23

$\gamma\gamma$ scatterings and the 'spin' of the photon

$\gamma\gamma$ collisions in e^+e^- process are known to be an important source of hadrons as the cross-section $e^+e^- \to e^+e^- +$ hadrons increases logarithmically with the energy while the annihilation process $e^+e^- \to$ hadrons decreases like $1/s$. The dominant contribution comes from two on-shell photons emitted at small angles using the so-called equivalent photon approximation [280].

23.1 OPE and moment sum rules

The subprocess:

$$\gamma + \gamma \to \text{hadrons} , \tag{23.1}$$

depicted in Fig. 23.1, where one photon is far off-shell (large Q^2) and the other almost on shell (small k^2), can be considered as a deep-inelastic scattering on a photon target with the kinematic variables:

$$\nu \equiv p_2 \cdot q , \quad \tilde{\nu} = k \cdot q , \quad Q^2 \equiv -q^2 , \quad x = Q^2/2\nu , \quad y = Q^2/2\tilde{\nu} , \tag{23.2}$$

and the DIS limit:

$$Q^2 , \ \nu , \ \tilde{\nu} \to \infty , \qquad -k^2/Q^2 \ll 1 . \tag{23.3}$$

One can also express these variables in terms of the energy E_1' and scattering angle θ_1 of the hard scattered electron, the energy E of the incident electron, the scattered angle θ_2 of the target electron and the invariant hadronic mass W. In this way, one has:

$$Q^2 = 4EE_1' \sin^2 \frac{\theta_1}{2} , \qquad -k^2 \simeq EE_2'\theta_2^2 , \tag{23.4}$$

and:

$$x = \frac{E_1' \sin^2(\theta_1/2)}{E - E_1' \cos^2(\theta_1/2)} , \qquad y = \frac{Q^2}{Q^2 + W^2} . \tag{23.5}$$

The formalism is very similar to the case of ep scattering discussed previously where the gluon is now replaced by a photon. The derivation of the moment sum rules is based on the

https://doi.org/10.1017/9781009290296.032 Published online by Cambridge University Press

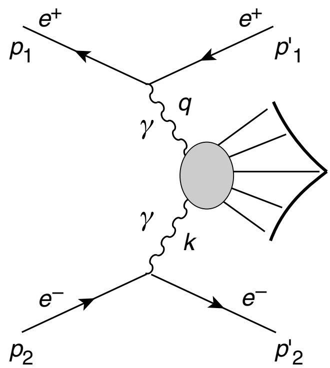

Fig. 23.1. $e^+e^- \to e^+e^- +$ hadrons process.

OPE of the T-product of two electromagnetic currents ($-q^2 \to \infty$):

$$\begin{aligned}
iJ_\mu(q)J_\nu(-q) \sim & \sum_{n=2,\text{even}} \sum_h \mathcal{O}^{h,n}_{\mu_1\ldots\mu_n}(0)\frac{2^n}{(-q^2)^{n+1}} \\
& \times \left[C_1^{h,n}(-q^2)q^{\mu_1}\ldots q^{\mu_n}(g^{\mu\nu}q^2 - q^\mu q^\nu)\right. \\
& \left. + C_2^{h,n}(-q^2)q^{\mu_3}\ldots q^{\mu_n}(g^{\mu\mu_1}q^2 - q^\mu q^{\mu_1})(g^{\nu\mu_2}q^2 - q^\nu q^{\mu_2})\right] \\
& + \sum_{n=1,\text{odd}} \sum_h \mathcal{O}^{h,n}_{3,\mu_1\ldots\mu_n}(0)\frac{2^n}{(-q^2)^n}C_3^{h,n}(-q^2)q^{\mu_2}\ldots q^{\mu_n}i\epsilon^{\mu\nu\alpha\mu_1}q_\alpha\ . \quad (23.6)
\end{aligned}$$

where $\mathcal{O}^{h,n}_{\mu_1\ldots\mu_n}$ and $\mathcal{O}^{h,n}_{3,\mu_1\ldots\mu_n}$ are set of even and odd parity, twist-2 operators (including photons) listed in Eqs. (15.60), (16.3) and (16.4). The sum h runs over non-singlet, singlet, gluon and photon operators. Introducing this expression into the four-point function $J_\mu J_\nu A_\lambda A_\rho$, one obtains:

$$\begin{aligned}
&\langle 0|\mathcal{O}^{h,n}_{\mu_1\ldots\mu_n}A_\lambda(k)A_\rho(-k)|0\rangle \\
&= \frac{1}{k^4}\hat{\mathcal{O}}^{h,n}(k^2)k_{\mu_3}\cdots k_{\mu_n}(k^2 g_{\lambda\mu_1}g_{\rho\mu_2} - k_\lambda k_{\mu_1}g_{\mu_2\rho} - k_\rho k_{\mu_2}g_{\mu_1\lambda} + k_{\mu_2}k_{\mu_1}g_{\lambda\rho}) \\
&\qquad\qquad (n \geq 2,\ \text{even})\ ,
\end{aligned}$$

and

$$\langle 0|\mathcal{O}^{h,n}_{3,\mu_1\ldots\mu_n}A_\lambda(k)A_\rho(-k)|0\rangle = \frac{1}{k^4}\hat{\mathcal{O}}^{h,n}_3(k^2)k_{\mu_2}\cdots k_{\mu_n}i\epsilon_{\lambda\rho\mu_1\alpha}k^\alpha \qquad (n \geq 2,\ \text{odd}) \quad (23.7)$$

Therefore, the moments of the photon structure functions read:

$$\begin{aligned}
\mathcal{M}_L^{(n)} &\equiv \int_0^1 dy\ y^{n-1}F_L^\gamma(y, Q^2, k^2) = \sum_h C_L^{h,n+1}(Q^2)\hat{\mathcal{O}}^{h,n+1}(k^2)\ , \\
\mathcal{M}_2^{(n)} &\equiv \int_0^1 dy\ y^{n-1}F_2^\gamma(y, Q^2, k^2) = \sum_h C_2^{h,n+1}(Q^2)\hat{\mathcal{O}}^{h,n+1}(k^2)\ , \\
\mathcal{M}_1^{(n)} &\equiv \int_0^1 dy\ y^{n-1}g_1^\gamma(y, Q^2, k^2) = \sum_h C_3^{h,n}(Q^2)\hat{\mathcal{O}}_3^{h,n}(k^2)\ , \quad (23.8)
\end{aligned}$$

https://doi.org/10.1017/9781009290296.032 Published online by Cambridge University Press

where $C_L \equiv C_2 - C_1$ and C_3 are Wilson coefficients and $\hat{\mathcal{O}}$ are reduced operators or form factors.

23.2 Unpolarized photon structure functions

One can introduce the 'electron' structure function F_2^e similarly to the case of the proton structure function in ep scattering given in Eq. (15.50) (the other structure functions F_1^e and F_L^e are defined in a similar way). In terms of which, the unpolarized cross-section reads [267]:

$$\sigma = 2\pi\alpha^2 \frac{1}{s} \int_0^\infty \frac{dQ^2}{Q^2} \int_0^1 \frac{dx}{x^2} \left[\left(\frac{xs}{Q^2} - 1 + \frac{Q^2}{2xs} \right) F_2^e - \frac{Q^2}{2xs} F_L^e \right] . \tag{23.9}$$

The 'electron' structure function can be related to the conventional photon structure function F_i^γ using the Altarelli–Parisi evolution equation [281]:

$$F_i^e(x, Q^2) = \frac{\alpha}{2\pi} \int_0^\infty \frac{dk^2}{k^2} \int_x^1 \frac{dy}{y} \frac{x}{y} P_{\gamma e}\left(\frac{x}{y}\right) F_i^\gamma(y, Q^2, k^2) , \tag{23.10}$$

where $i \equiv 2, L$ and:

$$P_{\gamma e} = \frac{1}{z}(1 - (1-z)^2) , \tag{23.11}$$

is the splitting function. Using the previous evolution equation into the expression of the cross-section, one can derive the x-moments of the cross-section:

$$\int_0^1 dx\, x^n \frac{d^3\sigma}{dQ^2 dx dk^2} = \frac{\alpha^3}{Q^4 k^2} \int_0^1 z^n\, P_{\gamma e}(z) \int_0^1 y^{n-1} F_i^\gamma(y, Q^2, k^2) . \tag{23.12}$$

