impacts of systematic errors in XNAV measurements. The results of simulations have shown that the proposed navigation system can reach a positioning accuracy of less than 100 m, an improved performance over separate XNAV and CNS.

KEY WORDS
Celestial navigation systems (CNS) using stellar angle measurement have been developed for autonomous navigation. They operate by measuring the angle between the line of sight from a given satellite to Earth and the line of sight from a reference star, and can be applied to all outer space. The sensors required by the method are of small size, and the corresponding measurement sampling period is short. However, the navigation performance of the method degrades as the orbit altitude of the satellite increases, and for high-orbit satellites, the best positioning accuracy achievable is of the order of around 1 km (Fang et al., 2006).

X-ray pulsars are rapidly rotating neutron stars that are distant from Earth and generate periodic electromagnetic radiation in the X-ray band (Graven et al., 2008a). The spin periods of X-ray pulsars are highly stable over long periods, and several millisecond pulsars can even match the quality of current atomic clocks for accuracy (Tian et al., 2012). Adopting pulsars as accurate beacons to fix the positions of Earth-orbiting satellites has been investigated since the 1970s (Downs, 1974; Hanson, 1996). In particular, the Advanced Concepts Team (ARIADNA) of the European Space Agency (ESA) studied the feasibility of spacecraft navigation relying on pulsar timing information in 2004, and the Defense Advanced Research Projects Agency (DARPA) of the United States initiated a program called X-ray Source-based Navigation for Autonomous Position Determination in 2005 (Sala et al., 2004; Graven et al., 2008b).

The signals from X-ray pulsars can be detected over the whole of outer space. It has been found that the navigation performance of an X-ray pulsar-based navigation system (XNAV) would not be significantly affected by the increase of orbit altitude, and thus XNAV is a good candidate to provide reliable and stable position information of high-orbit satellites. However, the flux of pulsars is usually very low, and the pulsed signal from pulsars is weak and discrete. A pulse Time of Arrival (TOA) at a satellite can only be calculated after a period of observation, which usually lasts for several minutes. Furthermore, the navigation error accumulating during the period of observation cannot be ignored, and it would degrade the performance of XNAV. Moreover, an X-ray detector with a large area, which was usually set to be 1 m², was assumed to be adopted to enhance the Signal Noise Ratio (SNR) of the signal detected (Xiong et al., 2009). However, it is impracticable for most satellites to carry three such X-ray detectors to fix position.

Using current techniques, many systematic errors exist in XNAV. Firstly, the direction of the pulsar cannot be determined with high accuracy (Liu et al., 2010a). Moreover, for Earth-orbiting satellites, in order to fulfil the time transfer equation, which is the principle of XNAV, the position of Earth predicted by planetary ephemeris such as DE405 should be adopted, and the error existing in the planetary ephemeris would affect the performance of XNAV (Wang et al., 2013a). Furthermore, considering that the pulse TOA is recorded by a satellite-borne atomic clock, the error in this clock is also a factor affecting the navigation performance. These factors have been separately analysed previously.

As XNAV and CNS are complementary to each other, the concept of integrated navigation using pulsar and stellar angle measurements has been proposed (Liu et al., 2010b; Wang et al., 2013b). In the work of Liu, XNAV and CNS work separately, and their results are fused by a federated unscented Kalman filter (FUKF). In addition, CNS operates during a period of pulsar observation, and only one X-ray detector with an area of 1 m² is employed. However, in the case that only a pulsar is observed over the whole navigation process, the observability of XNAV is poor, and it would deliver
an unsatisfactory navigation performance (Zheng et al., 2008). Furthermore, the output of the federated filter is not globally optimal, because the two navigation subsystems use the same dynamic model and their results are correlative (Yang, 2003; Yang, 2004a). In our prior work, a joint unscented Kalman filter (JUKF) was used to fuse the original measurements. Although a JUKF can reach a globally optimal estimation, the computation burden is high, numerical conditioning difficulties may result and only the impact of clock error is considered.

In this paper, a new autonomous navigation method for high-orbit satellites is proposed that integrates XNAV and CNS measurements. Three X-ray detectors with small areas are adopted and an improved kinematic and static filter is derived to obtain globally optimal navigation results. In addition, the factors affecting the performance of XNAV will be handled by the technique of separate-bias.

The integrated navigation system is described in Section 2, the improved kinematic and static filter is presented in Section 3, simulations are described in Section 4 and followed by the conclusion in Section 5.

2. OVERALL DESIGN OF INTEGRATED NAVIGATION SYSTEM

2.1 Dynamic Model. An Earth-centred inertial coordinate system is selected, and the dynamic model of a high-orbit satellite can be described as

\[
\begin{bmatrix}
\dot{r} \\
\dot{v}
\end{bmatrix} =
\begin{bmatrix}
v \\
\alpha
\end{bmatrix} +
\begin{bmatrix}
w_r \\
w_v
\end{bmatrix}
\]

where \( r \) is the position vector of satellite, \( v \) is the velocity vector of satellite, \( w = [w_r^T, w_v^T]^T \) is the process noise which can be assumed as a zero-mean Gaussian white noise process, and \( a = a_{TB} + a_{NS} + a_T + a_{H.O.T} \) is the acceleration of satellite about Earth including the following terms (Battin, 1999).

