11 The Minimal Supersymmetric Standard Model

We can now very easily construct a supersymmetric version of the Standard Model. For each gauge field of the usual Standard Model we introduce a vector superfield. For each fermion (quark or lepton) we introduce a chiral superfield with the same gauge quantum numbers. Finally, we need at least two Higgs doublet chiral fields; if we introduce only one, as in the simplest version of the Standard Model, the resulting theory possesses gauge anomalies and is inconsistent. So, the theory is specified by the gauge group $SU(3) \times SU(2) \times U(1)$ and enumeration the chiral fields,

$$Q_f, \quad \bar{u}_f, \quad \bar{d}_f, \quad L_f, \quad \bar{e}_f, \quad f = 1, 2, 3; \quad H_U, \ H_D. \tag{11.1}$$

The gauge-invariant kinetic terms, auxiliary D terms and gaugino–matter Yukawa couplings are completely specified by the gauge symmetries. The superpotential can be taken to be

$$W = H_U (\Gamma_U)_{f,f'} Q_f \bar{U}_{f'} + H_D (\Gamma_D)_{f,f'} Q_f \bar{D}_{f'} + H_D (\Gamma_E)_{f,f'} L_f \bar{e}_{f'}. \tag{11.2}$$

If the Higgs fields obtain suitable expectation values then $SU(2) \times U(1)$ is broken and quarks and leptons acquire mass, just as in the Standard Model.

There are other terms which can also be present in the superpotential. These include the μ term, $\mu H_U H_D$. This is a supersymmetric mass term for the Higgs fields; see Section 11.1.1. We will see later that we need $\mu \gtrsim M_Z$ to have a viable phenomenology. A set of dimension-four terms permitted by the gauge symmetries raise serious issues. For example, one can have the terms

$$\bar{u}_f \bar{d}_g \bar{d}_h \Gamma^{fgh} + Q_f L_g \bar{d}_h \lambda^{fgh}. \tag{11.3}$$

These couplings violate B and L! This is our first serious setback. In the Standard Model, there is no such problem. The leading B- and L-violating operators permitted by gauge invariance possess dimension six, and they will be highly suppressed if the scale of interactions which violates these symmetries is high, as in grand unified theories.

If we are not going to simply give up, we need to suppress B and L violation at the level of dimension-four terms. The simplest approach is to postulate additional symmetries. There are various possibilities one can imagine.

1. *Global continuous symmetries* It is hard to see how such symmetries could be preserved in any quantum theory of gravity, and in string theory there is a theorem which asserts that there are no global continuous symmetries. We will prove this statement, at least for a large subset of known string theories, later.

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2. *Discrete symmetries* As we will see later, discrete symmetries can be gauge symmetries. As such they will not be broken in a consistent quantum theory. They are common in string theory. These symmetries are often *R symmetries*, symmetries which do not commute with supersymmetry.

A simple (though not unique) solution to the problem of B- and L-violation by dimension-four operators is to postulate a discrete symmetry known as R-parity. Under this symmetry, all ordinary particles are even while their superpartners are odd. Imposing this symmetry immediately eliminates all the dangerous operators. For example,

$$\int d^2\theta\, \bar{u}\bar{d}\bar{d} \sim \psi_{\bar{u}}\psi_{\bar{d}}\tilde{\bar{d}} \tag{11.4}$$

(we have changed notation again: the tilde here indicates the superpartner of the ordinary field, i.e. the *squark*) is odd under the symmetry.

More formally, we can define this symmetry as the following set of transformations on superfields:

$$\theta_\alpha \to -\theta_\alpha, \tag{11.5}$$

$$(Q_f, \bar{u}_f, \bar{d}_f, L_f, \bar{e}_f) \to -(Q_f, \bar{u}_f, \bar{d}_f, L_f, \bar{e}_f), \tag{11.6}$$

$$(H_U, H_D) \to (H_U, H_D). \tag{11.7}$$

Alternatively, we can describe it as multiplication of the quark and lepton superfields by -1, multiplication of the Higgs fields by 1 and a 2π rotation in space (which rotates all fermions by -1). Because invariance under 2π rotations is automatic in Lorentz-invariant theories, we need only the overall multiplication of the superfields. With this symmetry the full, renormalizable, superpotential is just that in Eq. (11.2).

In addition to solving the problem of very fast proton decay, R-parity has another striking consequence: the lightest of the new particles predicted by supersymmetry, the Lightest supersymmetric particle (LSP), is *stable*. This particle can easily be neutral under the gauge groups. It is then, inevitably, very weakly interacting. This in turn means the following.

- The generic signature of R-parity-conserving supersymmetric theories is the occurrence of events with missing energy.
- Supersymmetry is likely to produce an interesting dark-matter candidate.

This second point is one of the principal reasons that many physicists have found the possibility of low-energy supersymmetry so compelling. If one calculates the dark-matter density then, as we will see in the chapter on cosmology, one automatically finds a density in the right range if the scale of supersymmetry breaking is about 1 TeV. Later, we will see an additional piece of circumstantial evidence for low-energy supersymmetry: the unification of the gauge couplings within the MSSM.

We can imagine more complicated symmetries which would have similar effects, and we will have occasion to discuss these later. We can also relax the assumption of exact R-parity conservation. If, for example, the lepton-number-violating couplings are forbidden then the restrictions on the baryon-number-violating couplings are not so severe and the phenomenological consequences are interesting. In most of what follows we will assume a conserved Z_2 R-parity.

11.1 Soft supersymmetry breaking in the MSSM

If supersymmetry is a feature of the underlying laws of nature then it is certainly broken. The simplest approach to model building with supersymmetry is to add soft-breaking terms to the effective Lagrangian in such a way that the squarks, sleptons and gauginos have sufficiently large masses that they have not yet been observed (or, in the event that they are discovered, to account for their values).

