Part 3

ASTEROID ORBIT DETERMINATION AND IMPACTS

Dynamics of Populations of Planetary Systems
Proceedings IAU Colloquium No. 197, 2005
Z. Knežević and A. Milani, eds.

DOI: 10.1017/S1743921304008683

Virtual asteroids and virtual impactors

Andrea Milani

Department of Mathematics, University of Pisa, Via Buonarroti 2, 56127 Pisa, Italy
email: milani@dm.unipi.it

Abstract. When a celestial body, e.g., an asteroid, has been observed only over a short time, its orbit is not well determined but may be anywhere in a *confidence region* where the astrometric residuals are acceptable. This region can be sampled by a swarm of *Virtual Asteroids (VA)* sharing the reality of the asteroid: one of them is real, but we do not know which one. The problem is how to sample the confidence region with a small number of VA, still being able to solve the main problems of asteroid recovery/identification and impact monitoring.

One class of methods uses *random sampling* of the confidence region to mimic with the VA population the probabilistic distributions of the orbits. This class includes the Monte Carlo and the Statistical Ranging methods. When it is critical to detect a very small probability (e.g., of a catastrophic impact) by computing a small number of VA orbits, and also when a large catalog of asteroids has to be handled, it is more efficient to sample the confidence region with a *geometric object*, such as a smooth manifold: it can be sampled uniformly, taking into account its dimension. Our group has developed in the last 6-7 years 1-dimensional sampling methods based upon a differentiable curve, the *Line Of Variations (LOV)*, which can represent, in suitable cases, the spine of the confidence region. The LOV is sampled by uniformly spaced VA, thus interpolation between consecutive VA is possible. This is the basis for the current algorithms of *Impact Monitoring*, used in Pisa and at JPL. The LOV method is also used for recovery of lost asteroids and for identification of independent discoveries of the same object.

When the asteroid has moved on the sky while being observed by $< 1^{\circ}$, the confidence region is wide in two directions and the LOV may be an inappropriate way of sampling it. We have recently developed 2-dimensional sampling methods based upon the concept of *Admissible Region*, a 2-dimensional manifold parameterized by a compact subset of the range/range-rate plane. This region is then sampled by *triangulation*, with each node used as a VA. This allows to define methods for asteroid identification/recovery and for impact monitoring starting from very poor data, such as the ones collected during a single night of observations.

Keywords. orbit determination, surveys, recovery, identification, impacts, multiple solutions

1. Population orbit determination

Modern orbit determination was born with the space age, thus its concepts, algorithms and ways of thinking were based on the problem of determining the orbit of a spacecraft by using the tracking data. The key hypothesis is not that the celestial body is artificial, but that it has a device answering the signals from Earth: this is what I call *collaborative orbit determination*. It is possible to connect a trasponder with a planet, e.g., the Viking landers on Mars allowed to solve for the orbit of the planet within tens of meters, and to determine the orbit of a planet by tracking an orbiting spacecraft, e.g., with BepiColombo the orbit of Mercury will be determined to an accuracy of the order of 10 cm.

The opposite case, *population orbit determination*, is the one in which the observations are the scarce resource. This can occur with natural objects (asteroids, transneptunians, comets) and with artificial ones (space debris). The total number of data points may be comparable to the spacecraft tracking case but, the population being large, the number of observations for each object is on average low.

As an example, we have now $\simeq 600,000$ independent detections of asteroids for a total of $\simeq 50,000,000$ scalar data points†. The problem is made much more difficult by the fact that the non-collaborative observations do not contain a signature identifying the observed body. Thus for of each couple of independent detections of asteroids we do not know whether they correspond to the same body: finding the independent detections of the same body and an orbit fitting all the data is the *asteroid identification problem*.

Additional difficulties arise when, given a population, e.g., of Near Earth Asteroids (NEA), we need to use the few available data for a very accurate prediction, as in the case in which we try to predict whether an impact on Earth in a given year is possible.

To solve these problems we need some new mathematics, and as it is often the case in mathematical research one way forward is an act of courage: if the problem is difficult, let us make it more difficult by generalizing it. If the population, e.g., of asteroids, is big, let us make it bigger: we shall replace each detected asteroid with a swarm of *Virtual Asteroids (VA)* sharing the reality of the asteroid among them, that is, one of them is a good approximation of the real object orbit, but we do not know which one, and the others simply do not correspond to any existing objects. Then we can use the augmented population of all VA for all asteroids to tackle in a more effective way both the identification and the impact prediction problem.

2. The quasi-linear algorithms

In collaborative orbit determination (also for the best observed asteroids, the numbered ones) there is a *nominal solution* X^*, selected according to the least squares principle, that is a point of minimum for the *cost function* Q, which is computed from the sum of squares of all the m observation residuals‡. The nominal solution, obtained by *differential corrections*, that is by iteration of the least squares algorithm, is surrounded in the space of orbital elements X by a *confidence region* where the *penalty* $\Delta Q = Q(X) - Q(X^*)$ has an acceptable value. Since the confidence region is small, it is well approximated by a *confidence ellipsoid* $Z(\sigma)$ of the form

$$m\Delta Q(X) \simeq (X - X^*) \cdot C\,(X - X^*) \leqslant \sigma^2$$

where C is the *normal matrix* and the parameter σ corresponds to some confidence level.