For $n = 1,3,\ldots$, the z integral is finite, to which corresponds the moment sum rules of the structure functions:

$$\mathcal{M}_i^{(n)} = \int_0^1 y^{n-1} F_i^\gamma(y, Q^2, k^2) = \sum_h C_i^{h,n+1}(Q^2) \hat{\mathcal{O}}^{h,n+1}(k^2) . \tag{23.13}$$

One can notice that for $n = 0$ (total cross-section), the z integration is logarithmically divergent. More explicitly, one can express the cross-section as:

$$\frac{d^2\sigma}{dQ^2 dy}(ee \to eeX) \simeq \frac{d^2\sigma}{dQ^2 dy}(e\gamma \to eeX)\Phi(E) . \tag{23.14}$$

where the photon flux factor is:

$$\Phi(E) \equiv \frac{\alpha}{2\pi} \int_0^1 z^n\, P_{\gamma e}(z) \int_0^\infty \frac{dk^2}{k^2} \approx 2\frac{\alpha}{\pi} \ln \frac{E}{E_{\min}} \ln \frac{E\theta_{2\,\max}}{m_e} \tag{23.15}$$

after taking the cuts:

$$-k^2_{\max} = E^2\theta^2_{2\,\max} , \quad -k^2_{\min} = m_e^2 , \quad E_\gamma \geq E_{\min} , \quad z_{\min} = E_{\min}/E . \tag{23.16}$$

Assuming that the photon structure function is crudely approximately constant, and using the differential cross-section:

$$\frac{d^2\sigma}{dQ^2dy}(e\gamma \to eX) = 2\pi\alpha^2\frac{1}{Q^4}\left(1 - t + \frac{t^2}{2}\right)\frac{1}{y}F_2^\gamma(y, Q^2)\,, \tag{23.17}$$

where $t = Q^2/ys$, one can deduce for $Q^2/xs \ll 1$:

$$\sigma = \alpha^3\frac{4}{Q^2_{\min}}\ln\frac{E}{E_{\min}}\ln\frac{E\theta_{2\,\max}}{m_e}\ln\frac{2E^2}{\tilde{\nu}_{\min}}F_2^\gamma\ , \tag{23.18}$$

where further cuts $\tilde{\nu}_{\max} = s/2$ and $\tilde{\nu}_{\min}$ have been taken for $\tilde{\nu} \equiv k \cdot q$. One recovers the result of [280] obtained using the equivalent photon approximation.

The parton model contribution to F_2^γ comes from the box diagram and dominates over the vector meson contribution. For large Q^2, the *parton model* expression reads:

$$F_2^\gamma(x, Q^2) = \left(N_c\sum_i Q_i^4\right)8\alpha^2 x P_q\gamma(x)\ln Q^2 \tag{23.19}$$

where $P_q\gamma(x)$ is the splitting function encountered previously in the case of *ep* scattering but the gluon is now replaced by a photon. Witten [282] pointed out that QCD corrections affect the parton model expression in Eq. (23.19), and his result has been extended to next order in [283]. The moments of the photon structure functions can be expressed in a similar way as in the case of gluons, where there is a mixing between the quark and photon operators. It reads [282]:

$$\int_0^1 dx\ x^{n-2}F_2^\gamma(x, Q^2) \sim \alpha\left[a_n\ln\frac{Q^2}{\Lambda^2} + \tilde{a}_n\ln\ln\frac{Q^2}{\Lambda^2} + b_n + \mathcal{O}\left(\frac{1}{\ln\frac{Q^2}{\Lambda^2}}\right)\right], \tag{23.20}$$

where the VDM contributions are included in the $1/\ln Q^2$ term. a_n, $\tilde{a}_n$ and b_n have been calculated in perturbation theory by the previous authors: a_n depends on the one-loop anomalous dimension and one-loop β-function; $\tilde{a}_n$ depends in addition on the two-loop β function. In addition to the previous dependences, b_n depends also on the two-loop anomalous dimensions and one-loop contribution to the Wilson coefficients, and is renormalization-scheme dependent. Extensive phenomenology of this process exists in the literature (see for example [49]).