Without a microscopic theory of supersymmetry breaking, all the soft terms are independent. It is of interest to ask how many soft-breaking parameters there are in the MSSM. More precisely, we will count the parameters of the model beyond those of the minimal Standard Model with a single Higgs doublet. Having imposed R-parity, the number of Yukawa couplings is the same in both theories, as are the numbers of gauge couplings and θ parameters. The quartic couplings of the Higgs fields are completely determined by the gauge couplings. So the "new" terms arise from the soft-breaking terms as well as the μ term for the Higgs fields. We will speak loosely of all of this as the soft-breaking Lagrangian. Suppressing flavor indices, we have

$$\begin{aligned}\mathcal{L}_{\rm sb} = {} & \widetilde{Q}^* m_Q^2 \widetilde{Q} + \tilde{\bar{u}}^* m_{\bar{u}}^2 \tilde{\bar{u}} + \tilde{\bar{d}}^* m_{\bar{d}}^2 \tilde{\bar{d}} + \tilde{L}^* m_L^2 \tilde{L} + \tilde{\bar{e}}^* m_{\bar{e}}^2 \tilde{\bar{e}} + H_U \widetilde{Q} A_u \tilde{\bar{u}} + H_D \widetilde{Q} A_d \tilde{\bar{d}} \\ & + H_D \widetilde{L}\, \widetilde{A}_l\, \tilde{\bar{e}} + \text{c.c.} + M_i \lambda\lambda + \text{c.c.} + m_{H_U}^2 |H_U|^2 + m_{H_D}^2 |H_D|^2 + \mu B H_U H_D \\ & + \ \mu \psi_H \psi_H. \end{aligned} \tag{11.8}$$

The matrices m_Q^2, $m_{\bar{u}}^2$ etc. are 3×3 Hermitian matrices, so they have nine independent entries. The matrices A_u, A_d etc. are general 3×3 complex matrices, so they each possess 18 independent entries. Each of the gaugino masses is a complex number, so these introduce six additional parameters. The quantities μ and B are also complex; they add four more. In total, then, there are 111 new parameters. As in the Standard Model, not all these parameters are meaningful; we are free to make field redefinitions. The counting is significantly simplified if we just ask how many parameters there are beyond the usual 18 of the minimal theory.

To understand what redefinitions are possible beyond the transformations on the quarks and leptons which go into defining the CKM parameters, we need to ask what are the symmetries of the MSSM before the introduction of the soft-breaking terms and the μ term (the μ term is more or less on the same footing as the soft-breaking terms, since it is of the same order of magnitude; as we will discuss later, it might well arise from the physics of supersymmetry breaking). Apart from the usual baryon and lepton numbers, there are two more. The first is a Peccei–Quinn symmetry, under which the two Higgs superfields rotate by the same phase while the right-handed quarks and leptons rotate by the opposite phase. The second is an R symmetry, a generalization of the symmetry we found in the Wess–Zumino model (see Section 9.6.1). It is worth describing this in some detail. By

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definition, an R symmetry is a symmetry of the Hamiltonian which does not commute with the supersymmetry generators. Such symmetries can be continuous or discrete. In the case of continuous R symmetries, by convention we can take the θs to transform by a phase $e^{i\alpha}$. Then the general transformation law takes the form

$$\lambda_i \to e^{i\alpha}\lambda_i \tag{11.9}$$

for the gauginos, while, for the elements of a chiral multiplet, we have

$$\Phi_i(x,\theta) \to e^{ir_i\alpha}\Phi(x,\theta e^{i\alpha}), \tag{11.10}$$

or, in terms of the component fields,

$$\phi_i \to e^{ir_i\alpha}\phi_i, \quad \psi_i \to e^{i(r_i-1)\alpha}\psi_i, \quad F_i \to e^{i(r_i-2)\alpha}F_i. \tag{11.11}$$

In order that the Lagrangian exhibit a continuous R symmetry, the total R charge of all terms in the superpotential must be 2. In the MSSM, we can take $r_i = 2/3$ for all the chiral fields.

The soft-breaking terms, in general, break two of the three lepton-number symmetries, the R-symmetry and the Peccei–Quinn symmetry. So there are four non-trivial field redefinitions which we can perform. In addition, the minimal Standard Model has two Higgs parameters. So from our 111 parameters, we can subtract a total of six, leaving 105 as the number of *new* parameters in the MSSM.

Clearly we would like to have a theory which predicts these parameters. Later, we will study some candidates. To get started, however, it is helpful to make an ansatz. The simplest thing to do is suppose that all the scalar masses are the same, all the gaugino masses the same and so on. It is necessary to specify also a scale at which this ansatz holds, since, even if true at one scale, it will not continue to hold at lower energies. Almost all investigations of supersymmetry phenomenology assume such a degeneracy at a large energy scale, typically the reduced Planck mass $M_{\rm p}$. It is often said that degeneracy is automatic in supergravity models, so this is frequently called the supergravity (SUGRA) model but, as we will see, supergravity by itself makes *no* prediction of degeneracy. Some authors, similarly, include this assumption as part of the definition of the MSSM, but in this text we will use the term MSSM to refer to the particle content and the renormalizable interactions. In any case, the ansatz consists of the following statements at the high-energy scale.

1. All the scalar masses are the same, $\widetilde{m}^2 = m_0^2$. This assumption is called the *universality* of the scalar masses.
2. The gaugino masses are the same, $M_i = M_0$. This is referred to as the *GUT relation*, since it holds in simple grand unified models.
3. The soft-breaking cubic terms are assumed to be given by

$$\mathcal{L}_{\rm tri} = A(H_U Q y_u \bar{u} + H_D Q y_d \bar{d} + H_D L y_l \bar{e}). \tag{11.12}$$

The matrices y_u, y_d etc. are the same as those which appear in the Yukawa couplings. This is the assumption of *proportionality.*

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Note that with this ansatz, if we ignore the various phases possibilities, five parameters are required to specify the model $(m_0^2, M_0, A, B_\mu, \mu)$. One of these can be traded for M_Z, so this is quite an improvement in predictive power. In addition, this ansatz automatically satisfies all constraints coming from rare processes. As we will explain, rare decays and flavor violation are suppressed ($b \to s + \gamma$ is not as strong a constraint, but it requires other relations among soft masses). However, we need to ask: just how plausible are these assumptions? We will try to address this question later.