\( a_{TB} = -\mu_E \frac{r}{||r||^3} \) is the two-body gravitational acceleration of Earth, \( \mu_E \) is the gravitational coefficient of Earth, and \( ||\cdot|| \) denotes the magnitude of vector.

\( a_{NS} = \partial U_{NSE}/\partial r \) is the non-spherical perturbation acceleration of Earth, and \( U_{NSE} \) can be expressed as

\[
U_{NSE} = -\frac{\mu_E}{||r||} \sum_{n=2}^{\infty} \left( \frac{R_e}{||r||} \right)^n J_n P_n \sin \phi \\
- \frac{\mu_E}{||r||} \sum_{n=2}^{\infty} \sum_{m=1}^{n} \left( \frac{R_e}{||r||} \right)^n J_{n,m} P_{nm} \sin \phi \cos m(\lambda - \lambda_{n,m})
\]

where \( R_e \) is the radius of Earth, \( \phi \) and \( \lambda \) are the latitude and longitude respectively, \( P_n \) and \( P_{nm} \) are the Legendre polynomials, \( J_n \) is the zonal coefficient, \( \lambda_{n,m} \) is the tesseral harmonic term, and \( J_{n,m} \) is the tesseral harmonic coefficient.

\( a_T = \sum_{i=1}^{2} \mu_i [r_i/||r_i||^3 - (r - r_i)/||r - r_i||^3] \) is the third-body gravitational perturbation acceleration including the impacts of Sun and Moon, \( \mu_i \) is the gravitational coefficient of the \( i \)th celestial body, and \( r_i \) is the position vector of the \( i \)th celestial body with respect to Earth.

\( a_{H.O.T} \) represents all higher-order terms that may affect acceleration but are nominally considered negligible compared to the remaining effects.
2.2 XNAV Measurement Model. It is of note that the time coordinate system of the whole paper is assumed as the Barycentric Dynamical Time (TDB). Through processing the pulsed signal from the $i$th pulsar, the pulse TOA at the satellite, $t_S^i$, can be measured, and its corresponding arrival time at the Solar System Barycenter (SSB), $t_{SSB}^i$, can be predicted by the pulse timing model. Considering the geometric and relativistic effects, the time transfer equation can be expressed as (Sheikh et al., 2006):

$$
t_{SSB}^i = t_S^i + \frac{n^i \cdot r_S}{c} + \frac{1}{2cD_0} \left[ (n^i \cdot r_S)^2 - \| r_S \|^2 + 2(n^i \cdot b)(n^i \cdot r_S) - 2(b \cdot r_S) \right] + \frac{2\mu_S}{c^3} \ln \left| \frac{n^i \cdot r_S + \| r_S \|}{n^i \cdot b + \| b \|} + 1 \right| \tag{3}
$$

where $n^i$ is the direction vector of the $i$th pulsar, $r_S$ is the position vector of the satellite relative to the SSB, $\mu_S$ is the gravitational coefficient of Sun, $b$ is the position of the SSB with respect to Sun, $c$ is the speed of light, and $D_0$ is the distance between the $i$th pulsar and Sun.

For an Earth-orbiting satellite, $r_S$ can be obtained by

$$
r_S = r + r_E \tag{4}
$$

where $r_E$ is the position vector of Earth relative to the SSB.

It can be seen from Equation (3) that the systematic errors including pulsar direction error, $\delta n^i$, Earth ephemeris position error, $\delta r_E$, and clock error, $\delta t_S$, are the main factors that would significantly affect the performance of XNAV. Therefore, those systematic errors should be taken into consideration during the derivation of the XNAV measurement model.

Then, Equation (3) can be transformed into

$$
t_{SSB}^i = t_S^i + \frac{\hat{n}^i \cdot (r + \hat{r}_E)}{c} + \frac{1}{2cD_0} \left[ (\hat{n}^i \cdot (r + \hat{r}_E))^2 - \| r + \hat{r}_E \|^2 + 2(\hat{n}^i \cdot b)(\hat{n}^i \cdot (r + \hat{r}_E)) - 2(b \cdot (r + \hat{r}_E)) \right] + \frac{2\mu_S}{c^3} \ln \left| \frac{\hat{n}^i \cdot (r + \hat{r}_E) + \| r + \hat{r}_E \|}{\hat{n}^i \cdot b + \| b \|} + 1 \right| \tag{5}
$$

where $\hat{n}^i$ is the measured direction vector of $i$th pulsar, $\hat{r}_E$ is the predicted position of Earth, which can be obtained by planetary ephemeris such as DE405, and $B_1^i(\delta r_E)$ and $B_2^i(\delta n^i)$ are the system biases caused by Earth ephemeris error and the error in $i$th pulsar direction vector.

