ENERGY DISSIPATION MECHANISMS IN THE SOLAR CORONA

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ABSTRACT

Present ideas concerning the electric heating of the solar corona are reviewed. We consider in some more detail the dissipation of MHD waves in strong horizontal gradients of the Alfvén velocity. Then we consider the evolution of DC currents in the solar corona. Some theories aiming at the evaluation of the net rate of energy dissipation by such mechanisms will be described. A short account will be given of a recent analytical study based on a generalization of Taylor's hypothesis concerning the evolution of magnetic helicity in plasma with a large magnetic Reynolds number.

1. INEFFICIENCY OF ACOUSTIC HEATING

The fact that the solar corona is made of a very hot ($2\ 10^6$ K), tenuous plasma ($n_e = 10^{10}$ cm^{-3}) has been an intriguing fact for a number of years. The heating of this outer atmosphere of the sun, and of other stars as well, poses a theoretical question that has not yet received an answer. This heating results from the dissipation of mechanical motions, as for example MHD waves generated by the subphotospheric convection zone of the star, or from the continuous release of magnetic energy pumped into the solar corona by the stresses exerted in the photosphere and convection zone on the low lying parts of coronal magnetic field lines. Up to some five years ago, the standard belief was that chromospheric and coronal heating were the result of the dissipation of sound waves emanating from the sun's convection zone, steepening into shocks when reaching the tenuous top of the atmosphere. This classical scenario (Schatzman, 1949) has been elaborated in great detail, and is still a very viable candidate for understanding the heating of the sun's chromosphere. For a review, see Kuperus et al. (1981) and Ulmschneider (1981). However, this view has been strongly modified, concerning the corona, when the Skylab X-ray pictures have shown a solar corona heavily structured by its magnetic field in the form of loops. Also, when reasonably reliable measures of the flux of accoustic energy emanating from the dense atmosphere towards the corona were made, they revealed that this energy flux is insufficient by a factor

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approximately 10 to feed the energy lost by the X and UV radiating corona Athay and White, 1978 ; Mein et al. 1980 ; Schmieder and Mein, 1980). Results by Mein's and Schmieder include phase information and give a measured flux of 4 10^3 ergs/cm^2/s at 1500 km, to be compared to the 3 10^5 ergs cm^{-2} s^{-1} needed to feed the radiative losses above this level.

2. DISSIPATION OF CURRENTS REQUIRES ENHANCED DAMPING

Currents are driven in the solar corona as a consequence of the motions of the heavy atmosphere ($\beta > 1$) which move the footpoints of coronal field lines more or less at random. If the characteric time of these motions is longer than the Alfvén transit time along closed loops, the coronal magnetic configuration continuously adapts to these changes at the boundary through series of magnetohydrostatic equilibria, at least if such equilibria can be found. These configurations normally carry currents, and the permanent dissipation of these permanently re-created currents could be the cause of coronal heating. If motions of the boundary are faster than an Alfvén transit time, MHD oscillations are set up in the corona. These carry (A.C.) currents too. Of course these oscillations may propagate, be evanescent or form standing waves according to the nature of the coronal environment. The dissipation of these oscillations could also give rise to coronal heating.

However, both of these ideas share a common difficulty: dissipation is impossible by "normal" dissipative processes. This is particularly obvious for DC currents. The time scale for dissipating currents by Joule effect is not less than 10^6 years. Similarly, MHD waves in an homogeneous medium having the properties of the solar corona are damped on very long time scales (Uchida and Kaburaki, 1974). In brief, if the corona is really heated by electric currents, nature must have invented some trick to hasten their dissipation. Much of the recent and past work on solar coronal heating has been concerned with the understanding of that trick.

It is not possible to review here most of the ideas which have been considered, some of which were marginally successful, as, far example the theory based on the non-linear interaction of counterstreaming Alfvén waves, coupling to form a slow mode wave which is efficiently damped (Wentzel, 1974 ; Kabuaki and Uchida, 1971, 1974 ; Chin and Wentzel, 1972). This theory was not pursued after Wentzel (1976), recognized that it could meet the observational requirements only for loops having magnetic fields less than 10 G., the effect being extremely sensitive to the value of the magnetic field.

In a similar way Hollweg et al. (1982) have studied the possibility that Alfvén waves in flux tubes could non-linearly steepen into shock waves in the chromosphere and enter in the corona as so called switch-on shocks. The heating due to trains of such shocks give rise to the following heating rate per unit volume (Hollweg, 1982):

$$Q_{\text{switch on shocks}} = B_o^2/(32\pi\tau) \; . \; (\Delta v/v_A)^4$$

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where Δv is the transverse velocity jump in the shock and τ is the period of wave trains. The effect scales as B^{-2} and is then found effective only in weak field regions, like coronal holes, if τ is small (10Cs) and Δv large enough (200 km/s, say) (Hollweg, 1983b). The situation in this respect is similar to that reported by Wentzel for weak turbulence effect on Alfvén waves. The present trend in research is to improve on the geometrical aspects of all these phenomena, because of the strong coronal structuration due to gravitationnal and magnetic fields. WKB, as a rule, is a very poor approximation, and quasi static evolution is heavily dominated by the magnetic configuration.