11.1.1 The μ term

One puzzle in the MSSM is the μ term, the supersymmetric mass term for the Higgs fields. This term is not forbidden by the gauge symmetries, so the first question is: why is it small, of order a few TeV rather than of order M_p or M_gut? One possibility is that there is a symmetry which accounts for this. There might, for example, be a discrete symmetry forbidding $H_U H_D$ in the superpotential, spontaneously broken by the fields which also break supersymmetry. Another possibility is related to the non-renormalization theorems. If for some reason, there is no mass term at lowest order for the Higgs fields, one will not be generated perturbatively. The μ term, then, might be the result of the same non-perturbative dynamics, for example, those responsible for supersymmetry breaking. In string theories, as we will see later, it is quite common to find massless particles at tree level, simply "by accident". Such a phenomenon can also be arranged in grand unified theories.

In the absence of a large, tree level, μ term, supersymmetry breaking can quite easily generate a μ term of order $m_{3/2}$. Consider, for example, the Polonyi model. The operator

$$\int d^4\theta \frac{1}{M_\mathrm{p}} Z^\dagger H_U H_D \tag{11.13}$$

would generate a μ term of just the correct size. In simple grand unified theories, such a term is often generated.

When we discuss other models for supersymmetry breaking, such as gauge mediation, we will see that the μ term sometimes poses additional challenges.

11.1.2 Cancelation of quadratic divergences in gauge theories

We have already seen that soft supersymmetry-violating mass terms receive only logarithmic divergences. While not essential to our present discussion, it is perhaps helpful to see how the cancelation of quadratic divergences for scalar masses arises in gauge theories like the MSSM.

Take, first, a $U(1)$ theory, with (massless) chiral fields ϕ^+ and ϕ^-. Without doing any computation it is easy to see that, provided we work in a way which preserves supersymmetry, there can be no quadratic divergence. In the limit where the mass term vanishes, the theory has a chiral symmetry under which ϕ^+ and ϕ^- rotate by the same phase,

$$\phi^\pm \to e^{i\alpha} \phi^\pm. \tag{11.14}$$

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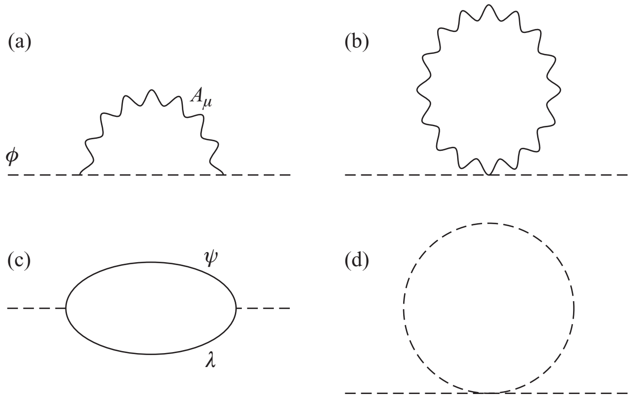

Fig. 11.1 One-loop diagrams contributing to scalar masses in a supersymmetric gauge theory.

This symmetry forbids a mass term $\Lambda\phi^+\phi^-$ in the superpotential the only from in which a supersymmetric mass term could appear. The actual diagrams we need to compute are shown in Fig. 11.1. Since we are interested only in the mass, we can take the external momentum to be zero. It is convenient to choose the Landau gauge for the gauge boson. In this gauge the gauge boson propagator is

$$D_{\mu\nu} = -i\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\frac{1}{q^2}, \tag{11.15}$$

so the first diagram vanishes. The second, third and fourth are straightforward to work out from the basic Lagrangian. One finds:

$$I_\text{b} = g^2 i \times i\frac{3}{(2\pi)^4}\int \frac{d^4k}{k^2}, \tag{11.16}$$

$$I_\text{c} = g^2 i \times i\frac{(\sqrt{2})^2}{(2\pi)^4}\int \frac{d^4k}{k^4}\,\text{Tr}(k_\mu\sigma^\mu k_\nu\bar{\sigma}^\nu) \tag{11.17}$$

$$= -\frac{4g^2}{(2\pi)^4}\int \frac{d^4k}{k^2}, \tag{11.18}$$

$$I_\text{d} = g^2(i)(i)\frac{1}{(2\pi)^4}\int \frac{d^4k}{k^2}. \tag{11.19}$$

It is easy to see that the sum $I_\text{a} + I_\text{b} + I_\text{c} + I_\text{d} = 0$.

Including a soft-breaking mass for the scalars only changes I_d:

$$\begin{aligned} I_\text{d} &\to \frac{g^2}{(2\pi)^4}\int \frac{d^4k}{k^2 - \widetilde{m}^2} \\ &= -i\frac{g^2}{(2\pi)^4}\int \frac{d^4k_E}{k_E^2 + \widetilde{m}^2} \\ &= \widetilde{m}^2_\text{independent} + \frac{ig^2}{16\pi^2}\widetilde{m}^2 \ln\frac{\Lambda^2}{\widetilde{m}^2}. \end{aligned} \tag{11.20}$$

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We have worked here in Minkowski space and have indicated the factors i to assist the reader in obtaining the correct signs for the diagrams. In the second line of Eq. (11.20) we have performed a Wick rotation. In the third line we have separated off the mass-independent part, since we know that this is canceled by the other diagrams.