Ignoring the second and higher order terms, $B_1^i(\delta r_E)$ and $B_2^i(\delta n^i)$ can be modeled by the following equations.

$$
B_1^i(\delta r_E) = \hat{n}^i \cdot \delta r_E + \frac{1}{D_0} \left[ -(r + \hat{r}_E) \cdot \delta r_E + [\hat{n}^i \cdot (r + \hat{r}_E)](\hat{n}^i \cdot \delta r_E) - b \cdot \delta r_E \right] + (\hat{n}^i \cdot b)(\hat{n}^i \cdot \delta r_E) + \frac{2\mu_S}{c^3} \left[ \frac{\hat{n}^i \cdot \delta r_E + [(r + \hat{r}_E) \cdot \delta r_E]/\| r + \hat{r}_E \|}{\hat{n}^i \cdot (r + \hat{r}_E) + \| r + \hat{r}_E \| + \hat{n}^i \cdot b + \| b \|} \right] \tag{6}
$$
The state model of \( w_B \) where measurement noise \( \eta \) was modelled as slow time-varying processes (Wang et al., 2013a; Liu et al., 2010a). Therefore, the impacts of system bias \( \tilde{r}_S \) and the term, \( r + \tilde{r}_E \), in Equation (6) can be approximated by \( \tilde{r}_E \).

In earlier works, the impacts of \( B_1(\delta r_E) \) and \( B_2(\delta n^i) \) have been modelled as slow time-varying processes (Wang et al., 2013a; Liu et al., 2010a). Moreover, for high-orbit satellites, the pulsar observation period is usually several minutes, and the drift of clock error can be ignored (Chen et al., 2011). Therefore, the impacts of \( B_1(\delta r_E) \), \( B_2(\delta n^i) \), and \( \delta t_S^i \) can be handled together.

Assume

\[
B^i = B_1^i(\delta r_E) + B_2^i(\delta n^i) + c\delta t_S^i
\]

The state model of \( B^i \) can be described by

\[
\dot{B}^i = 0 + w_B
\]

where \( w_B \) is the process noise.

Assume the measurement \( Z_p = [c(t_{SSB}^1 - t_S^1), \ldots, c(t_{SSB}^i - t_S^i), \ldots, c(t_{SSB}^N - t_S^N)]^T \), the measurement noise \( V_p = [v^1, \ldots, v^i, \ldots, v^N]^T \), and the system bias \( B_p = [B^1, \ldots, B^i, \ldots, B^N]^T \), where \( 1 \leq i \leq N \) and \( N \) is the number of observed pulsars. The measurement model can be presented as:

\[
Z_p = h_p(x) + B_p + V_p
\]
where $h_p(x) = [h_p^1(x), \ldots, h_p^i(x), \ldots, h_p^N(x)]^T$, and $h_p^i(x)$ is the measurement equation observing $i$th pulsar. The expression of $h_p^i(x)$ is:

$$h_p^i(x) = \tilde{n}_i \cdot (r + \tilde{r}_E) + \frac{1}{2D_0^i} \left( [\tilde{n}_i \cdot (r + \tilde{r}_E)]^2 - \| r + \tilde{r}_E \|^2 + 2(\tilde{n}_i \cdot b)(\tilde{n}_i \cdot (r + \tilde{r}_E)) \right) - 2[b \cdot (r + \tilde{r}_E)] + \frac{2\mu_S}{c^2} \ln \left( \frac{\tilde{n}_i \cdot (r + \tilde{r}_E) + \| r + \tilde{r}_E \|}{\| \tilde{n}_i \cdot b + \| b \|} + 1 \right)$$

In Equation (11), $V_p$ can be modeled as a zero-mean Gauss white noise whose standard deviation can be given by (Sheikh, 2005):

$$\sigma_{TOA} = \frac{W \sqrt{[B_X + F_X(1 - p_f)]d + F_X p_f}}{2F_X p_f \sqrt{A t_m}}$$

where $W$ is the width of the pulse, $B_X$ is the X-ray background radiation flux, $F_X$ is the X-ray flux of the pulsar, $p_f$ is the pulsed fraction of the pulsar, $d$ is the ratio of the pulse width to the pulse period $P$, $A$ is the area of the X-ray detector, and $t_m$ is the period of XNAV measurement.

2.3 CNS Measurement Model. Figure 1 shows the stellar angle measurement, $\alpha$. In Figure 1, $s$ is the direction vector of the reference star with respect to the satellite. Then, the CNS measurement is (Qi et al, 2006):

$$Z_{st} = h_{st}(x) + v_\alpha = \arccos \left( -\frac{r \cdot s}{\| r \|} \right) + v_\alpha$$

where $v_\alpha$ is a zero-mean Gaussian measurement noise with covariance $R_{st}$, which is determined by the accuracies of measurement sensors.

3. IMPROVED KINEMATIC AND STATIC FILTER

3.1 Review of Kinematic and Static Filter. To avoid the problem that the output of a federated filter is suboptimal, Yang (2003) proposed an information fusion filter
named “kinematic and static filter” and proved that the output of the proposed filter is globally optimal. The kinematic and static filter is composed of a kinematic filter and a static filter. The kinematic filter works based on the data from the dynamic model and the measurement from one measurement sensor, and the static filter works based on the output of the kinematic filter and the measurements from the other sensors. Therefore, the data from the dynamic model is just used once and the output of the static filter is globally optimal. For the case that two sensors are used, Figure 2 shows the structure of the kinematic and static filter.