3. WAVE ACCESS TO THE SOLAR CORONA: REFRACTION AND REFLECTION

Ignoring for conciseness the problem of wave generation (Unno, 1964 ; Stein, 1968 ; Ulmschneider, 1981), it is important to recognize the extreme importance of wave reflection and refraction at the chromosphere-corona interface. In a low β plasma, for example, fast modes obey the dispersion relation $\omega^2 = k^2 v_A^2$. Splitting k into an horizontal (conserved) component k_H and a vertical component k_v yields $k_v^2 = \omega^2/v_V^2 - k_H^2$. This implies wave evanescence for large enough v_A, which is actually the case for the solar corona. Fast waves are trapped in the sun's low atmosphere. Slow mode and Alfvén waves on the other hand are chanelled by the magnetic field.

However, even for field-aligned waves, non WKB effects produce strong reflection. Actually the wave length of oscillations of any given period increases enormously as it propagates into a more tenuous plasma, because c_S and v_A increase. As a rule, the wavelength becomes much larger than the scale height, and a full wave description should be considered. These effects have been considered in a series of papers by B. Leroy, S. Schwarz and N. Bel (Leroy and Bel (1979); Leroy (1980), (1981); Bel and Leroy (1981); Leroy and Schwarz (1982); Schwarz and Leroy (1982) in a model including gravity and a vertical magnetic field. These authors found that the escaping flux does not exceed some 10^6 ergs cm^{-2} s^{-1} in the form of slow modes for large vertical photospheric motions of 100m/s in a field of 3000 G, and is smaller for weaker fields. They similarly found a high reflectivity for Alfvén waves of periods in excess of 100 to 1000 seconds.

The medium has also large horizontal gradients and coronal loops have two feet. Hollweg (1983) has studied wave propagation in loop models consisting of tubes of constant Alfvén velocity (the coronal part) and two ends where the Alfvén velocity increases exponentially to infinity. He considers a transmission-reflection problem, with outgoing waves only on one side. The interesting new result is that the transmission coefficient in the region of incoming waves is very large at resonances of the coronal section, defined by $\omega_{Res} = n \pi v_{Acor}/L$. The transmission resonances have a width which reflects the rate at which energy leaks out of the Alfvén-resonant cavity. The associated quality $Q = \omega_{Res}/\Delta\omega$, is found equal to $L/4\pi h$, where h is the scale height of the atmosphere bordering the coronal section, and is typically of the order of 50. Hollweg estimates

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that, though the loop picks a small part of the incoming spectrum, due to the resonance, the energy flux entering it may be sufficient for feeding its radiative losses. Actually if P is the incident power, with bandwith B, T_{max} the maximum transmission coefficient, the loop receives a power :

$$F = P\, T_{max} \frac{\Delta\omega}{B} \frac{\pi}{2}$$

P may be estimated from photospheric fluctuations and flux divergence:

$$P = 2\, (\rho_{phot}\, v^2_{phot})\, v_{Aphot} \frac{Bcor}{Bphot} \simeq 1.5\ 10^8 \text{ ergs cm}^{-2} \text{ s}^{-1}$$

With $B = 0.1\ s^{-1}$, $Q = 1/50$, $F/P = 0.02$ and $2\pi/\omega_{Res} = 1000$ seconds, Hollweg finds $F \simeq 3\ 10^6$ ergs cm^{-2} s^{-1} for the first resonance, which is large enough.

An interesting effect of refraction of fast mode waves in the corona has been discussed by Habbal, Leer and Holtzer (1979). These authors postulate the presence of coronal fast modes with very short periods, of the order of several seconds, and show that these waves tend to focus into regions of smaller v_A. So, even if the wave flux were uniform at the base of the corona, it could converge to heat selective regions. This would solve the apparent paradox that an isotropically propagating wave-mode could produce field-aligned heterogeneities. This is beautifully shown by Zweibel (1980). However, as shown before, the presence of a high flux of fast modes in the corona is questionable.

4. WAVES IN MAGNETICALLY STRUCTURED MEDIA

Much of the recent literature has been concerned with wave motions in a magnetically structured corona, with large horizontal gradients of the Alfvén velocity. The new feature, in its simplest expression, is that two media with different Alfvén velocity, in contact at a discontinuity may propagate surface waves along their interface. These modes decay exponentially on both sides of the interface, and propagate along the interface, in the limit $c_S >> v_A$ at a velocity given by:

$$\frac{\omega^2}{k^2_{\parallel}} = \frac{B_1^2 + B_2^2}{\mu_o(\rho_1+\rho_2)}$$

See Uberoy (1972), Wentzel (1979 a-b), Ionson (1978). More elaborate structures (sheets or tubes) have been examined for wave properties (Spruit (1981), Roberts (1981); Edwin and Roberts (1982); Roberts and Webb (1979); Webb and Roberts (1980a-b)). These waves are dispersive in the presence of gravity, and obey a Klein-Gordon type of equation (Rae and Roberts (1982). They turn non-linearly into solitons (Roberts and Mangeney, 1982), which propagate at velocities in excess of $c_T = v_A c_S/(v_A^2+c_S^2)^{1/2}$. This

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phenomenon may delay shock formation. But for effects considered in the next chapter, surface waves dissipate little in the body of the plasma itself (Gordon and Hollweg, 1983).