Summarizing, the one-loop mass shift is

$$\delta \widetilde{m}^2 = -\frac{g^2}{16\pi^2}\widetilde{m}^2 \ln \frac{\Lambda^2}{\widetilde{m}^2}. \tag{11.21}$$

Note that the mass shift is proportional to $\widetilde{m}^2$, the supersymmetry-breaking mass, which we expect since supersymmetry is restored as $\widetilde{m}^2 \to 0$. In the context of the Standard Model we see that the scale of supersymmetry breaking cannot be much larger than the Higgs mass scale itself without fine tuning. Roughly speaking, it cannot be much larger than this scale than by a factor of order $1/\sqrt{\alpha_W}$, i.e. of order six. We also see that the correction has a logarithmic sensitivity to the cutoff. So, just as for the gauge and Yukawa couplings, the soft masses run with the energy.

11.2 $SU(2) \times U(1)$ breaking

In the MSSM there are a number of general statements which can be made about the breaking of $SU(2) \times U(1)$. The only quartic couplings of the Higgs fields arise from the $SU(2)$ and $U(1)$ D^2 terms. The general form of the soft-breaking mass terms has been described above. So, before we worry about any detailed ansatz for the soft breakings, we note that the Higgs potential is given quite generally by

$$\begin{aligned} V_{\text{Higgs}} &= m_{H_U}^2|H_U|^2 + m_{H_D}^2|H_D|^2 - m_3^2(H_U H_D + \text{h.c.}) \\ &\quad + \frac{1}{8}(g^2 + g'^2)(|H_U|^2 - |H_D|^2)^2 + \frac{1}{2}g^2|H_U H_D|^2. \end{aligned} \tag{11.22}$$

This potential by itself conserves CP; a simple field redefinition removes any phase in m_3^2. (As we will discuss shortly, there are many other possible sources of CP violation in the MSSM.) The physical states in the Higgs sector are usually described by assuming that CP is a good symmetry. In that case there are two CP-even scalars, H^0 and h^0, where, by convention, h^0 is the lighter of the two. There are a CP-odd neutral scalar A and charged scalars $H^{\pm}$. At tree level, one also defines a parameter which is the ratio of the vevs of H_U and H_D or v_1 and v_2:

$$\tan\beta = \frac{|\langle H_U\rangle|}{|\langle H_D\rangle|} \equiv \frac{v_1}{v_2}. \tag{11.23}$$

Note that, with this definition, as $\tan\beta$ grows so does the Yukawa coupling of the b quark.

To obtain a suitable vacuum, there are two constraints which the soft breakings must satisfy.

1. Without the soft-breaking terms, $H_U = H_D$ ($v_1 = v_2 = v$) makes the $SU(2)$ and $U(1)$ D terms vanish, i.e. there is no quartic coupling in this direction. So the energy

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is unbounded below, unless

$$m_{H_U}^2 + m_{H_D}^2 - 2|m_3|^2 > 0. \tag{11.24}$$

2. In order to obtain symmetry breaking, the Higgs mass matrix must have a negative eigenvalue. This gives the requirement

$$\left|m_3^2\right|^2 > m_{H_U}^2 m_{H_D}^2. \tag{11.25}$$

When these conditions are satisfied, it is straightforward to minimize the potential and determine the spectrum. One finds that

$$m_A^2 = \frac{m_3^2}{\sin\beta\cos\beta}. \tag{11.26}$$

It is conventional to take m_A^2 as one parameter. Then one finds that the charged Higgs masses are given by

$$m_{H^\pm}^2 = m_W^2 + m_A^2, \tag{11.27}$$

while the neutral Higgs masses are

$$m_{H^0,h^0}^2 = \frac{1}{2}\left[m_A^2 + m_Z^2 \pm \sqrt{\left(m_A^2 + m_Z^2\right)^2 - 4m_Z^2 m_A^2 \cos 2\beta}\,\right]. \tag{11.28}$$

Note the inequalities

$$m_{h^0} \le m_A, \qquad m_{h^0} \le m_Z, \qquad m_{H^\pm} \ge m_W. \tag{11.29}$$

With the discovery of the Higgs at 125 GeV, it would appear that the MSSM is ruled out. However, these are tree level relations. We will shortly turn to the issue of radiative corrections and will see that these can be quite substantial. We will also see, however, that accounting for a Higgs mass of 125 GeV appears to require a significant fine tuning of the parameters.

11.3 Embedding the MSSM in supergravity

In the previous chapter we introduced $N = 1$ supergravity theories. These theories are not renormalizable and must be viewed as effective theories, valid below some energy scale which might be the Planck scale or unification scale (or something else).

The approach we have introduced to model building is quite useful when we are considering models for the origin of supersymmetry breaking in the MSSM. The basic assumptions of this approach were as follows.

- The theory consists of two sets of fields the *visible sector fields* y_a, which in the context of the MSSM would be the quark and lepton superfields, and the *hidden sector fields* z_i, responsible for supersymmetry breaking.

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- The superpotential was taken to have the form

$$W(z, y) = W_z(z) + W_y(y). \tag{11.30}$$

- For the Kahler potential we took the simple ansatz

$$K = \sum_a y_a^\dagger y_a + \sum_i z_i^\dagger z_i. \tag{11.31}$$

In this case, we saw that if the supersymmetry-breaking scale was of order

$$M_{\text{int}} = m_{3/2} M_{\text{p}} \tag{11.32}$$

then there was an array of soft-breaking terms of order $m_{3/2}$. In particular, there were universal masses and A terms,

$$a m_{3/2}^2 |y_a|^2 + b m_{3/2} W_{ab} y_a y_b + c m_{3/2} W_{abc} y_a y_b y_c. \tag{11.33}$$

Here $W_{ab} = \partial_a \partial_b W$ and $W_{abc} = \partial_a \partial_b \partial_c W$.

Given that the MSSM is at best an effective-low-energy theory, one can ask how natural are our assumptions, and what would be the consequences of relaxing them? The assumption that there is some sort of hidden sector, and that the superpotential breaks up as we have hypothesized, is, as we will see, a reasonable one. It can be enforced by symmetries. The assumption that the Kahler potential takes this simple (often called "minimal") form is a strong one, not justified by symmetry considerations. It turns out not to hold in any general sense in string theory, the only context in which presently we can compute it. If we relax this assumption, we lose the universality of scalar masses and the proportionality of the A terms to the superpotential. As we will see later in this chapter, without these or something close the MSSM is not compatible with experiment.