As is shown in Figure 2, the predicted satellite state at time $t_k$, $x_k^-$, and the corresponding error covariance matrix, $P_k^-$, are provided by the dynamic model. The measurement from sensor 1 is $y_{1,k}$, whose error covariance matrix is $R_{1,k}$. The solution of the kinematic filter can be derived as:

$$x_{1,k}^+ = [(P_k^-)^{-1} + H_{1,k}^T(R_{1,k})^{-1}H_{1,k}]^{-1}[(P_k^-)^{-1}x_k^- + H_{1,k}^T(R_{1,k})^{-1}y_{1,k}]$$  \(\text{(15)}\)

$$P_{1,k}^+ = (I - K_{1,k}H_{1,k})P_k^-$$  \(\text{(16)}\)

where

$$K_{1,k} = P_k^-H_{1,k}^T(H_{1,k}P_k^-H_{1,k}^T + R_{1,k})^{-1}$$  \(\text{(17)}\)

In Equations (15)–(17), $H_{1,k}^T$ is the corresponding measurement matrix of $y_{1,k}$.

At the static filter stage, the satellite state predicted by the dynamic model is no longer used. Instead the result of the kinematic filter, $x_{1,k}^+$ and $P_{1,k}^+$, are utilized to fuse with the measurement from sensor 2, $y_{2,k}$, whose error covariance matrix is $R_{2,k}$.

The state model in static filter is

$$x_{2,k}^- = x_{1,k}^+ \quad P_{2,k}^- = P_{1,k}^+$$  \(\text{(18)}\)

Based on Equation (18), the result of the static filter stage can be presented as:

$$x_{2,k}^+ = [(P_{2,k}^-)^{-1} + H_{2,k}^T(R_{2,k})^{-1}H_{2,k}]^{-1}[(P_{2,k}^-)^{-1}x_{2,k}^- + H_{2,k}^T(R_{2,k})^{-1}y_{2,k}]$$  \(\text{(19)}\)

$$P_{2,k}^+ = (I - K_{2,k}H_{2,k})P_{2,k}^-$$  \(\text{(20)}\)

where

$$K_{2,k} = P_{2,k}^-H_{2,k}^T(H_{2,k}P_{2,k}^-H_{2,k}^T + R_{2,k})^{-1}$$  \(\text{(21)}\)
Considering only two sensors are adopted, $x_{2,k}^+$ and $P_{2,k}^+$ are the final estimated state and its corresponding error covariance matrix at time $t_k$.

3.2 Improvement on Kinematic and Static Filter. As is shown in section 3.1, the kinematic and static filter can provide a more flexible structure to fuse the data from the dynamic model and that from different sensors. However, it requires that the sampling period of sensors should be the same; in practice, the period of pulsar observation is usually much longer than that of CNS measurement. Furthermore, the kinematic and static filter is designed for a linear or linearized system; the dynamic model, the XNAV measurement model and the CNS measurement model are all nonlinear equations. Moreover, to enhance the performance of an integrated navigation system, the impact of systematic error in XNAV must be reduced. The common method of reducing systematic error is the augmented-state method, but it increases the computation burden. Therefore, it is necessary to improve the kinematic and static filter.

3.2.1 Improvement with Different Sampling Periods. Suggested by the structure of the kinematic and static filter, the process velocity of the static filter can be slower than that of the kinematic filter. Considering that the sampling period of CNS measurement is much shorter than that of XNAV measurement, XNAV measurement should be handled by the static filter. Otherwise, the CNS measurements obtained during one pulsar observation period are not used in navigation but just wasted. Then, we improve the kinematic and static filter to be an integrated filter with two different sampling periods. Figure 3 shows the structure of the integrated filter.

3.2.2 Improvement with Unscented Transformation. Unscented transformation (UT) is a method of approximating the way that the mean and covariance of a random variable changes when the random variable undergoes a nonlinear transformation (Julier et al., 2000). The unscented Kalman filter (UKF), which utilizes a set of sigma points produced by an unscented transformation to capture the mean and covariance of the state, would not cause linearization error, and can be used to improve the kinematic and static filter.

Thus the kinematic and static filters can both be improved in the form of a UKF. Assume that the system model of integrated navigation system is:

$$x_k = f(x_{k-1}) + w_k$$  \hspace{1cm} (22)
\[
\begin{bmatrix}
Z_{st,k} \\
Z_{p,k}
\end{bmatrix} = \begin{bmatrix}
h_{st}(x_k) \\
h_p(x_k) + B_p
\end{bmatrix} + \begin{bmatrix}
v_{st,k} \\
v_{p,k}
\end{bmatrix}
\] (23)

where
\[
E(w_k) = 0 \quad E(w_kw_k^T) = Q_k
\] (24)
\[
E(v_{st,k}) = 0 \quad E(v_{st,k}v_{st,k}^T) = R_{st,k}
\] (25)
\[
E(v_{p,k}) = 0 \quad E(v_{p,k}v_{p,k}^T) = R_{p,k}
\] (26)

In Equation (22), \( x_k \) is the state of the satellite at time \( t_k \). In Equation (23), \( Z_{st,k} \) and \( Z_{p,k} \) are the measurements of CNS and XNAV at time \( t_k \), \( h_{st}(x_k) \) and \( h_p(x_k) \) are the measurement equations of CNS and XNAV at time \( t_k \), and \( v_{st,k} \), \( v_{p,k} \) are the corresponding measurement noises.

Assume that the estimated satellite state and its corresponding error covariance matrix at time \( t_{k-1} \) are:
\[
\hat{x}_{k-1} = E(x_{k-1}) \quad P_{k-1} = E[(x_{k-1} - \hat{x}_{k-1})(x_{k-1} - \hat{x}_{k-1})^T]
\] (27)

Then, the improved filter is given by the following equations.