Under these *quasi-linear* conditions, even the most complex problems of population orbit determination would have a comparatively simple solution.

2.1. *Identifications*

Given two independent sets of observations, let $m_j, X_j, Q_j, C_j, j = 1, 2$ be the number of residuals, the nominal solution, the minimum of the cost function and the 6×6 normal matrix (at some common epoch) for each of them. Under the hypothesis that they belong to the same object, the increase in the cost function for both sets fitted together is

$$(m_1 + m_2)\Delta Q(X) = (m_1 + m_2)\,Q(X) - (m_1\,Q_1 + m_2\,Q_2),$$

which can be computed in the linear approximation as

$$(X - X_1) \cdot C_1\,(X - X_1) + (X - X_2) \cdot C_2\,(X - X_2) = (X - X_0) \cdot C_0\,(X - X_0) + K$$

† These numbers refer to the published data. The detections kept secret by some observers are believed to amount to additional hundreds of thousands. Two thirds of the published data points refer to the $\simeq 80,000$ numbered asteroids.

‡ Q is either the sum of squares divided by m, or more generally a quadratic function of the residuals, taking into account uneven precisions, correlations and biases (Carpino *et al.* 2003).

where X_0 is the best "compromise solution" and the minimum K of the combined quadratic form is the *orbit identification penalty*. This algorithm has a geometric interpretation in terms of intersections of the two families of confidence ellipsoids. Selecting the couples with low orbit identification penalty as candidate for identification, then checking them by differential correction with X_0 as first guess is a very effective procedure to find identifications when the quasi-linear conditions apply (Milani *et al.* 2000a).

2.2. *Close approaches*

The close approach predictions can be handled in quasi-linear conditions by computing the *covariance matrix* $\Gamma = C^{-1}$ and then propagating it to the *target plane* of some future encounter with a planet (a plane perpendicular to the geocentric velocity at closest approach). The confidence ellipsoid in the space of initial conditions projects onto a *confidence ellipse* on the target plane, an impact is possible if this ellipse touches the impact cross section of the planet; the probability of the impact can be estimated by a classical Gaussian formalism (Milani *et al.* 2002, Milani and Valsecchi, 1999).

3. Sampling the confidence region

When the observational data are few and limited to a very short time span the orbit determination is strongly nonlinear, that is Q cannot be approximated by a quadratic function and $Z(\sigma)$ has a shape very different from an ellipsoid. Then, although the quasi-linear algorithms can be computed, they do not provide reliable predictions†.

If observations are scarce and computers are powerful, the logical approach is to use computationally intensive methods to provide fully nonlinear predictions. Thus the confidence region is sampled by a number of VA and for each of them the orbit is propagated: to the time of a potential recovery observation, to the time close to another detection candidate for identification, to the time of a possible close approach. The problem is how to select the VA in such a way that they are representative of all the possible outcomes and still the number of orbits to be computed is compatible with the available resources.

One class of methods uses *random sampling* of the confidence region to mimic with the VA population the probability density of some observation error model (e.g., Gaussian). This class includes the *Monte Carlo (MC)* and the *Statistical Ranging (SR)* methods.

3.1. *Monte Carlo*

The MC method uses the probabilistic interpretation of the least squares principle. If the residuals with respect to the "true" orbit are random observation errors distributed according to some known probability density, e.g., they are Gaussian, then this distribution can be sampled providing a set of equally probable VA (Milani *et al.* 2002).

In the quasi-linear case, the distribution of X can be approximated by a multivariate Gaussian with the nominal solution as mean and Γ as covariance matrix; this is the version of the MC method most often used. However, in the strongly nonlinear case there is no simple analytic way to compute the probability density of X, and the distribution needs to be sampled in the residuals space; this is done in a *nonlinear* version of MC method. Anyway, in the space of the orbital elements X the VA will be distributed in a non-uniform way, more densely packed near the nominal solution.

The MC method has been successfully used to compute nonlinear predictions, including computations of collision probabilities (Chodas and Yeomans 1996, Chodas and

† This does not occur in collaborative orbit determination: the tracking is planned at the mission definition stage, in such a way that the amount of data is enough to ensure convergence of the quasi-linear algorithms.

 https://doi.org/10.1017/S1743921304008683

Yeomans 1999). However, when it is critical to detect a very small probability (e.g., of a catastrophic impact) the MC method may not be efficient enough: to detect a possible impact it requires that one of the VA has an impact orbit. E.g., to detect an impact with a probability of 10^{-8} we would need to compute a number of orbits of the order of 10^8.

The MC method is not used to handle at once a large real population, as in the asteroid identification problem, because the resulting VA population would be unpractically large.

