# SELF-COMPLEMENTARY VERTEX-TRANSITIVE GRAPHS 

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(Received 30 March 2016; first published online 7 June 2016)

2010 Mathematics subject classification: primary 05C25; secondary 05E18.
Keywords and phrases: self-complementary graph, vertex-transitive graph, metacirculant, Cayley graph.

A graph is self-complementary if its complement is isomorphic to the graph itself. A graph is vertex-transitive if its full automorphism group is transitive on its vertex set. This dissertation is intended to present our research results on self-complementary vertex-transitive graphs. In particular, we studied the following problems: constructions of self-complementary vertex-transitive graphs, self-complementary vertex-transitive graphs of order a product of two primes, selfcomplementary metacirculants and self-complementary vertex-transitive graphs of prime-cubed order. The main analysis on these problems relies on two pivotal results due to Guralnick et al. [1] and Li and Praeger [2], which characterise the full automorphism group of a self-complementary vertex-transitive graph in the primitive and the imprimitive cases, respectively.

For constructions of self-complementary vertex-transitive graphs, there are generally three known methods: construction by partitioning the complementing isomorphism orbits, construction using the coset graphs and the lexicographic product. In this dissertation we developed various alternative construction methods. As a result, we find a family of self-complementary Cayley graphs of non-nilpotent groups and a new construction for self-complementary metacirculants of $p$-groups.

A complementing isomorphism of a self-complementary graph is an isomorphism between the graph and its complement. For the self-complementary vertex-transitive graphs whose automorphism groups are of affine type, we have obtained a characterisation of all their complementing isomorphisms. Furthermore, we provide a construction of self-complementary metacirculants which are Cayley graphs and have insoluble automorphism groups. This is the first known example with this property in the literature.

[^0]For the self-complementary vertex-transitive graphs of order a product of two primes, we give a complete classification of these graphs: they are either a lexicographic product of two self-complementary vertex-transitive graphs of prime order or a normal Cayley graph of an abelian group. This result has been published in [4].

A graph is called a metacirculant if its full automorphism group contains a transitive metacyclic subgroup. We show that the full automorphism group of these graphs is either soluble or contains the only insoluble composition factor $\mathrm{A}_{5}$. This extends a result due to Li and Praeger [3], which says that the full automorphism group of a self-complementary circulant is soluble. This result has been published in [5].

Finally, we investigate self-complementary vertex-transitive graphs of prime-cubed order. It is well known that vertex-transitive graphs of prime-cubed order are Cayley graphs [6]. We successfully show that for each type of the groups of primecubed order, there exist self-complementary Cayley graphs of the corresponding groups. Moreover, we also gain a characterisation of all the self-complementary vertex-transitive graphs of prime-cubed order: they are normal Cayley graphs, or a lexicographic product of two smaller self-complementary vertex-transitive graphs or their automorphism group is soluble.

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[^0]:    Thesis submitted to the University of Western Australia in September 2014; degree approved 4 September 2015; supervisors Cai Heng Li, Gordon Royle and Shu Jiao Song.
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