**Kinematic filter:**

Step 1. Structure of sigma points and weights

The set of sigma points, \( \{\chi_{i,k-1} | i = 0, \ldots, 2n, k \geq 1 \} \), is
\[
\begin{align*}
\chi_{0,k-1} &= \hat{x}_{k-1} \\
\chi_{i,k-1} &= \hat{x}_{k-1} + \sqrt{n + \xi} \cdot (\sqrt{P_{k-1}})_i, \quad i = 1, 2, \ldots, n \\
\chi_{i+n,k-1} &= \hat{x}_{k-1} - \sqrt{n + \xi} \cdot (\sqrt{P_{k-1}})_i, \quad i = n + 1, n + 2, \ldots, 2n
\end{align*}
\] (28)

where \( n \) is the number of components contained in state vector, \( \xi = \alpha^2(n + \kappa) \), in which \( \alpha \) is used to control the distribution of sigma points and its value is between 0 and 1, besides \( \kappa \) equals 3-\( n \), and \( \sqrt{P_{k-1}} \) is the Cholesky factor of \( P_{k-1} \).

The weights of mean values and covariance values are
\[
\begin{align*}
\omega_0^m &= \frac{\xi}{n + \xi} \\
\omega_0^c &= \frac{\xi}{n + \xi} + (1 - \alpha^2 + \beta) \\
\omega_i^m &= \omega_i^c = \frac{1}{2(n + \xi)}
\end{align*}
\] (29)

where \( \beta \) is the parameter related with the prior distribution of the state and it is usually set to be 2 in the case of Gaussian distribution.

Step 2. Time update
\[
\chi_{i,k|k-1} = f(\chi_{i,k-1}) \quad x_k^- = \sum_{i=0}^{2n} \omega_i^m \chi_{i,k|k-1}
\] (30)
\[
P_k^- = \sum_{i=0}^{2n} \omega_i^c [\chi_{i,k|k-1} - x_k^-][\chi_{i,k|k-1} - x_k^-]^T + Q_k
\] (31)

where \( x_k^- \) and \( P_k^- \) are the predicted satellite state and its corresponding error covariance matrix at time \( t_k \).
Step 3. Measurement update

\[ Z_{i,k|k-1}^u = h_s(x_{i,k|k-1}) \]
\[ Z_{st,k}^- = \sum_{i=0}^{2n} \omega_i^m Z_{i,k|k-1}^u \]  \hspace{1cm} (32)

\[ P_{st,k} \hat{z}_{st,k} = \sum_{i=0}^{2n} \omega_i^s [Z_{i,k|k-1}^u - Z_{st,k}^-] [Z_{i,k|k-1}^u - Z_{st,k}^-]^T + R_{st,k} \]  \hspace{1cm} (33)

\[ P_{st,k} \hat{z}_{st,k} = \sum_{i=0}^{2n} \omega_i^s [Z_{i,k|k-1} - x_k^-] [Z_{i,k|k-1} - x_k^-]^T \]  \hspace{1cm} (34)

\[ x_{st,k}^+ = x_{st,k}^- + K_{st,k} (Z_{st,k} - \hat{Z}_{st,k}^-) \]
\[ P_{st,k}^- = P_{st,k}^- - K_{st,k} P_{st,k} \hat{z}_{st,k} \hat{z}_{st,k} K_{st,k}^T \]  \hspace{1cm} (35)

\[ K_{st,k} = P_{st,k} \hat{z}_{st,k} P_{st,k}^{-1} \]  \hspace{1cm} (36)

where \( x_{st,k}^+ \) and \( P_{st,k}^- \) are the results of the kinematic filter at time \( t_k \).

Static filter:

In the stage of static filter, \( B_p \) in Equation (23) is ignored and it will be handled in section 3.2.3.

Based on Equation (18), we have

\[ x_{p,k}^- = x_{st,k}^+ \]
\[ P_{p,k}^- = P_{st,k}^+ \]  \hspace{1cm} (37)

Another UT is applied by

\[ \xi_{0,k-1} = x_{p,k}^- \]
\[ \xi_{i,k-1} = x_{p,k}^- + \sqrt{n + \zeta} \cdot \left( P_{p,k}^- \right)^i \]
\[ \xi_{i+n,k-1} = x_{p,k}^- - \sqrt{n + \zeta} \cdot \left( P_{p,k}^- \right)^i \]

where \( \sqrt{P_{p,k}^-} \) is the Cholesky factor of \( P_{p,k}^- \).