11.4 Radiative corrections to the Higgs mass limit

We have seen that, in the MSSM, the Higgs mass at tree level is less than the Z mass. This bound is clearly violated in nature. In this section and the next, we will see that a 125 GeV Higgs particle can be accommodated within the MSSM, though it requires either a large scale of supersymmetry breaking or the introduction of new degrees of freedom.

In the MSSM, at tree level, the form of the Higgs potential is highly constrained because the quartic couplings are completely determined by the gauge interactions. Once supersymmetry (susy) is broken, however, there can be corrections to the quartic terms from radiative corrections.These corrections are soft, in that the susy-violating four-point functions vanish rapidly at momenta above the susy-breaking scale. Still, they are important in determining the low-energy properties of the theory, such as the Higgs vacuum expectation values (vevs) and the spectrum.

The largest effect of this kind comes from loops involving top quarks or their scalar partners, the *stops*. It is not hard to get a rough estimate of the effect. In the limit $\widetilde{m}_t \gg m_t$, the effective Lagrangian is not supersymmetric below $\tilde{m}_t$. As a result, there can be

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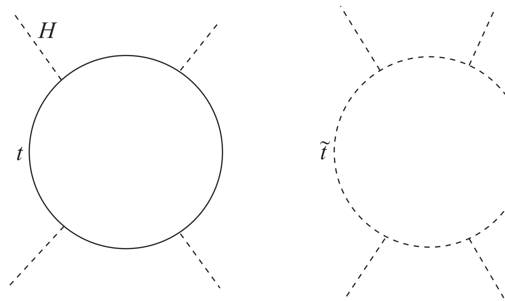

Fig. 11.2 Corrections to quartic Higgs couplings from top loops.

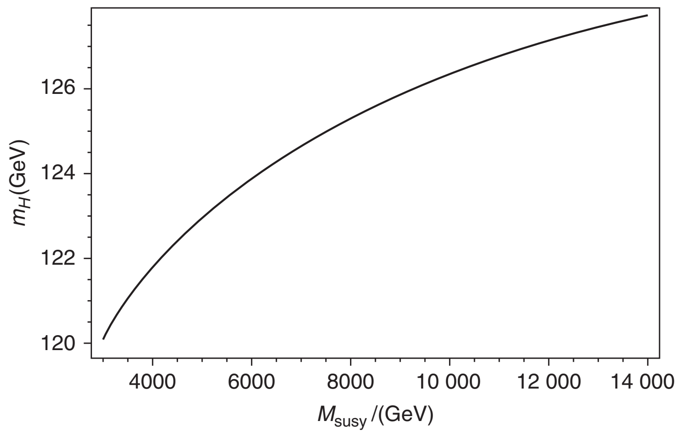

Fig. 11.3 Higgs mass as a function of susy-breaking parameters.

corrections to the Higgs quartic couplings. Consider the diagrams of Fig. 11.2. In this limit we can get a reasonable estimate by just keeping the top quark loop. The result will be logarithmically divergent, and we can take the cutoff to be $\widetilde{m}_t$. So we have

$$\delta\lambda = (-1)y_t^4 \times 3 \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr} \frac{1}{(\not{k} - m_t)^4} \tag{11.34}$$

$$= -\frac{12iy_t^4}{16\pi^2} \ln \frac{\widetilde{m}_t^2}{m_t^2}. \tag{11.35}$$

One can get a better estimate by keeping finite terms and higher-order corrections. There exist online tools to perform these calculations (mentioned in the references at the end of this chapter). For large $\tan\beta$ these corrections are most effective; this corresponds precisely to the decoupling limit discussed in Chapter 3, where the Higgs is principally H_U. A typical plot of m_H as a function of $\tilde{m}_t$, for small values of the A parameter for the stops and for large $\tan\beta$, is that of Fig. 11.3. We see that, for moderate values of the A parameter, a Higgs of 125 GeV corresponds to a stop mass of order 10 TeV. As we will see in the next section, this, in turn, implies a significant tuning of the Higgs mass.

11.5 Fine tuning of the Higgs mass

We saw earlier that in the Wess–Zumino model at one loop there is a negative renormalization of the soft-breaking scalar masses. This calculation can be translated to the MSSM, with a modification for the color and $SU(2)$ factors. One obtains

$$m_{H_U}^2 = (m_{H_U})_0^2 - \frac{6y_t^2}{16\pi^2} \ln \frac{\Lambda^2}{m^2} \left(\widetilde{m}_t^2 + m_t^2\right), \tag{11.36}$$

$$\widetilde{m}_t^2 = (\widetilde{m}_t)_0^2 - \frac{4y_t^2}{16\pi^2} \ln \frac{\Lambda^2}{m^2} \widetilde{m}_H^2. \tag{11.37}$$

So, we see that loop corrections involving the top quark Yukawa coupling reduce both the Higgs and the stop masses. If $\widetilde{m}_t^2 = 10$ TeV, and if $\Lambda \sim M_{\rm p}$, the correction to the stop mass is of order one but the correction to the Higgs mass is of order $8000 m_H^2$! This suggests a tuning of the parameter $(m_{H_U})_0^2$ at nearly the one part in 10 000 level, and a more refined renormalization group analysis supports this.

Such a tuning of parameters is troubling, given that we introduced supersymmetry in order to avoid such problems with naturalness. It is, at least, not as extreme as the situation without supersymmetry. It is also consistent with the data. In the next section, we will mention a few ideas to ameliorate this tuning.

