

## SELF-COMPLEMENTARY VERTEX-TRANSITIVE GRAPHS

GUANG RAO✉

(Received 30 March 2016; first published online 7 June 2016)

2010 *Mathematics subject classification*: primary 05C25; secondary 05E18.

*Keywords and phrases*: self-complementary graph, vertex-transitive graph, metacirculant, Cayley graph.

A graph is *self-complementary* if its complement is isomorphic to the graph itself. A graph is *vertex-transitive* if its full automorphism group is transitive on its vertex set. This dissertation is intended to present our research results on self-complementary vertex-transitive graphs. In particular, we studied the following problems: constructions of self-complementary vertex-transitive graphs, self-complementary vertex-transitive graphs of order a product of two primes, self-complementary metacirculants and self-complementary vertex-transitive graphs of prime-cubed order. The main analysis on these problems relies on two pivotal results due to Guralnick *et al.* [1] and Li and Praeger [2], which characterise the full automorphism group of a self-complementary vertex-transitive graph in the primitive and the imprimitive cases, respectively.

For constructions of self-complementary vertex-transitive graphs, there are generally three known methods: construction by partitioning the complementing isomorphism orbits, construction using the coset graphs and the lexicographic product. In this dissertation we developed various alternative construction methods. As a result, we find a family of self-complementary Cayley graphs of non-nilpotent groups and a new construction for self-complementary metacirculants of  $p$ -groups.

A *complementing isomorphism* of a self-complementary graph is an isomorphism between the graph and its complement. For the self-complementary vertex-transitive graphs whose automorphism groups are of affine type, we have obtained a characterisation of all their complementing isomorphisms. Furthermore, we provide a construction of self-complementary metacirculants which are Cayley graphs and have insoluble automorphism groups. This is the first known example with this property in the literature.

---

Thesis submitted to the University of Western Australia in September 2014; degree approved 4 September 2015; supervisors Cai Heng Li, Gordon Royle and Shu Jiao Song.

© 2016 Australian Mathematical Publishing Association Inc. 0004-9727/2016 \$16.00

For the self-complementary vertex-transitive graphs of order a product of two primes, we give a complete classification of these graphs: they are either a lexicographic product of two self-complementary vertex-transitive graphs of prime order or a normal Cayley graph of an abelian group. This result has been published in [4].

A graph is called a *metacirculant* if its full automorphism group contains a transitive metacyclic subgroup. We show that the full automorphism group of these graphs is either soluble or contains the only insoluble composition factor  $A_5$ . This extends a result due to Li and Praeger [3], which says that the full automorphism group of a self-complementary circulant is soluble. This result has been published in [5].

Finally, we investigate self-complementary vertex-transitive graphs of prime-cubed order. It is well known that vertex-transitive graphs of prime-cubed order are Cayley graphs [6]. We successfully show that for each type of the groups of prime-cubed order, there exist self-complementary Cayley graphs of the corresponding groups. Moreover, we also gain a characterisation of all the self-complementary vertex-transitive graphs of prime-cubed order: they are normal Cayley graphs, or a lexicographic product of two smaller self-complementary vertex-transitive graphs or their automorphism group is soluble.

## References

- [1] R. Guralnick, C. H. Li, C. E. Praeger and J. Saxl, 'On orbital partitions and exceptionality of primitive permutation groups', *Trans. Amer. Math. Soc.* **356** (2004), 4857–4872.
- [2] C. H. Li and C. E. Praeger, 'On partitioning the orbitals of a transitive permutation group', *Trans. Amer. Math. Soc.* **355** (2003), 637–653.
- [3] C. H. Li and C. E. Praeger, 'Finite permutation groups with a transitive cyclic subgroup', *J. Algebra* **349** (2012), 117–127.
- [4] C. H. Li and G. Rao, 'Self-complementary vertex-transitive graphs of order a product of two primes', *Bull. Aust. Math. Soc.* **89**(2) (2014), 322–330.
- [5] C. H. Li, G. Rao and S. J. Song, 'On finite self-complementary metacirculants', *J. Algebraic Combin.* **40**(4) (2014), 1135–1144.
- [6] D. Marušič, 'Vertex transitive graphs and digraphs of order  $p^k$ ', in: *Cycles in Graphs* (Burnaby, BC, 1982), North-Holland Mathematical Studies, 115 (North-Holland, Amsterdam, 1985), 115–128.

GUANG RAO, School of Science and Engineering,  
The Chinese University of Hong Kong (Shenzhen),  
2001 Longxiang Road, Longgang District,  
Shenzhen, PR China  
e-mail: [raoguang@cuhk.edu.cn](mailto:raoguang@cuhk.edu.cn)