The static filter only contains the measurement update step, and it can be shown as

\[ Z_{i,k|k-1}^p = h_p(\xi_{i,k|k-1}) \]
\[ Z_{p,k}^- = \sum_{i=0}^{2n} \omega_i^m Z_{i,k|k-1}^p \]  \hspace{1cm} (39)

\[ P_{p,k} \hat{z}_{p,k} = \sum_{i=0}^{2n} \omega_i^s [Z_{i,k|k-1}^- - Z_{p,k}^-] [Z_{i,k|k-1}^- - Z_{p,k}^-]^T + R_{p,k} \]  \hspace{1cm} (40)

\[ P_{p,k} \hat{z}_{p,k} = \sum_{i=0}^{2n} \omega_i^s [\xi_{i,k|k-1} - x_{p,k}^-] [Z_{i,k|k-1}^- - Z_{p,k}^-]^T \]  \hspace{1cm} (41)

\[ x_{p,k}^+ = x_{p,k}^- + K_{p,k} (Z_{p,k} - \hat{Z}_{p,k}^-) \]
\[ P_{p,k}^+ = P_{p,k}^- - K_{p,k} P_{p,k} \hat{z}_{p,k} \hat{z}_{p,k} K_{p,k}^T \]  \hspace{1cm} (42)

\[ K_{p,k} = P_{p,k} \hat{z}_{p,k} P_{p,k}^{-1} \]  \hspace{1cm} (43)

where \( x_{p,k}^+ \) and \( P_{p,k}^- \) are the outputs of the static filter at time \( t_k \).

Although the JUKF and the kinematic and static filter with UT can both achieve a globally optimal estimation, the computation burdens are different. Assume that the dimensions of satellite state, CNS measurements, and XNAV measurements are \( n \), \( m_{s,t} \), and \( m_p \), respectively. We select the times of multiplication processed in a filter as
an index to scale the computation cost by a filter. The indices of JUKF and the kinematic and static filter with UT are listed in Table 2.

To obtain a desired navigation performance, \( m_{si} + m_p \) should be greater than 3. In this case, the index of the kinematic and static filter with UT is less than that of the JUKF. Furthermore, compared with the JUKF, the kinematic and static filter with UT handles smaller matrices. Therefore, the kinematic and static filter with UT can reduce the computation burden and enhance the numerical stability.

3.2.3. Improvement with Separate-bias Estimation. The system bias \( B_p \) is handled in this subsection.

A common way to reduce the impact of systematic error is augmenting it into the state of the navigation system. However, the technique of augmented-state would cause the filter implementation to require computation with large matrices, which increases the likelihood of numerical conditioning difficulties (Friedland, 1969). To ensure the numerical stability of the implementation of the kinematic and static filter, we adopt the technique of separate-bias proposed by Friedland to reduce the impacts of systematic errors.

The essence of the technique of separate-bias is to decouple the augmented filter into two parallel filters. The first filter, called the bias-free filter, works based on the assumption that the system bias does not exist. The second filter, called the bias filter, works to estimate the bias. Finally, the outputs of the bias-free filter and bias filter can be used to reconstruct the original system state.

In the part of the static filter provided in section 3.2.2, the impact of \( B_p \) is ignored and the static filter can be used as the bias-free filter. The bias filter and the final estimation result are given as follows.

Assume that the estimated system bias and its corresponding error covariance matrix at time \( t_{k-1} \) are

\[
\hat{B}_{p,k-1} = E(B_{p,k-1}) \quad P_{k-1} = E[(B_{p,k-1} - \hat{B}_{p,k-1})(B_{p,k-1} - \hat{B}_{p,k-1})^T]
\]  

(44)

The part of system bias estimation is shown by the following equations.

\[
B_{p,k}^- = \hat{B}_{p,k-1} \quad P_{B,k}^- = \hat{P}_{B,k-1} + Q_{B,k-1}
\]  

(45)

\[
P_{B,\tilde{Z},k} = P_{B,k}^T \quad P_{B,Z,k} = SP_{B,k}^T + P_{p}\tilde{Z}_{p,k} \tilde{Z}_{p,k}^T
\]  

(46)

\[
\hat{B}_{k} = B_{k}^- + K_{B,k}(Z_{p,k} - \hat{Z}_{p,k} - SB_{k}) \quad \hat{P}_{B,k} = P_{B,k}^- - K_{B,k}P_{B,Z,k} K_{B,k}^T
\]  

(47)

\[
K_{B,k} = P_{B,\tilde{Z},k} P_{B,\tilde{Z},k}^{-1} \quad K_{B,mp} = P_{B,\tilde{Z},k} P_{B,\tilde{Z},k}^{-1} \quad K_{B,sp} = P_{B,\tilde{Z},k} P_{B,\tilde{Z},k}^{-1}
\]  

(48)
And then, the final estimations at time $t_k$ are:

$$\hat{x}_k = x^+_{p,k} + V_k \hat{B}_{p,k} \quad \hat{P}_k = P^+_{p,k} + V_k \hat{B}_{p,k} V_k^T$$ (49)

In Equations (45)–(49),

$$S_k = H_k U_k + I_{m_p \times m_p} \quad V_k = U_k - K_{p,k} S_k$$ (50)

$$U_k = V_{k-1} \quad H_k = \left. \frac{\partial h_p(x)}{\partial x} \right|_{x = x^+_{p,k}}$$ (51)

where $I$ is a unit matrix.

Considering that the dimension of system bias is the same as that of XNAV measurement, the indices of the augmented-state method and the separate-bias method are shown in Table 3.

As shown in Table 3, based on the kinematic and static filter with UT, the separate-bias method costs less computation burden and handles smaller matrices compared to the augmented-state method.

4. SIMULATION AND RESULTS. To verify the performance of the proposed integrated navigation system, some simulations are described in this section.

