

SELF-COMPLEMENTARY VERTEX-TRANSITIVE GRAPHS

GUANG RAO✉

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A graph is *self-complementary* if its complement is isomorphic to the graph itself. A graph is *vertex-transitive* if its full automorphism group is transitive on its vertex set. This dissertation is intended to present our research results on self-complementary vertex-transitive graphs. In particular, we studied the following problems: constructions of self-complementary vertex-transitive graphs, self-complementary vertex-transitive graphs of order a product of two primes, self-complementary metacirculants and self-complementary vertex-transitive graphs of prime-cubed order. The main analysis on these problems relies on two pivotal results due to Guralnick *et al.* [1] and Li and Praeger [2], which characterise the full automorphism group of a self-complementary vertex-transitive graph in the primitive and the imprimitive cases, respectively.

For constructions of self-complementary vertex-transitive graphs, there are generally three known methods: construction by partitioning the complementing isomorphism orbits, construction using the coset graphs and the lexicographic product. In this dissertation we developed various alternative construction methods. As a result, we find a family of self-complementary Cayley graphs of non-nilpotent groups and a new construction for self-complementary metacirculants of p -groups.

A *complementing isomorphism* of a self-complementary graph is an isomorphism between the graph and its complement. For the self-complementary vertex-transitive graphs whose automorphism groups are of affine type, we have obtained a characterisation of all their complementing isomorphisms. Furthermore, we provide a construction of self-complementary metacirculants which are Cayley graphs and have insoluble automorphism groups. This is the first known example with this property in the literature.

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For the self-complementary vertex-transitive graphs of order a product of two primes, we give a complete classification of these graphs: they are either a lexicographic product of two self-complementary vertex-transitive graphs of prime order or a normal Cayley graph of an abelian group. This result has been published in [4].

A graph is called a *metacirculant* if its full automorphism group contains a transitive metacyclic subgroup. We show that the full automorphism group of these graphs is either soluble or contains the only insoluble composition factor A_5 . This extends a result due to Li and Praeger [3], which says that the full automorphism group of a self-complementary circulant is soluble. This result has been published in [5].

Finally, we investigate self-complementary vertex-transitive graphs of prime-cubed order. It is well known that vertex-transitive graphs of prime-cubed order are Cayley graphs [6]. We successfully show that for each type of the groups of prime-cubed order, there exist self-complementary Cayley graphs of the corresponding groups. Moreover, we also gain a characterisation of all the self-complementary vertex-transitive graphs of prime-cubed order: they are normal Cayley graphs, or a lexicographic product of two smaller self-complementary vertex-transitive graphs or their automorphism group is soluble.

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GUANG RAO, School of Science and Engineering,
The Chinese University of Hong Kong (Shenzhen),
2001 Longxiang Road, Longgang District,
Shenzhen, PR China
e-mail: raoguang@cuhk.edu.cn