CORRIGENDUM

Pseudoprime Reductions of Elliptic Curves – CORRIGENDUM

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Unfortunately, there are two inaccuracies in the argument of [CLS]. First, the statements of Lemmas 3, 4, 6, and 7 of [CLS] hold only under the additional condition \( \gcd(m, M_E) = 1 \) for some integer \( M_E \geq 1 \) depending only on \( E \). Second, the divisibility condition (3·6) in [CLS] implies that \( t_b(\ell) \mid n_E(p) - 1 \) (rather than \( t_b(\ell) \mid n_E(p) \)), as it was erroneously claimed on p. 519 in [CLS]). In particular, instead of the divisibility \( \ell t_b(\ell) \mid n_E(p) \) (see the last displayed formula on p. 519 in [CLS]), we conclude that for every prime \( \ell \mid L \) there is an integer \( a_\ell \) such that

\[
n_E(p) \equiv a_\ell \pmod{\ell t_b(\ell)}.
\]

However, the final result is correct and can easily be recovered. To do so, we remark that under the condition \( \gcd(m, M_E) = 1 \), we have full analogues of Lemmas 6, 7, 9, and 10 of [CLS] for the function \( \Pi(x; m, a) \) defined as the number of primes \( p \leq x \) with \( n_E(p) \equiv a \pmod{m} \) (rather than just for \( \Pi(x; m) = \Pi(x; m, 0) \) as in [CLS]). Define \( \rho^\star(n) \) as the largest square-free divisor of \( n \) which is relatively prime to \( M_E \). We then derive from (0·1) above that

\[
n_E(p) \equiv a_\ell \pmod{\ell \rho^\star(t_b(\ell))}.
\]

Therefore

\[
\# T \leq \sum_{y < \ell \leq z} \Pi(x; \ell \rho^\star(t_b(\ell)), a_\ell).
\]

Since

\[
\rho^\star(n) \mid \rho(n) \quad \text{and} \quad \rho^\star(n) \geq \rho(n)/M_E,
\]

we see that (0·2) above implies the bound (3·7) from [CLS], and the result now follows without any further changes.

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REFERENCES