Three existing satellite orbits of NTS 2, DOGE 1, and ATS 1 are investigated, and the initial orbital elements of the true orbits are shown in Table 4. The navigation pulsars adopted are selected from the X-ray pulsar database provided by Microcosm Incorporated, and the parameters are listed in Table 5. Fifty stars, distributed on the whole celestial sphere (visual magnitude $\leq 2^m$), are selected as reference stars. The data of star and orbit are generated by the Satellite Tool Kit (STK).
Assume that the navigation process lasts for 7 days, and the position errors of pulsars are 0.001 arcsec. The XNAV measurement noise standard deviation is determined by Equation (13), with the specified parameters \( A = 0.3 \, \text{m}^2 \), \( t_m = 0.5 \, \text{h} \), and \( B_X = 0.005 \, \text{ph/cm}^2/\text{s} \). The accuracy of star sensor is \( 3''(1\sigma) \), and the accuracy of the optical camera is \( 0.05^\circ(1\sigma) \).

For the three satellites listed in Table 4, the percentages of the time interval when the pulsars are visible for the whole navigation period are provided in Table 6. In the visibility analysis, the impacts of Sun, Moon, and Earth are considered.

It can be seen from Table 6 that most of the pulsars are visible during the whole navigation process and the visibility grows as the orbit altitude increases. The result is consistent with the analysis provided by Mao (2009). Among the pulsars that are visible during the whole navigation process, we selected navigation pulsars based on the following criteria: (1) navigation pulsars should not be binary pulsars whose measurement models are not considered in the paper; (2) the distributions of the navigation pulsars should not be close; (3) the flux of navigation pulsar should be as large as possible. Thus PSR B0531+21, PSR B1821-24, and PSR B1509-58 are selected as the navigation pulsars adopted in the following simulations.

Assume that the navigation errors existing in the initial state of the satellite are [1 km, 1 km, 1 km] and [1 m/s, 1 m/s, 1 m/s], and the clock error is 1 \( \mu \text{s} \). Furthermore,
DE405 ephemeris is used to predict the Earth position used in the navigation process, and the error contained in DE405 is approximated by the difference between DE405 and DE421, which was created after DE405. The covariance of the state process noise $Q = \text{diag}(q_1^2, q_1^2, q_1^2, q_2^2, q_2^2, q_2^2)$, where $q_1$ is $2 \times 10^{-5}$ m, $q_2$ is $6 \times 10^{-4}$ m/s (Xiong et al., 2009). The covariance of the bias process noise $Q_B = \text{diag}(q_3^2, q_3^2, q_3^2)$, where $q_3$ is chosen as 0·01 m.

Figure 4 shows the performance comparison of the integrated navigation system, XNAV, and CNS. As is illustrated in Figure 4, the three navigation methods can all converge for the three satellites, but the integrated navigation system performs best. Table 7 gives the position accuracy comparison among the three navigation methods over 300 Monte Carlo trials. For the three satellites, we can see that the positioning RMS (Root Mean Square) of the integrated navigation systems are all less than 100 m but the RMS of the other two methods are greater than 200 m. Thus the integrated navigation system behaves better than XNAV and CNS. The reason for this is that the integrated navigation system works by fusing the XNAV measurement and CNS measurement. Moreover, it can be learnt from Figure 4 and Table 7 that the performance of the integrated navigation system is little affected by the increment of the orbit altitude. Thus the proposed integrated navigation system is suitable for high-orbit satellite autonomous navigation.
From the above results, we can also see that the NTS 2 has the lowest position accuracy among three satellites. To demonstrate the effectiveness of the proposed integrated navigation system, only the result of NTS 2 is considered in the remainder of this paper.

Figure 5 depicts the estimated position error and its corresponding $3\sigma$ positioning outlier for the integrated navigation system and XNAV. We can see that the errors of the estimated positions of the proposed integrated navigation system are smaller than $3\sigma$ during the whole navigation process, but some of the errors of estimated positions of XNAV are larger than that boundary. This means that the integrated navigation system can reduce the impacts of the systematic errors in XNAV.

Only the impact of satellite-borne clock error is considered. Figure 6 shows the navigation performance with different satellite-borne clock errors. We can see that the positioning accuracies of two navigation systems decline as the satellite-borne clock error increases but the integrated navigation system outperforms with the same clock error. This means that the performance of the proposed navigation system is little affected by the increment of systematic error. The impacts of pulsar direction error and Earth ephemeris error can also be analysed in the same way.

The impact of linearization error that might occur in the kinematic and static filter is investigated. The navigation schemes designed are listed in Table 8, and the
corresponding positioning accuracies over 300 Monte Carlo trials are provided in Table 9.

As shown in Table 9, linearization error would degrade the navigation performance. Compared with the linearization of measurement models, the linearization of the dynamic model would more easily degrade navigation performance. Therefore, to ensure the performance of the integrated navigation system, the dynamic model should not be linearized, but the XNAV measurement model and CNS measurement model can be linearized to reduce computation burden if the corresponding navigation performances are acceptable.

Finally, without consideration of systematic errors, Figure 7 presents the performance comparison between the integrated navigation system using three X-ray detectors. The area of each detector is 0.3 m², and the integrated navigation system using one X-ray detector with area of 1 m², which observes PSR B0531 + 21 during the whole navigation process. From Figure 7, it can be seen that the integrated navigation system using three small detectors outperforms the integrated navigation system using one detector with area of 1 m². Although it can be seen from Equation (13) that the

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Table 8. Navigation schemes adopted to analyze the impact of linearization error.