23.3 Polarized process: the 'spin' of the photon

23.3.1 Moments and cross-section

We will be interested here in the polarized $\gamma\gamma$ process, in which one can test the idea of the universality of the topological charge screening discussed in the previous chapter. An approach similar to the case of the unpolarized $\gamma\gamma$ process gives the results in terms of the

g_1 structure function as defined in Eq. (19.1):

$$g_1^e(x, Q^2) = \frac{\alpha}{2\pi}\int_0^\infty \frac{dk^2}{k^2}\int_x^1 \frac{dy}{y}\frac{x}{y}\Delta P_{\gamma e}\left(\frac{x}{y}\right) g_1^\gamma(y, Q^2, k^2)\,, \tag{23.21}$$

where:

$$\Delta P_{\gamma e} = 2 - z\,, \tag{23.22}$$

is the splitting function. The ratio of the polarized over the unpolarized cross-section is:

$$\frac{\Delta\sigma}{\sigma} = \frac{1}{2}\frac{\tilde a_n}{a_n}\frac{Q^2_{\rm min}}{s}\ln\frac{Q^2_{\rm max}}{Q^2_{\rm min}}\left[1 + \ln\frac{Q^2_{\rm max}}{\Lambda^2}\left(\ln\frac{Q^2_{\rm min}}{\Lambda^2}\right)^{-1}\right], \tag{23.23}$$

where one can approximately take $a_n \simeq \tilde a_n$. The moment is given in Eq. (23.8). The Wilson coefficients have a 3×3 anomalous dimension matrix γ_n^{hh} in the hadron sector and another $\gamma_n^{h\gamma}$ reflecting the mixing of the photon and singlet hadron operators. It explicitly reads:

$$\begin{aligned}\mathcal{M}_1^{(n)}(Q^2, k^2) = {}& C_3^{h,n}(1, \alpha_s(Q^2))\mathcal{T}\exp - \int_0^t dt'\gamma_n^{hh}(\alpha_s(t'))\hat{\mathcal{O}}_3^{h,n}(k^2, \alpha_s(\mu), \alpha)\\ &+ \left[C_3^{h,n}(1, \alpha_s(Q^2))\mathcal{T}\exp - \int_0^t dt'\gamma_n^{h\gamma}(\alpha_s(t')) + C_3^{\gamma,n}(1, \alpha_s(Q^2))\right]\\ &\times \hat{\mathcal{O}}_3^{\gamma,n}(k^2, \alpha_s(\mu), \alpha)\,.\end{aligned} \tag{23.24}$$

To leading order, one obtains:

$$\mathcal{M}_1^{(n)}(Q^2, k^2) = \frac{\alpha}{4\pi}\tilde a_n \ln\frac{Q^2}{\Lambda^2} \qquad n \geq 3 \text{ odd.} \tag{23.25}$$

For $n = 1$, there is no operator $R_{\gamma,1}$, such that the lowest twist 2 operator is the axial current $J_{\mu 5}$. To, leading order, one can write

$$\begin{aligned}\mathcal{M}_1^{(n)}(Q^2, k^2) = {}& \sum_{a\neq 0} 2\mathrm{Tr}\,(Q^2\lambda^a)\hat{\mathcal{O}}_3^{a,1}(k^2, \alpha_s, \alpha)\\ &+ n_f^{-1}\mathrm{Tr}\; Q^2\exp\left\{-\int_0^t dt'\gamma(\alpha_s(t'))\hat{\mathcal{O}}_3^{\gamma,n}(k^2, \alpha_s(\mu), \alpha)\right\}\,.\end{aligned} \tag{23.26}$$