5 EFFECT OF A FINITE GRADIENT OF ALFVEN VELOCITY

New important effects arise if we consider wave propogation in a region where the Alfvén velocity is inhomogeneous. The standard case considered is an inhomogeneity in the x-direction, with straight field lines in the z-direction. If we consider approximately incompressible motions, the wave equation splits into two parts, according to the polarization of velocity fluctuation (along $\vec{ey}$ or along $\vec{ex}$).

(a) $$\vec{v} = \vec{ey}\ \hat{v}(x) \exp i\ (k_{\perp} y + k_{\parallel} z - \omega t)$$

$$(\omega^2 - k_{\parallel}^2 v_A^2(x))\ \hat{v}(x) = 0$$

(b) $$\vec{v} = \vec{ex}\ \hat{v}(x) \exp i\ (k_{\perp} y + k_{\parallel} z - \omega t)$$

$$\frac{d}{dx} (\omega^2 - k_{\parallel}^2 v^2(x)) \frac{d\hat{v}}{dx} = (k_{\perp}^2 + k_{\parallel}^2)\ (\omega^2 - k_{\parallel}^2 v_A^2(x))\ \hat{v}$$

We refer to (a) as shear Alfvén waves and (b) as MHD waves. The solution for case (a) is simply:

$$\hat{v}(x) = v_{\omega}\ \delta\ (x - x_{\omega})$$

where x_{ω} is the location of the Alfvén resonance, defined by:

$$\omega = k_{\parallel}\ v_A(x_{\omega})$$

This means that each magnetic surface oscillates at its own eigenfrequency, independently of the neighbouring surface. For a given $k_{\parallel}$, there is then a continuum of (singular) modes, with frequencies between $\omega_- = k_{\parallel} v_A(-\infty)$ and $\omega_+ = k_{\parallel} v_A(+\infty)$. The case (b) (MHD waves) is more tricky; because the wave equation is singular in the vicinity of the Alfvén resonant point. It is noteworthy that this equation also arises in the study of plasma oscillations in an inhomogeneous plasma. Simply $k_{\parallel} v_A(x)$ is replaced by the local plasma frequency, $\omega_p(x)$. Barston (1964) has shown that this equation admits also a continuum of modes for a given $k_{\parallel}$, with ω between ω_- and ω_+. These modes, however, are singular in the vicinity of their Alfvén resonant point, where they behave like

$$\hat{v} = \log \mid x - x_{\omega} \mid \qquad (x > x_{\omega})$$

$$\hat{v} = \log \mid x - x_{\omega} \mid + i\pi \quad (x < x_{\omega})$$

As in other similar problems, these singular modes can be used to construct mode packets which are perfectly regular. Now, how do we recover the surface mode in the limit of an infinitely sharp interface? This point has given rise to some controversy in the literature. It all started with

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the work of Sedlacek, who solved the initial value problem for the equation:

$$\frac{\partial}{\partial x}\left(\frac{\partial^2}{\partial t^2}+\Omega^2(x)\right)\frac{\partial\hat{}}{\partial x} = k^2\left(\frac{\partial^2}{\partial t^2}+\Omega^2(x)\right)\hat{v}$$

by Laplace transform method. Disregarding mathematical details, it suffices here to say that the continuum of modes introduces cuts in the complex Laplace variable plane. On inverting the Laplace transform of the solution, the Bromwich contour is displaced from the original plane into other Riemann sheets connected to it. Sedlacek has found that one of these sheets contains a pole given approximately, for a sharp but finite transition $(k_{/\!/} a \ll 1)$, by:

$$\omega = k_{/\!/}\sqrt{\frac{B_1^2+B_2^2}{\mu_o(\rho_1\ \rho_2)}}\left(1-i\pi\left(\frac{\rho_1}{\rho_2}+\frac{\rho_2}{\rho_1}-2\right)^{-1} k_{/\!/}\, a\ \frac{|v_{A1}-v_{A2}|}{\sqrt{\frac{B_1^2+B_2^2}{\mu_o(\rho_1+\rho_2)}}}\right)$$