11.6 Reducing the tuning: the NMSSM

We have seen that in the MSSM the effective Higgs quartic coupling is small because it is determined by the gauge couplings; this is what accounts for the tree level Higgs mass bound. The requirement of a large stop mass was driven by the need to enhance the quartic coupling. One might also hope to enhance the quartic coupling by introducing additional fields with superpotential couplings to the Higgs. The simplest approach yields the Next to Minimal Supersymmetric Standard Model, or NMSSM. In its simplest version the field content of the model is that of the MSSM plus an additional singlet, S. The superpotential includes a term

$$W_{\rm NMSSM} = \lambda S H_U H_d \tag{11.38}$$

in addition to the Yukawa couplings of the Higgs. This superpotential leads to a quartic coupling

$$\delta V = |\lambda H_U H_d|^2, \tag{11.39}$$

which can increase the Higgs mass. However, λ cannot be arbitrarily large otherwise perturbation theory would break down. Requiring that there be no Landau pole for λ typically implies that $\lambda < 0.7$.

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One difficulty with this proposal is that the maximum effect occurs when $\tan\beta \sim 1$, so that H_U and H_D are more or less aligned. In this limit the top quark corrections to the quartic coupling are less effective. Adding other terms to the superpotential, such as $\frac{1}{2}m_S S^2$ and S^3 as well as the various possible soft breakings, yields a large parameter space to explore. One typically finds that fine tuning can be significantly improved over the MSSM, but because of the constraints on λ it is still significantly worse than 10%.

There are other proposals to reduce the tuning of the MSSM by introducing additional degrees of freedom. Additional gauge interactions, for example, can help. Perhaps a compelling model may yet emerge. As we will see in the following sections, however, direct searches for supersymmetric particles, especially with the LHC, have placed stringent lower limits on the masses of supersymmetric partners of ordinary particles.

11.7 Constraints on low-energy supersymmetry: direct searches and rare processes

Naturalness points to supersymmetry at a scale below the TeV scale – arguably of order M_Z. We have already discussed how the Higgs mass points towards a significantly higher scale, somewhere around 10 TeV. Direct searches for supersymmetric particles, as we will briefly review here, also point to a high scale. Current limits on squarks and gluinos are, over much of the parameter space, larger than a TeV and they will become stronger (or evidence for supersymmetry will emerge) during future LHC runs. The limits on leptons, charginos and neutralinos (see below) are significant, though not quite as strong.

There are also strong constraints on the supersymmetry parameters (the 101 parameters we counted in the MSSM, for example) from rare processes.

11.7.1 Direct searches for supersymmetric particles

As mentioned above, direct searches for supersymmetric particles at LEP, the Tevatron and the LHC have placed significant limits on their masses. Among the states in the MSSM which are possible discovery channels for supersymmetry, are the *charginos*, linear combinations of the partners of the $W^\pm$ and $H^\pm$, and the *neutralinos*, linear combinations of the partners of the Z and γ (B and W^3) and the neutral Higgs. The mass matrix for the charginos, $w^\pm$ and $\tilde{h}^\pm$ is given by

$$\begin{aligned}\mathcal{L}_{\rm sb} = {} & \widetilde{Q}^* m_Q^2 \widetilde{Q} + \tilde{\bar{u}}^* m_{\bar{u}}^2 \tilde{\bar{u}} + \tilde{\bar{d}}^* m_{\bar{d}}^2 \tilde{\bar{d}} + \tilde{L}^* m_L^2 \tilde{L} + \tilde{\bar{e}}^* m_{\bar{e}}^2 \tilde{\bar{e}} + H_U \widetilde{Q} A_u \tilde{\bar{u}} + H_D \widetilde{Q} A_d \tilde{\bar{d}} \\ & + \langle H_U \rangle^2 + m_{H_D}^2 \langle H_D \rangle^2 + \mu B H_U H_D \\ & + \mu \psi_H \psi_H. \qquad (11.40)\end{aligned}$$

The matrices m_Q^2, $m_{\bar{U}}^2$ and so on that give mass to the scalar partners of quarks and leptons (*squarks* and *sleptons*) are 3×3 Hermitian matrices, so they have nine independent entries. The matrices A_u, A_d etc. are general 3×3 complex matrices, so they each possess

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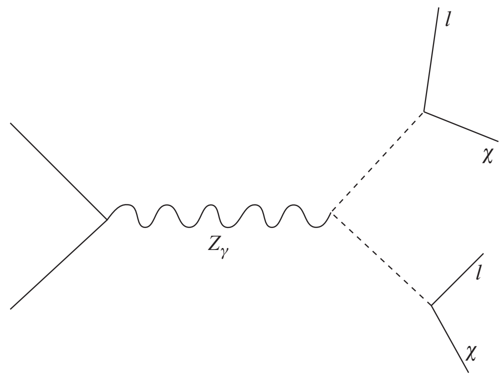

Fig. 11.4 Slepton production in $e^+ e^-$ annihilation.

18 independent entries. Each gaugino mass is a complex number, so these introduce six additional parameters; M_1, M_2 and M_3 are Majorana mass terms for the $U(1)$, $SU(2)$ and $SU(3)$ gauginos. The quantities μ and B are also complex and so introduce four more parameters. In total, then, there are 111 new parameters. As in the Standard Model, they are not all meaningful since we are free to make field redefinitions. The counting is significantly simplified if we just ask how many parameters there are beyond the usual 18 of the minimal theory.

For the neutralinos, w^0, b, $\tilde{h}_U^0$, $\tilde{h}_D^0$, there is a 4×4 mass matrix. We will leave the study of these for the exercises. Conventionally, the charginos are denoted $\tilde{\chi}_1^+$, $\tilde{\chi}_1^-$, $\tilde{\chi}_2^+$, $\tilde{\chi}_2^-$, where the label 2 indicates a chargino having greater mass. The neutralinos are denoted $\tilde{\chi}_1^0$, $\tilde{\chi}_2^0$, $\tilde{\chi}_3^0$, $\tilde{\chi}_4^0$, again ordered by increasing mass. The lightest of these states is stable if R-parity is conserved and is a natural dark-matter candidate.