23.3.2 The g_1^γ sum rule and the axial anomaly

The AVV vertex and chiral Ward identities

Let us define the vertices:

$$\begin{aligned}\Gamma^a_{\mu\lambda\rho}(p, k_1, k_2) &\equiv \langle 0|J^a_{\mu 5}(p)J_\lambda(k_1)J_\rho(k_2)|0\rangle \;:\; J^a_{\mu 5} = \bar\psi\gamma_\mu\gamma_5\lambda^a\psi\,,\\ \Gamma^a_{5\lambda\rho}(p, k_1, k_2) &\equiv \langle 0|J^a_5(p)J_\lambda(k_1)J_\rho(k_2)|0\rangle \;:\; \Phi^a_5 = i\bar\psi\gamma_5\lambda^a\psi\,,\\ \Gamma_{\mathcal{Q}\lambda\rho}(p, k_1, k_2) &\equiv \langle 0|Q(p)J_\lambda(k_1)J_\rho(k_2)|0\rangle \;:\; \mathcal{Q} = (\alpha_s/8\pi)\,\mathrm{Tr}\,\tilde G_{\mu\nu}G^{\mu\nu}\,,\end{aligned} \tag{23.27}$$

https://doi.org/10.1017/9781009290296.032 Published online by Cambridge University Press

where λ^a are $SU(3)$ matrices. The conservation of the electromagnetic currents implies:

$$k_1^\lambda \Gamma^a_{\mu\lambda\rho}(p, k_1, k_2) = 0 = k_2^\rho \Gamma^a_{\mu\lambda\rho}(p, k_1, k_2)\,. \tag{23.28}$$

The vertices obey the anomalous chiral Ward identities.

$$\begin{aligned} &ip^\mu \Gamma^a_{\mu\lambda\rho} - 2m\Gamma^a_{5\lambda\rho} + \frac{N_c}{4\pi^2} l_a \epsilon_{\lambda\rho\alpha\beta} k_1^\alpha k_2^\beta = 0 \quad (a \neq 0) \\ &ip^\mu \Gamma^0_{\mu\lambda\rho} - 2m\Gamma^0_{5\lambda\rho} - 2n_f \Gamma_{Q\lambda\rho} + \frac{N_c}{4\pi^2} l_a \epsilon_{\lambda\rho\alpha\beta} k_1^\alpha k_2^\beta = 0\,, \end{aligned} \tag{23.29}$$

where: $l_a = \mathrm{Tr} Q^2 \lambda^a$ is related to the quark charge Q in units of e. Then, for $n_f = 3$, $l_0 = 2/3,\ l_3 = 1/6$, and $l_8 = 1/(6\sqrt{3})$. The AVV vertex function has the general Lorentz decomposition:

$$\begin{aligned} -i\langle 0|J^a_{\mu 5}(p) J_\lambda(k_1) J_\rho(k_2)|0\rangle = {} & A_1^a \epsilon_{\mu\lambda\rho\alpha} k_1^\alpha + A_2^a \epsilon_{\mu\lambda\rho\alpha} k_2^\alpha \\ & + A_3^a \epsilon_{\mu\lambda\alpha\beta} k_1^\alpha k_2^\beta k_{2\rho} + A_4^a \epsilon_{\mu\rho\alpha\beta} k_1^\alpha k_2^\beta k_{1\lambda} \\ & + A_5^a \epsilon_{\mu\lambda\alpha\beta} k_1^\alpha k_2^\beta k_{1\rho} + A_6^a \epsilon_{\mu\rho\alpha\beta} k_1^\alpha k_2^\beta k_{2\lambda}\,, \end{aligned} \tag{23.30}$$

where $A_i^a(p^2, k_1^2, k_2^2)$ are invariants. In the case of the $n = 1$ sum rule with $p = 0,\ k_1 = -k_2 = k$, one can deduce away from the chiral limit $m_\pi^2 \neq 0$:

$$\hat{\mathcal{O}}_3^{a,1}(k^2) = 4\pi\alpha \left(A_1^a - A_2^a\right)(0, k^2, k^2)\,. \tag{23.31}$$

Rewriting:

$$\left(A_1^a - A_2^a\right)(0, k^2, k^2) = \frac{N_c}{4\pi^2} \mathrm{Tr}(Q^2 \lambda^a) F_a(k^2, \mu^2)\,, \tag{23.32}$$

one obtains for $n_f = 3$:

$$\begin{aligned} \mathcal{M}_1^{(1)}(Q^2, k^2) &\equiv \int_0^1 dy\ g_1^\gamma(y, Q^2, k^2) \\ &= \frac{\alpha}{18\pi}[3F_3(k^2) + F_8(k^2) + 8F_0(k^2, Q^2)]\,, \end{aligned} \tag{23.33}$$

where the singlet form factor F_0 has a non-trivial Q^2 dependence due the anomalous dimension γ.