In the limit $k_{/\!/} a \to o$, this pole obviously represents the surface wave. So any surface wave propagating on a finite thickness discontinuity damps in time. There has been some controversy in the literature concerning the question as to whether this damping coefficient really represents "true" damping (Lee, 1980) because the equations from which we start are non dissipative. It is fair to say that this controversy is now quite completely resolved. Perhaps the best account of it is the paper by Rae and Roberts (1981) which gives a detailed description of the evolution of an initial disturbance. It is shown there that the surface wave decays because it couples to ordinary, field aligned, Alfvén modes in the inner part of the interface. There is no dissipation of the wave strictly speaking, but a conversion to the continuum of Alfvén waves. However these waves rapidly phase-mix, and eventually damp, even if the dissipative coefficients like viscosity are very small. This behaviour is well known to laboratory plasma physicists (see for example Tataronis and Grossmann, 1973). We come back to wave damping by phase mixing, for the simpler case of shear Alfvén waves, below. At this point, it is perhaps useful to trace a parallel between these phenomena and the Van Kampen treatment of plasma waves propagation in hot plasmas (see Ecker, 1972, Ch. III). In this case too, eigenmodes form a continuum for a given k. They can be used to solve initial value problems, where the Laplace method discovers a damping (Landau damping) which is not dissipation in the thermodynamical sense of the word, but results from phase mixing of Van Kampen modes. Of course, for most practical purposes, Landau damping behaves like true damping. So does the dissipationless damping found here. Barston modes play the role of Van Kampen modes. Phase mixing is due to a spread of phase velocity with x, instead of v in the Landau problem. Sedlacek's solution is akin to Landau's.

Phase mixing in inhomogeneous structures is an ubiquitous phenomenon, which is in no way related exclusively to surface wave damping. The really relevant problem to treat is to find the response of an inhomogeneous structure excited at its base by random given motions. This problem is considered in the next section in more detail, for the simple case of shear Alfvén waves (Heyvaerts and Priest, 1983).

6. HEATING BY PHASE MIXING OF SHEAR ALFVEN WAVES

We consider an equilibrium which is laterally stratified, with an Alfvén velocity $v_A(x)$, and a field $B_o(x)\,\vec{e}_z$. Modeling open structures we consider a boundary surface at $z = o$, where the fluid motion, in the y direction, is assumed to be prescribed, or, when we model closed loops, we add a second boundary at $z = \ell$, where the fluid motion is also prescribed. We assume the coronal fluid (region $z > o$ or $o < z < \ell$ for, resp., open and closed configurations) to be weakly dissipative. The equation of motion, for small dissipation is then found to write:

$$\frac{\partial^2 v}{\partial t^2} = v_A^2(x)\,\frac{\partial^2 v}{\partial z^2} + \nu\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right)\frac{\partial v}{\partial t}$$

where ν is the sum of kinematic viscosity and ohmic diffusity. In the case of an open configuration, it is convenient to consider an harmonic excitation, coherent on the inhomogeneity scale, at $z = o$. Neglecting dissipation, this excitation will propagate at the velocity $v_A(x)$, and as the altitude z grows these oscillations will become more and more out of phase. In closed configurations, it is the parallel wavelength which is fixed, and stationnary oscillations excited in this resonant cavity progressively phase-mix when time elapses. These oscillators are coupled by weak friction. It is possible to approximately solve equation above. The most interesting case is perhaps that of a closed structure with imposed boundary motions. Separating the boundary motion by putting:

$$v(x,z,t) = v(x,o,t) + \frac{z}{\ell}\,(v(x,\ell,t) - v(x,o,t)) + w(x,z,t)$$

and analyzing w in Fourier series in z on the interval (o,ℓ) (only sine terms contribute), we get for the time dependent coefficients $b_n(x,t)$ the equation ($\Omega_n = 2n\pi v_A(x)/\ell$):

$$\frac{\partial^2 b_n}{\partial t^2} + \Omega_n^2(x)\,b_n - \nu\,\frac{\partial^2}{\partial x^2}\,\frac{\partial}{\partial t}\,b_n = \frac{(-1)^{n+1}}{n\pi}\,\frac{\partial^2}{\partial t^2}\,(v(x,o,t) - v(x,\ell,t))$$

This is the equation of a continuum of forced oscillators (indexed by x), weakly coupled by friction. An approximate solution can be found, because in the limit of small damping and large phase mixing ($\Omega_n(x)t \gg 1$), the Green function is approximately given by:

$$G(t) = \frac{\sin\Omega(x)t}{\Omega(x)t}\,\exp\left(-\frac{\nu}{6}\,t^3\left(\frac{d\Omega}{dx}\right)^2\right)$$

Waves of frequency Ω_n damp in a time

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$$\tau_{Damp} = 6^{1/3} \left(\nu\left(\frac{d\Omega}{dx}\right)^2\right)^{-1/3}$$

Putting $d\Omega/dx = \Delta\omega/a$, where a is the thickness of the inhomogeneous region, we see that the damping time is a multiple of the phase coherence loss time, $(\Delta\omega)^{-1}$:

$$\tau_{Damp} = \left(\frac{6\Delta\omega a^2}{\nu}\right)^{1/3} \frac{1}{\Delta\omega}$$

The dimensionless quantity in parentheses above is a Reynolds number, $R_e \simeq 6\omega a^2/\nu$, approximately equal to 10^5, typically. The damping time is then around 10 periods, and the quality factor near 10-20. However, as shown below, this laminar calculation gives an upper limit to the damping time. Having obtained the solution for b_n's, and hence for $v(x,z,t)$ in terms of its drive $v(x,o,t)$ and $v(x,\ell,t)$, it is a simple matter to calculate the average rate of energy dissipation:

$$\overline{W} = \frac{1}{2}\,\rho(x)\;\nu \int_o^{\ell} \frac{dz}{\ell} \left(\frac{\partial v}{\partial x}\right)^2$$