The direct searches are easy to describe, and production and decay rates can be computed given a knowledge of the spectrum since the couplings of the fields are known. If R-parity is conserved then the LSP is stable and weakly interacting, so the characteristic signal for supersymmetry is *missing energy*. For example, in e^+e^- colliders one can produce slepton pairs, if they are light enough, through the diagram of Fig. 11.4. These then decay, typically, to a lepton and a neutralino, as indicated. So the final state contains a pair of acoplanar leptons and missing energy. The LEP ran at center of mass energies as high as $\sqrt{s} = 209$ GeV, setting limits of order 90 GeV on sleptons and 103.5 GeV on charginos. The LHC has strengthened these limits in some regions of the parameter space.

In hadron colliders at high energies, one has the potential to produce colored hadrons – squarks and gluinos – at high rates. As a result the most dramatic limits on supersymmetric particles have been set by the LHC (following earlier searches at the Tevatron). The LHC has run at 7 and 8 GeV, collecting 20 $(\text{femtobarns})^{-1}$ of data per detector at the higher energy, Setting limits, however, on gaugino and squark masses (and those of other states) is a model-dependent process. For example, if gauginos are heavier than squarks, they will first decay to a gluon and a squark; the squark may decay to a quark and a neutralino or to a quark and a chargino, with the chargino in turn decaying by a variety of possible channels. If the squarks are heavier than gluinos, there are alternative decay chains.

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Many analyses employ the ansatz we called SUGRA (see Section 11.1), with five parameters. Quite stringent limits can then be set on these different parameters, and correspondingly on the masses of the various superparticles. In recent years this model has been refined somewhat and rebranded as the *Constrained Minimal Supersymmetric Standard Model*, or CMSSM. A more phenomenological variant with assumptions which are not quite as restrictive is the PMSSM. The strategy, in this framework, is to allow the maximum (or close to the maximum) number of parameters consistent with the various facts of low-energy physics. An alternative approach, adopted by many theorists and employed in many experimental analyses, is referred to as the "simplified model" method. Here one focuses on signals, i.e. particular production and decay possibilities, rather than on fitting to models. From all these types of analysis one finds lower limits on gluinos of order 1.2–1.7 TeV and similar limits for squarks.

11.7.2 Constraints from rare processes

Rare processes provide another set of strong constraints on the soft-breaking parameters. In the simple ansatz, all the scalar masses are the same at some very high energy scale. However, even if this is assumed to be true at one scale, it is not true at all scales, i.e. these relations are *renormalized*. Indeed, all 105 parameters are truly parameters and it is not obvious that the assumptions of universality and proportionality are *natural.* However, there are strong experimental constraints which suggest some degree of degeneracy.

As one example, there is no reason, a priori, why the mass matrix for the $\tilde{L}$s (the partners of the lepton doublets) should be diagonal in the same basis as the charged leptons. If it is not then there is no conservation of separate lepton numbers, and the decay $\mu \to e\gamma$ will occur (Fig. 11.5). To see that we are potentially in serious trouble, we can make a crude estimate. The muon lifetime is proportional to $G_F^2 m_\mu^5$. The decay $\mu \to e\gamma$ occurs owing to the operator

$$\mathcal{L}_{\mu e\gamma} = eCF_{\mu\nu}\bar{u}\sigma^{\mu\nu}e. \tag{11.41}$$

If there is no particular suppression, we might expect that

$$C = \frac{\alpha_w}{\pi}\frac{m_\mu}{m_{\rm susy}^2}. \tag{11.42}$$

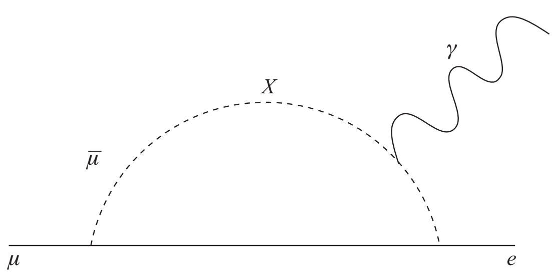

Fig. 11.5 Contribution to $\mu \to e\gamma$.

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Therefore the branching ratio, i.e. the ratio of the rate of decay to e_γ and the rate for all decays, would be of order

$$BR = \frac{\Gamma(\mu \to e + \gamma)}{\Gamma(\mu \to \text{all})} = \left(\frac{\alpha_w}{\pi}\right)^2 \left(\frac{M_W}{M_{\text{susy}}}\right)^4. \tag{11.43}$$

This ratio might become as small as 10^{-8}–10^{-9} if the supersymmetry-breaking scale is large, 1 TeV or so. But the current experimental limit is 1.2×10^{-11}. So even in this case it is necessary to suppress the off-diagonal terms. More detailed descriptions of the limits are found in the suggested reading at the end of the chapter.

Another troublesome constraint arises from the neutron and electron electric dipole moments, d_n and d_e. Any non-zero value of these quantities signifies CP violation. Currently, one has $d_n < 2.9 \times 10^{-26} e$ cm and $d_e < 18.7 \times 10^{-29} e$ cm. The soft-breaking terms in the MSSM contain many new sources of CP violation. Even with the assumptions of universality and proportionality, the gaugino mass and the A, μ and B parameters are all complex and can violate CP. At the quark level, the issue is that one-loop diagrams can generate a quark dipole moment, as in Fig. 11.6. Note that this particular diagram is proportional to the phases of the gluino and the A parameter. It is easy to see that, even if $m_{\text{susy}} \sim 500$ GeV, these phases must be smaller than about 10^{-2}. More detailed estimates can be found in the suggested reading at the end of the chapter.

In the real world CP is violated, so it is puzzling that all the soft-supersymmetry-violating terms should preserve CP to such a high degree. This is in contrast with the minimal Standard Model, with a single Higgs field, which can reproduce the observed CP violation with phases of order 1. It is thus a serious challenge to understand why CP should be such a good symmetry if nature is supersymmetric. Various explanations have been offered. We will discuss some of these later, but it should be kept in mind that the smallness of CP violation suggests that either the low-energy supersymmetry hypothesis is wrong or there is some interesting physics which explains the surprisingly small values of the dipole moments.