Non-singlet form factors

Using the conservation of the electromagnetic current on the AVV (amputated) vertex in Eq. (23.28), one can derive:

$$A_1^a = A_3^a k_2^2 - A_5^a \frac{1}{2}\left(k_1^2 + k_2^2 - p^2\right)\,. \tag{23.34}$$

Assuming a smooth behaviour of the form factors in the limit $p \to 0$ and $k_1 \to -k_2 \to \pm k$, one obtains:

$$A_1^a(0, k^2, k^2) = k^2\left(A_3^a - A_5^a\right)(0, k^2, k^2)\ \mathcal{O}(k^2)\,, \tag{23.35}$$

https://doi.org/10.1017/9781009290296.032 Published online by Cambridge University Press

by assuming in addition that there is no $1/k^2$ pole in the form factors A_i ($i \equiv 3, 5$) (and $i \equiv 4, 6$ if one assumes that a similar result holds for A_2^a). Defining the form factor $\mathcal{F}_a$:

$$2m\Gamma^a_{5\lambda\rho} = \mathcal{F}_a \epsilon_{\lambda\rho\alpha\beta} k_1^\alpha k_2^\beta \ , \tag{23.36}$$

and considering the previous Ward identities, one obtains:

$$\left(A_1^a - A_2^a\right)(0, k^2, k^2) = -\mathcal{F}_a(k^2) + \frac{N_c}{4\pi^2} l_a \ . \tag{23.37}$$

Identifying with the result in Eq. (23.32), one obtains:

$$F_a(k^2) = 1 - \frac{\mathcal{F}_a(k^2)}{\mathcal{F}_a(0)} \ . \tag{23.38}$$

Expressing the PVV vertex in terms of the pion field and coupling to $\gamma\gamma$, one obtains the leading-order relation:

$$\mathcal{F}_a(k^2) = \frac{1}{8\pi\alpha} f_\pi g_{\pi_a\gamma^*\gamma^*}(k^2) \ : \quad f_\pi = 92.4 \text{ MeV} \ , \tag{23.39}$$

which gives:

$$F_a(k^2) = 1 - \frac{g_{\pi_a\gamma^*\gamma^*}(k^2)}{g_{\pi_a\gamma\gamma}(0)} \ . \tag{23.40}$$

Using an OPE of the PVV vertex for large k^2, one obtains:

$$\langle \pi_a | J_\lambda(k) J_\rho(-k) | 0 \rangle \sim 2\epsilon_{\lambda\rho\alpha\mu} \frac{k^\alpha}{k^2} C_3^{a,1}(k^2) \langle \pi_a | J_5^{a\mu}(0) | 0 \rangle \ . \tag{23.41}$$

Therefore, one can deduce:

$$F_a(k^2) = 1 - \frac{16\pi^2}{N_c} \frac{f_\pi^2}{(-k^2)} + \cdots \tag{23.42}$$

Combining this result with the one in Eq. (23.40), one can deduce that the form factor interpolates smoothly from 0 to 1 when k^2 varies from zero to infinity. We can parametrize this behaviour as:

$$F_a(k^2) = \frac{-k^2}{-k^2 + M_a^2} \ , \tag{23.43}$$

where:

$$M_a^2 \simeq \left(\frac{16\pi^2}{N_c}\right) f_\pi^2 \simeq 0.67^2 \text{ GeV}^2 \ , \tag{23.44}$$

is a characteristic hadronic mass scale indicative of the non-perturbative realization of the AVV vertex in the spontaneously broken chiral symmetry phase of QCD. It can be related to the quark vacuum condensate in the QCD spectral function analysis of the vertex function.

https://doi.org/10.1017/9781009290296.032 Published online by Cambridge University Press

Singlet form factors

The situation is much more involved here due to the presence of the $U(1)$ anomaly [255,256]. Defining the form factor as:

$$\Gamma_{Q\lambda\rho} = \frac{1}{n_f}\mathcal{F}_0\epsilon_{\lambda\rho\alpha\beta}k_1^\alpha k_2^\beta \,, \tag{23.45}$$

and using the fact that $A_1^0 - A_2^0 = \mathcal{O}(k^2)$, one can write:

$$F_0(k^2) = 1 - \frac{\mathcal{F}_0(k^2,\mu^2)}{\mathcal{F}_0(0)} \,. \tag{23.46}$$