The calculation gives the time Fourier transform of $(\partial v/\partial x)$ in terms of that of its drive in the form of a response-function:

$$\left(\frac{\partial v}{\partial x}\right)_\omega = Z(x,\omega)\;(v(z=o) - v\;(z=\ell))_\omega$$

When $(v(z=o)-v(z=\ell))$ is a stationnary random process with a power spectrum $P_v(x,\omega)$ the time average of $\overline{W}$ can be expressed as:

$$\overline{\overline{W}} = 2 \int_o^{\infty} d\omega\;|\;Z(x,\omega)\;|^2\;P_v(x,\omega)$$

After some algebra, we get the total heating rate of a slab of width L in the y direction:

$$W = \sum_{n=1}^{\infty} \left(\ell L \int_{-\infty}^{+\infty} dx\right) \frac{\mu}{\pi}\;\rho(x)\;\Omega_n^2(x)\;\;P_v(x,\Omega_n(x))$$

μ is a pure number of order unity, and P_v is the power spectrum of velocity at the border. This is an important result. It shows that in this limit of strong phase mixing and weak damping, the average rate of energy dissipation is independent of viscosity, confirming earlier results of Ionson (1982). The heating rate, in that case, depends only on the properties of the photospheric drive. Ionson's view sheds another light on this question. We review it now.

7. THE LRC CIRCUIT ANALOGY

Ionson (1982) took another, simplified, look at the same problem. His starting point is also our basic equation, with two-sided boundary conditions, except that he expressed the equation for the z component of the electric current, instead of velocity. As we do, he also separates that part which describes the corona from that part which describes the photospheric drive. For the coronal part, he obtains the equation:

$$\frac{\partial^2 j}{\partial t^2} = v_A^2(x) \frac{\partial^2 j}{\partial z^2} + \nu \frac{\partial^2}{\partial x^2} \frac{\partial j}{\partial t}$$

which is just the same as Heyvaerts and Priest's (1983). He then proceeds to average this equation over the entire loop volume. Taking into account the photospheric driving terms, he reduced his wave equation into an oscillating LRC equivalent circuit equation.

$$L \frac{d^2 I}{dt^2} + R \frac{dI}{dt} + \frac{I}{C} = \frac{dE}{dt}$$

where $L = \mu_0 \ell$, $R = R_{photosphere} + \mu_0 \ell \nu / a^2$, $C = \ell / \mu_0 \pi^2 v_A^2$, and $E = \pi a\, v_{phot} B_{phot}$. These are respectively the inductance of the loop circuit, its total resistance (photospheric plus coronal), its capacity (mainly storage of mechanical energy), and the applied equivalent driving voltage. The circuit has an eigen frequency $\omega_0 = (LC)^{-1/2}$, obviously the basic period of coronal Alfvén waves, and it has a quality factor due to dissipation:

$$Q = \frac{1}{R} \sqrt{\frac{L}{C}}$$

Note that the effect of the radial structure in the loop (different Alfvén velocity as a function of radius, effects of phase mixing) have been blurred in the averaging procedure. However the formulation has the advantage of simplicity and globality. For example the average rate of energy dissipation in the loop is given by:

$$\bar{\bar{W}} = \langle R_{loop} I^2 \rangle = R_{loop}\, 2 \int_0^\infty P_I(\omega)\, d\omega$$

where $P_I(\omega)$ is the power spectrum of I, which is related to that of the drive by the resonant circuit equation:

$$P_I(\omega) = |\, I(\omega)\, |^2 = \left| \frac{E(\omega)}{1 + i\, Q \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)} \right|^2$$

Putting this is the expression for $\bar{W}$, and taking into account the fact that the resonance function has bandwith which scales as R, we find again the result that underdamped loops dissipate at a rate independent of viscosity. Both these results show that, for underdamped loops, the velocity power spectrum of the photosphere maps into the loop properties. Given such a spectrum and solving the thermal equilibrium problem we obtain satisfactory results (Ionson, 1982), which obey the well known scalling laws (Rosner et al. 1978; Serio et al. 1981). Martens and Kuperus (1982) show that such a heating causes a surprising behaviour of loops thermal equilibrium, with catastrophic changes in the X-ray visibility of loops as they lengthen.

8. PHASE MIXING AND TURBULENCE

The phase-mixed flow in the Alfvén waves may well not remain laminar especially when the mixing is quite complete. This flow has been examined for stability to the Kelvin-Helmholtz and tearing perturbations. The analysis has been restricted to perturbations developing faster than the wave period, and was local. Without going into details, it suffices to say that propagating waves are stable, but that standing waves very easily suffer the K.H. instability at velocity antinodes. The order of magnitude of the growth rate is (Heyvaerts and Priest, 1983):

$$\gamma_{KH} = k_{\perp} v$$

where v is the velocity in the wave and $k_{\perp}$ the scale length in the cross-loop direction. This flow ceases then to be laminar when $k_{\perp}$ has increased due to phase mixing up to the point $\gamma_{KH} > \omega$. This defines naturally a Kelvin-Helmholtz stability time. After that time the flow becomes turbulent. For any reasonable set of parameters this time is less than, or equal to a couple of wave periods. The important consequence is that standing Alfvenic oscillations in inhomogeneous loops must be turbulent. This of course should reduce quite substantially the effective damping time (Heyvaerts and Priest, 1983). Hollweg (1983) has estimated the volumetric heating rate to be expected from a Kolmogoroff cascade. For an observed $\langle v^2 \rangle^{1/2}$ in the corona of 30 km/s, this rate is 8 10^{-4} ergs cm^{-3} s^{-1}, or an equivalent flux of 8 10^6 ergs cm^{-2} s^{-1}, of the right order of magnitude to heat active region coronal loops.