<table>
<thead>
<tr>
<th>Number</th>
<th>Dynamic model</th>
<th>CNS measurement model</th>
<th>XNAV measurement model</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Nonlinear</td>
<td>Nonlinear</td>
<td>Nonlinear</td>
</tr>
<tr>
<td>2</td>
<td>Nonlinear</td>
<td>Linearized</td>
<td>Linearized</td>
</tr>
<tr>
<td>3</td>
<td>Linearized</td>
<td>Nonlinear</td>
<td>Nonlinear</td>
</tr>
<tr>
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<td>Linearized</td>
<td>Linearized</td>
<td>Linearized</td>
</tr>
<tr>
<td>5</td>
<td>Linearized</td>
<td>Nonlinear</td>
<td>Nonlinear</td>
</tr>
<tr>
<td>6</td>
<td>Nonlinear</td>
<td>Linearized</td>
<td>Linearized</td>
</tr>
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</tr>
<tr>
<td>8</td>
<td>Linearized</td>
<td>Linearized</td>
<td>Linearized</td>
</tr>
</tbody>
</table>

Figure 6. Error of estimated positions with different satellite-borne clock error.
measurement noise would grow if the area of the detector decreases, three detectors can receive the signals from three pulsars at the same time, and can provide a better geometric structure compared to using only one detector. The impact of the increment of measurement noise can be offset by the performance improvement of a better geometric structure.

5. CONCLUSION. In this paper, a high-orbit satellite autonomous navigation method by integrating X-ray pulsar-based navigation (XNAV) and celestial navigation system (CNS) is proposed. An improved kinematic and static filter is derived to fuse data. In the filter, the unscented transformation (UT) is used to reduce linearization error, and the separate-bias technique is adopted to reduce the impact of systematic errors contained in XNAV, considering that the sampling periods of the sensors are different. Compared with the XNAV and CNS, the proposed navigation system has an improved performance and can reduce the impact of systematic error effectively. The proposed navigation system is suitable for high-orbit satellite autonomous navigation.

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REFERENCES


In this appendix, we demonstrate that the result of the federated filter is suboptimal and that the result of the kinematic and static filter is globally optimal.

A1. THE RESULT OF THE FEDERATED FILTER IS SUBOPTIMAL. For simplification, the federated filter with the measurements from two sensors is analysed, and the result can be easily expanded to the case that the federated filter works based on the measurements form numbers of sensors.

The structure of the federated filter with two sensors is depicted in Figure A1. In Figure A1, $y_{1,k}$ and $y_{2,k}$ are the measurements from sensors 1 and 2 at time $t_k$; $x^-$ is the predicted state obtained by the dynamic model; $x_{1,k}^+$ and $x_{2,k}^+$ are the estimations from local filters 1 and 2, and $P_{1,k}$ and $P_{2,k}$ are the corresponding covariance matrices. The outputs of two local filters would be fused in the master filter. And the global estimation is as follows (Yang, 2004b; Yang, 2006).

$$x_{g,k} = [P_{1,k}^{-1} + P_{2,k}^{-1}]^{-1} [P_{1,k}^{-1} x_{1,k}^+ + P_{2,k}^{-1} x_{2,k}^+]$$ (A1)

$$P_{g,k} = [P_{1,k}^{-1} + P_{2,k}^{-1}]^{-1}$$ (A2)

Then, the global estimation is fed back to two local filters by the following equations.

$$x_{i,k} = x_{g,k}(i = 1, 2)$$ (A3)

$$P_{i,k} = P_{g,k} \beta_i$$ (A4)

$$\beta_i = \frac{\|P_{i,k}\|^{-1}}{\|P_{1,k}\|^{-1} + \|P_{2,k}\|^{-1}}$$ (A5)

The expressions of $x_{1,k}^+$ and $x_{2,k}^+$ can be presented as

$$x_{i,k}^+ = K_{i,k} y_{i,k} + (I - K_{i,k} H_{i,k}) x_{-}^k (i = 1, 2)$$ (A6)

where $K_i$ is the Kalman gain, $H_i$ is the measurement matrix of $i$th local filter, $I$ is a $n \times n$ unit matrix, and $n$ is the dimension of state $x$.

Therefore, the correlation matrix between $x_{1,k}^+$ and $x_{2,k}^+$ is

$$\text{cov}(x_{1,k}^+, x_{2,k}^+) = (I - K_{1,k} H_{1,k}) P_{k}^- (I - K_{2,k} H_{2,k})$$ (A7)
As is shown in Equation (A7), the outputs of local filters correlate. However, the master filter fuses the outputs based on the least square criterion which requires the inputs to be uncorrelated. Therefore, the output of the master filter is not globally optimal.

**A2. The Result of the Kinematic and Static Filter is Globally Optimal.** It is learnt that the output of the federated filter is suboptimal because the same dynamic model is used in the local filters. However, in terms of the kinematic and static filter, the dynamic model is just used in the kinematic filter, and the static filter works based on the output of the kinematic filter and the measurement of sensor 2. Therefore, the dynamic model is used once, and the result of the static filter is globally optimal.