So far, we have discussed constraints on slepton degeneracy and CP-violating phases. There are also constraints on the squark masses, arising from various flavor-violating processes. In the Standard Model the most famous of these are strangeness-changing processes such as $K\bar{K}$ mixing. One of the early triumphs of the Standard Model was that it successfully explained why this mixing is so small. Indeed, the Standard Model gives

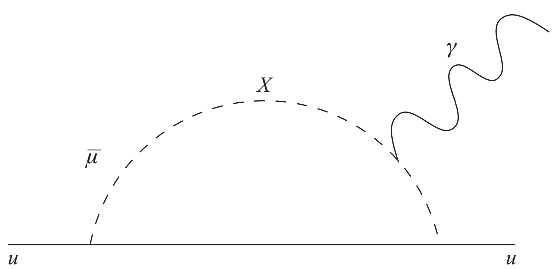

Fig. 11.6 Contribution to d_n in supersymmetric theories.

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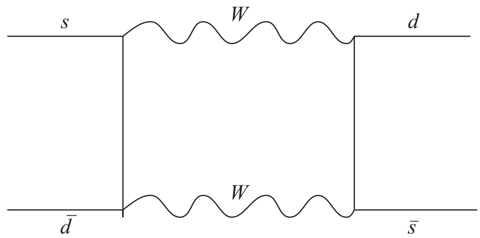

Fig. 11.7 Contribution to $K \leftrightarrow \bar{K}$ in the Standard Model.

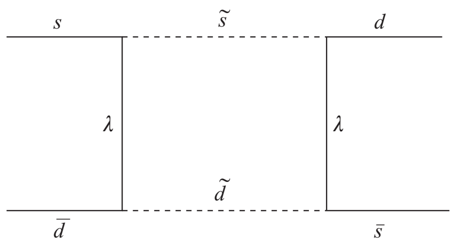

Fig. 11.8 Gluino exchange contribution to $K\bar{K}$ mixing in the MSSM.

a quite good estimate for the mixing. This was originally used to predict – amazingly accurately – the charm quark mass. The mixing receives contributions from box diagrams such as that shown in Fig. 11.7. If we consider only the first two generations and ignore the quark masses (compared with M_W), we have that

$$\mathcal{M}(K^0 \to \bar{K}^0) \propto (V_{di}V^\dagger_{is})(V^\dagger_{sj}V_{jd}) = 0. \tag{11.44}$$

Including fermion masses leads to terms in the low-energy effective action $\mathcal{L}_{\rm eff}$ of order

$$\frac{\alpha_W}{4\pi}\frac{m_c^2}{M_W^2}G_{\rm F}\ln\frac{m_c^2}{m_u^2}(\bar{s}\gamma^\mu\gamma_5 d)(\bar{d}\gamma^\mu\gamma^5 s) + \cdots. \tag{11.45}$$

The matrix element of the operator appearing here can be estimated in various ways, and one finds that this expression roughly saturates the observed value (this was the origin of the prediction by Gaillard and Lee of the value of the charm quark mass). Similarly, the CP-violating parameter in the kaon system (the "ϵ" parameter) is in rough accord with observation for reasonable values of the CKM parameter δ.

In supersymmetric theories, if squarks are degenerate then there are similar cancelations. However, if they are not then there are new, very dangerous, contributions. The most serious is that indicated in Fig. 11.8, arising from the exchange of gluinos and squarks. This is nominally larger than the Standard Model contribution by a factor $(\alpha_s/\alpha_W)^2 \approx 10$. Also, the Standard Model contribution vanishes in the chiral limit whereas the gluino exchange does not, and this leads to an additional enhancement of nearly an order of magnitude. However, the diagram is highly suppressed in the limit of exact universality and proportionality. Proportionality means that the A terms in Eq. (11.8) are suppressed by factors of order the light quark masses, while universality means that the squark propagator $\langle\tilde{q}^*\tilde{q}\rangle$ is proportional to the unit matrix in flavor space. So, on the one hand, there are no

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appreciable off-diagonal terms which can contribute to the diagram. On the other hand, there is surely some degree of non-degeneracy. One finds that, even if the characteristic susy scale is 1 TeV, one needs degeneracy in the down squark sector at the one part in 30 level.

So the CP-preserving part of the $K\overline{K}$ mass matrix already tightly constrains the down squark mass matrix and the CP-violations part provides even more severe constraints. There are also strong limits on $D\overline{D}$ mixing, which significantly restrict the mass matrix in the up squark sector. Other important constraints on soft breakings come from other rare processes such as $b \to s\gamma$. Again, more details can be found in the references given in the suggested reading.

Suggested reading

The minimal supersymmetric Standard Model is described in most reviews of supersymmetry. Probably the best place to look for up-to-date reviews of the model and the experimental constraints is the Particle Data Group website. A useful collection of renormalization group formulas for supersymmetric theories is provided in the review by Martin and Vaughn (1994). Limits on rare processes are discussed in a number of articles, such as that by Masiero and Silvestrini (1997). The status of the NMSSM, including questions of tuning, is discussed in Hall *et al.* (2012).

Exercises

(1) Derive Eqs. (11.24)–(11.27).

(2) Verify the formula for the top quark corrections to the Higgs mass. Evaluate y_t in terms of m_t and $\sin\beta$. Show that, to this level of accuracy,

$$m_h^2 < m_Z^2 \cos 2\beta + \frac{12g^2}{16\pi^2}\frac{m_t^4}{m_W^2} \ln\left(\widetilde{m}^2 m_t^2\right).$$

(3) Estimate the sizes of the supersymmetric contributions to the quark electric dipole moment, assuming that all the superpartner masses are of order m_{susy} and that δ is a typical phase. Assuming, as well, that the neutron electric dipole moment is of order the quark electric dipole moment, how small do the phases have to be if $m_{\text{susy}} = 500$ GeV?

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