9. HEATING BY D.C. CURRENTS: GLOBAL ENERGY BALANCE

An alternative class of theories of coronal heating is based on the idea that a part of the energy stored in the corona as a result of slow, quasi-static evolution be permanently released. It is easy to convince one self that the energy flux which flows to the corona in that form (it is actually a Poynting flux) may be just of the right order of magnitude, if the corona were able to get rid of that energy into heat at the same pace. Consider for example a loop of length ℓ, radius R and longitudinal field $B_{\parallel}$. Assume that as a result of boundary motions this loop is twisted by an angle χ. It acquires an azimutal component of B, B_{θ} say, approximately equal to $\chi B_{\parallel} (R/\ell)$ and the stored magnetic energy augments by $\Delta W = (B_{\parallel}^2/8\pi)\ (R^2/\ell^2)\ \pi R^2 \ell \chi^2$. This process takes place in a time $t = R\chi/\Delta v$ where Δv is the difference in velocity between the two sides of the loop, i.e. that part which induces twist. Then $t = \chi R/(R|\nabla v|) \simeq |\nabla v|^{-1}$, and the rate of energy transfer to that loop is (ϕ is the loop's flux):

$$\dot{W} = \Delta W/t = \frac{\chi}{8\pi} B_{\parallel}^2 \frac{\pi R^4}{\ell} \ |\nabla v| = \frac{1}{t} \frac{\chi^2}{8\pi^2} \frac{\phi^2}{\ell^2}$$

which corresponds to an average flux:

$$F = \dot{W}/\pi R^2 = (\chi/8\pi)\ B_{\parallel}^2 (R/\ell)\ R|\nabla v|$$

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For B = 100 G, R = 5000 km, 1 = 50 000 km, $|\nabla v|$ = (1km s^{-1})/1000 km = $10^{-3}s^{-1}$, $\chi = 2\pi$, we get F = 10^7 ergs cm^{-3} s^{-1}. The energy is sufficient. The problem is how to dissipate it quickly enough. Some authors simply assume the possibility to exist. Sturrock and Uchida (1981) elaborate on the preceding derivation by recognizing that boundary motions are not persistent but stochastic, so that loops are successively twisted and untwisted. If the correlation time tc of the boundary motion is short as compared to the dissipation time tD, the state of twist of the loop random walks instead of increasing continunously. The stored magnetic energy of a loop twisted by an angle χ, is:

$$W(\chi) = \frac{1}{8\pi^2} \frac{\phi^2}{\ell} \chi^2$$

as this quantity is quadratic in χ, it increases in time when χ random walks at the rate:

$$\frac{d\langle w\rangle}{dt} = \frac{1}{8\pi^2} \frac{\phi^2}{\ell} \langle\frac{\Delta\chi^2}{\Delta t}\rangle$$

where $\langle\Delta\chi^2/\Delta t\rangle$ is given in terms of the correlation function of the local angular velocity of the fluid at the base of the loop. If the coherence time of these motions is t_c:

$$\langle \frac{\Delta\chi^2}{\Delta t} \rangle = 2 \langle\omega^2\rangle t_c$$

Hence the heating rate is:

$$\frac{d\langle w\rangle}{dt} = \frac{\phi^2}{\pi\mu_o\ell} \langle\omega^2\rangle t_c$$

When entered into a thermal balance equation, scaling laws in agreement with data are also obtained.

Parker (1981a-b,1983a-b) makes it convincing that complex boundary motions must tangle the coronal field in such a way that a great many current sheets spontaneously form in the overlying corona. These sheets must suffer rapid reconnection, so relaxing continuously the amount of energy brought to the corona as a result of work done on the foot points of magnetic field lines. The estimation of this work is straightforward. If B is the vertical component of the field and $B_\perp$ its horizontal component which, after stresses have accumulated for a time t, is of order $B_\perp = Bxt/\ell$, where w is a typical velocity in the photosphere, then the work done per unit surface against the stress $B_\perp/\mu_o$ is:

$$F = w\frac{BB_\perp}{\mu_o} = \frac{B^2}{\mu_o\ell} w^2 t$$

This is the vertical component of the Poynting flux as well. Parker estimates, from observation w = 0.4 km/s, using Smithon studies (1973) of the motion of field knots in the photosphere, and he estimates the time by the condition that reconnection has time enough to relax the stress

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due to field line tangling. He finds, putting $t = h/v_{Rec}$, where h is the tube thickness and v_{Rec} some reconnection velocity, an equivalent heating flux (Parker 1983b):

$$F = w \frac{B^2}{\mu_o} \left(\frac{w}{v_{Rec}}\right) \left(\frac{h}{\ell}\right)$$

Perhaps the most interesting contribution of this series of papers is the detailed demonstration that complex boundary motions must create current singularities and lead to fast dissipation. These results should be put in parallel to those of Syrovatskii (1978) and Bobrova and Syrovatskii (1979) who have obtained similar conclusion for force free fields. In interchange (Uchida and Sakurai, 1977) or the tearing in closed magnetic structures (Galeev et al. 1981) could concur to create fine scale structures and provoque fast magnetic energy release by reconnection.