One can introduce the OZI Nambu–Goldstone boson η_0 associated with the singlet pseudoscalar field Φ_5^0 defined in Eq. (23.27) and its decay constant f_{η_0}, with $\eta^0 = 2\langle\Phi\rangle^{-1} f_{\eta_0}\Phi_5^0$. The latter being related to the first moment of the topological susceptibility:

$$f_{\eta_0} = 2n_f\sqrt{\chi'(0)} \,, \tag{23.47}$$

with:

$$\chi(p^2) = i\int d^4x\ e^{ipx}\langle 0|\mathcal{T}Q(x)Q^\dagger(0)|0\rangle \,. \tag{23.48}$$

An approach similar to the case of the non-singlet current gives:

$$F_0(k^2,\mu^2) = 1 - \frac{g_{\eta_0\gamma^*\gamma^*}(k^2)}{g_{\eta_0\gamma\gamma}(0)} \,. \tag{23.49}$$

However, the situation is more complicated as η_0 is not a physical state, while $g_{\eta_0\gamma^*\gamma^*}$ and $g_{\eta_0\gamma\gamma}$ are not RG invariants. Therefore, we approximate the η_0 by the η' and replace the difference of the couplings by their OZI limit $g_{\eta'\gamma^*\gamma^*} - g_{\eta'\gamma\gamma}$ which is RG invariant. We also replace the anomaly coefficient using the relation:

$$f_{\eta'}g_{\eta'\gamma\gamma} = 2N_c\frac{2}{3}\frac{\alpha}{\pi} \,, \tag{23.50}$$

where:

$$f_{\eta'} = \frac{1}{M'_\eta}2\langle\Phi\rangle\left[i\int d^4x\ e^{ipx}\langle 0|\mathcal{T}\Phi_5^0(x)\Phi_5^0(0)|0\rangle\right]^{-1/2} \,. \tag{23.51}$$

The form factor reads [281]:

$$\begin{aligned} F_0(k^2,\mu^2) \sim \frac{f_{\eta_0}(0,\mu^2)}{f_{\eta'}}\Bigg[1 - &\frac{8\pi^2}{N_c n_f}f_{\eta'}f_{\eta_0}(0,\mu^2) \\ &\times\left(\mathcal{T}\exp - \int_0^t dt'\ \gamma(\alpha_s(t'))\right)\frac{1}{(-k^2)} + \cdots\Bigg] \,. \end{aligned} \tag{23.52}$$

https://doi.org/10.1017/9781009290296.032 Published online by Cambridge University Press

The associated scale for interpolating the singlet form factor from 0 to 1 is:

$$M_0^2 \simeq \frac{8\pi^2}{N_c n_f} f_{\eta'} f_{\eta_0}(0, k^2) \, . \tag{23.53}$$

The explanation of the proton spin proposed in the previous section requires a small value of $f_{\eta_0}(0, Q^2 = 11 \text{ GeV}^2)$ compared to its OZI value of $\sqrt{6} f_\pi$.

Implications for the moment sum rules

Introducing the previous behaviour of the form factors, one can deduce from Eq. (23.33):

$$\int_0^1 dy \, g_1^\gamma(y, Q^2, k^2 = 0) = 0 \, , \tag{23.54}$$

and for $M_\rho^2 \ll -k^2 \ll Q^2$:

$$\int_0^1 dy \, g_1^\gamma(y, Q^2, k^2) \simeq N_c \frac{\alpha}{\pi} Q_f^4 \left(1 - c + c \frac{f_{\eta_0}(0, Q^2)}{f_{\eta'}}\right) \, , \tag{23.55}$$

with:

$$c = \frac{1}{n_f} \left(\sum_f Q_f^2\right)^2 \Big/ \sum_f Q_f^4 \, , \tag{23.56}$$

where the deviation from the naïve leading-order value comes from the effect of the $U(1)$ anomaly. Related phenomenology of the $U(1)$ anomaly but on the $\eta'(\eta) \to \gamma\gamma$ decays is reviewed in [284].