10. HEATING BY COMPLEX RECONNECTION: THE ROLE OF GLOBAL INVARIANTS

Very turbulent fusion machines were built in the past. May be one of the most intriguing are so called reversed z-pinches, which after a turbulent phase, show a reversal of the sense of the axial magnetic field as compared to the initial situation. Surprisingly enough, the field profile, in the final state is just a constant-α force-free field. Obviously reconnection operates in such discharges during some first phase efficiently enough to change totally the topology of magnetic configuration, but still the field does not relax to a potential field. Only a certain fraction of the magnetic energy can be released quickly to accomodate the topological restructuration of the magnetic field, but not all of it. How much ? Taylor (1974) has suggested that the field will rearrange internally, in such a way as to find itself in the minimum energy state compatible with the conservation of some global invariants, or, more precisely, quasi-invariants, because, as we shall see shortly the quantity which Taylor advocated should be conserved, is in fact not a strict invariant. That quantity is the so called total magnetic helicity:

$$K = \int_{vol} (\vec{A}.\vec{B})\, d^3r$$

where $\vec{A}$ is a vector potential for $\vec{B}$. In perfect MHD, the magnetic helicity is an invariant for each closed flux tube (Woltjer, 1958), and it has the property that the state of minimum magnetic energy subject to a prescribed distribution of magnetic helicity is some general force-free field. It has not been mathematically proved that the helicity should be more conserved in a dissipative system than other quantities conserved in perfect MHD, though some arguments can be given in favour of this idea (see Heyvaerts and Priest, 1983b, and Norman and Heyvaerts (1983)). Taylor's conjecture has led to a success in explaining the behaviour of the Zeta machine, because the state of lowest magnetic energy having a given total helicity is just a constant-α (linear) force-free field, as observed. Heyvaerts and Priest (1983b) have adapted the same idea to the calculation of the heating of the corona by DC currents. The simplest way is to first assume, that, at each moment of its evolution, complex reconnection phenomena occur very rapidly in the magnetic configuration, so that it is also always observed in the lowest energy state compatible with helicity. The latter evolves according to the equation:

$$\frac{dK}{dt} = \int_{Boundary} (\vec{A}.\vec{v})\ (\vec{B}.d\vec{S})$$

Care must be taken of the fact that for an open-ended flux tube K is not a gauge invariant quantity, so that some precise gauge condition has to be imposed. We have treated in detail the specific example of the evolution through the series of force-free configurations given by:

$$B_x = - B_o\ (\ell/k)\ \cos kx\ \exp - \ell z$$

$$B_y = - B_o\ (1-\ell^2/k^2)^{1/2}\ \cos kx\ \exp - \ell z$$

$$B_z = + B_o\ \sin kx\ \exp - \ell z$$

This represents a series of linear force-free fields endowed with translational symetry in the y-direction. Some shear motion along y is prescribed at the boundary. So, the flux distribution keeps the same at the base, as it is (for fixed k and different ℓ) for the series of equilibria described above. The evolution of helicity is then calculated and used to find the parameter ℓ as a function of time. Then, comparing the rate of increase of the stored magnetic energy with the integral of the Poynting flux through the boundary gives the heating rate, which is just the difference between the two. It has been found that this difference is systematically zero! This is a consequence of the time it takes for stresses to relax by dissipation being implicitely taken to be zero in this formulation. This result can be understood: Assume that the system performs an ideal MHD displacement during a very small time tD, and then relaxes by reconnection. During the perfect MHD step, the magnetic energy increases due to the work performed by boundary motions by an amount which is $O(\xi)$ where ξ is the fluid displacement ($\xi \simeq v.tD$). In the ensuing relaxation, the displacement will also be $O(\xi^2)$ but, as the final state is an equilibrium, the change in potential energy is only $O(\xi^2)$. In the limit $\xi \to o$, $(tD \to o)$, a negligible part of the energy increase goes into heat.

It was then felt necessary to develop a second order calculation based on the scheme sketched above: small perfect MHD displacements, lasting a phenomenological "stress relaxation time" tD are followed by episodes of relaxation to the minimum energy state compatible with the new value of the magnetic helicity. The change in magnetic energy of the configuration in the last step be then calculated. It represents the heating during the time tD. This procedure (see details in the paper) yields an explicit analytical expression for the rate of heating for the particular displacement field considered. This expression is not useful in itself, but in its structure. It shows the following features. a - If the boundary motion is such as to keep the field force-free with constant-α (this may happen if the scale of the magnetic field ℓ_B is much less than that of the velocity, ℓ_v, no heating ensues. b - The rate of heating is proportional to the small parameters: $(tD.v/\ell_B)$, where tD is the phenomenological relaxation time, v the velocity of boundary motions, and ℓ_B the scale of magnetic structures on the boundary. As we noted, heating vanishes when this parameter vanishes. On the other hand boundary motions such that $v \gg \ell_B/tD$ just do not relax, and evolve according to perfect MHD, also with no heating. c - The equivalent heating flux may be written in order of magnitude:

$$F = \frac{B^2}{\mu_o} \; v \; \left(\frac{\ell_B}{\ell_B+\ell_v}\right)^2 \left(\frac{t_{D}.v}{\ell_B}\right)$$

This analysis gives again the order of magnitude estimates of other theories namely $B^2/\mu_o.v$, but this maximum is limited by the rapidity of stress-relaxation (factor $tD.v/\ell_B$),and by the geometrical factor, which measures the importance of relaxable stresses accumulated in each step. The same idea of relaxation with conservation of global invariants can also be used to predict the end result of catastrophic phenomena, like flares (Norman and Heyvaerts, 1983): after a flare, the coronal magnetic configuration should be a constant-α force-free field with the helicity of the configuration prior to flare. Also this analysis traces an interesting border between flares and coronal heating. Both phenomena would result from magnetic stress relaxation by reconnection, but heating achieves this result continuously, because boundary motions are slow and stresses never accumulate very much, while flares occur in configurations which have evolved so quickly that stresses could not relax at the same pace as they were built, and ultimately relax very violently.

11. UNIFICATION: THE LRC CIRCUIT ANALOGUE AGAIN

There is something common to the various processes considered in this review. They all deal with the dissipative response of coronal electric circuits driven at various low or high frequencies by the photospheric driver. So it should be possible to formulate a unique simplified theory incorparating them all. Ionson (1984), also in these proceedings, proposes such an unifying scheme: the LRC circuit analogue. See his communication.

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DISCUSSION

D. Smith: What do you consider the best candidate for coronal heating on the basis of your calculations?

Heyvaerts: Our calculations of DC current heating contain a phenomenological parameter which is unfortunately very ill defined. The maximum conceivable equivalent flux from such mechanisms ($B^2v/8\pi$) is of a confortable order of magnitude, but the factors which reduce this efficiency, and which appear in all theories developed so far (geometical factors ℓ_B/ℓ_V, loop aspect ratio, R/ℓ, dissipation time scale ($\tau_D v/\ell_B$)) are all not known with enough precision. If we include the most trivial geometrical factors, the predicted flux still keeps larger than the necessary flux by a factor between 5-10 (Heyvaerts and Priest 1983b). So τ_D should really be just a bit shorter than (ℓ_B/v) to meet the requirement, which is not impossible if we judge from linear tearing times. To reach a firmer conclusion we need simultaneous observations of all the physical parameters of some coronal loop, a more detailed understanding of the dissipation time τ_D, and also a more detailed knowledge of the spectrum of the horizontal velocity which enters in the resonant heating theory. Probably both type of heating are active simultaneously (see Jim Ionson's communication, but in different structures).

Sturrock: Most of your talk has been concerned with loop structures. How are we to understand coronal heating in open-field structures such as coronal holes?

Heyvaerts: Any variation imposed at the foot point of an open field line must result in propagating or evanescent oscillations in the overlying corona. Hence wave mechanisms should be responsible for coronal hole heating. It is important to stress that due to solar wind losses, the energy needs of coronal holes are by no means smaller than those of closed field line regions. The subject of coronal hole heating is less developed than that of loop heating. Nevertheless some specific suggestions have been made (Hollweg 1983), namely the conversion of twist Alfvén wave trains in flux tubes into some type of perturbations which dissipate. In a more speculative way, we suggested (Heyvaerts and Priest, 1983) that phase mixing of evanescent MHD waves in the corona could drive the Kelvin-Helmholtz instability, and trigger a turbulent cascade. If that idea is correct, holes and loops could be heated by similar mechanisms.

Drake: Most of the energy absorbed by Alfvén waves in closed loops will flow down into the photosphere. How is the energy transferred to the open field regions?

Heyvaerts: The open field regions must be subject to special heating mechanisms, evocated in my answer to Dr. Sturrock's question. Loop heating can definitely not be transferred to open regions, at least not deep into them.

Benford: How can observations best distinguish between direct current heating and resonant, or AC heating?

Heyvaerts: The scale of structures involved in subtelescopic; resonant heating by waves should give rise to motions detectable as "microturbulent" line broadening, i.e. to broadening in excess of Doppler. Unfortunately, it is quite difficult to measure lines high enough in the corona, and, on the other hand, the reconnections associated with DC current dissipation also show up as motions. These, however, are expected to be more bursty, and perhaps it would be possible to detect the effect of some modest particle acceleration likely to be associated with this reconnection process (permanent weak X-ray noise, small, but bursty, component on top of the thermal radio emission).