3.2. *Statistical Ranging*

The SR method has been developed to improve upon the MC method by exploiting our understanding of the properties of the confidence region, especially in the case of *Too Short Arcs (TSA)*, that is sets of observations insufficient to compute an orbit according to the least squares principle. The method can work with just two observations, when there are only 4 equations in the six unknown coordinates of the initial conditions and there is no way to define "the orbit" satisfying the observations. The basic idea is that the two-dimensional manifold of orbits exactly satisfying two observations can be parameterized by the unknown values r_1, r_2 of the corresponding distances from the observer. If the 2-vector (r_1, r_2) is assumed, then the position of the asteroid is known at two different times (Lambert's problem) and a Keplerian orbit can be computed.

The SR method (Virtanen *et al.* 2001, Virtanen *et al.* 2003) generates a large population of VA by sampling at random the (r_1, r_2) plane and computing the corresponding keplerian elements; the elements corresponding to very unlikely orbits (e.g., hyperbolic) are discarded. To take into account the observational errors, the two observations used are changed at random by sampling the (supposedly known) error model; thus the SR method can be considered an implementation of the nonlinear MC method. This population of VA samples the confidence region, defined as the set of orbits not contradicting the observations, even when its shape has nothing to do with an ellipsoid. This is why with few observations the SR methods is more reliable than the MC: indeed it can be used, even starting from a TSA, to compute a nonlinear confidence region for interesting predictions, such as the position in the sky for recovery observations (Granvik *et al.* 2003) and the target plane positions for detecting a possible impact (Muinonen *et al.* 2001).

However, both the SR and the MC methods are limited in the resolution of the VA sampling of the confidence region. By design they attempt sampling of the entire confidence region, which has dimension 6, thus to decrease by an order of magnitude the typical distance among VA in the elements space the number of VA needs to increase by a factor one million. The SR method itself, with its 2-dimensional space of undetermined parameters (r_1, r_2), suggests that, for a TSA, the confidence region is "flat", that is it has a 2-dimensional spine. The confidence region, in such a strongly nonlinear case, is a thin tubular neighborhood of a 2-dimensional submanifold in the elements space. Thus it should be possible to devise a method of sampling in which we can decrease the typical distance by a factor d with an increase in the number of VA of the order of $1/d^2$.

4. Sampling by strings: the Line Of Variations

If the orbit of an asteroid is propagated for a time interval much longer than the time span of the observations, then the confidence region for the elements at the new epoch has a shape very different from an ellipsoid, even if the one at the original epoch was well approximated by an ellipsoid†. Given a generic set of VA sampling the confidence

† By the quasi-linear algorithms, the normal matrix at the new epoch is obtained by using the state transition matrix as a coordinate change in the tangent space: it defines a very elongated confidence ellipsoid, too large, especially in the along track direction, to be a good approximation.

region at an epoch close to the observations, as time goes by the separation increases along track (because different VA have different semimajor axis); they thus become like a string of pearls, "the wampum of the night" (Dickinson 1859). This suggests that, at least when predictions have to be provided for a time remote from the ones of the available observations, a sampling following a suitable one dimensional curve could be more effective than random sampling.

The idea of a *Line Of Variations (LOV)* representing somehow the spine of the confidence region is recurrent in the history of Celestial Mechanics, the oldest reference apparently being Le Verrier 1844. A simpler version, the orbits with all the keplerian elements equal but for the mean anomaly, has been in use for long time; also the method of solving for only five orbital parameters, e.g., by assuming the eccentricity, was used.

Recently this idea has been revived (Milani 1999, Milani *et al.* 2004a) by giving a rigorous definition of the LOV as a differentiable curve. At a point X in the elements space (at an epoch near the observations) the orbit determination has a *weak direction* $V_1(X)$, corresponding to the long axis of the confidence ellipsoid computed with the normal matrix $C(X)$. The point X is on the LOV if the cost function Q restricted to the hyperplane normal to the weak direction $V_1(X)$ has a local minimum.

The advantage of this definition is that there are effective algorithms to compute VA on the LOV, starting from a nominal solution when it is available. It is also possible to compute a LOV solution starting from a rough preliminary orbit, even in cases in which the quasi-linear differential corrections fail to provide a nominal solution; thus it is possible to sample the LOV for more asteroids than those for which it is possible to compute a nominal solution!

The main problem is that this definition of the LOV depends upon the metric of the orbital elements space, it is not invariant with respect to changes of coordinates and units. However, when the observed arc is not too short, the direction of the LOV changes very little with the coordinates used (see Milani *et al.* 2004a, Figures 3 and 4). Thus, if the LOV is sampled uniformly by a moderate number of VA, years after the observations the "pearls" are still evenly spaced along track (provided there has not been a very close approach), and they sample in an effective way the confidence region at a later epoch.

4.1. *Recovery*

LOV sampling is effective in organizing the recovery of a *lost asteroid*, an object such that the confidence region for the next possible observation is very large, e.g., spanning many degrees on the sky. In such cases the nominal prediction may be far from the real position, and the confidence ellipsoid computed by the standard linear theory may fail to identify the region to be scanned. A set of VA, regularly spaced along the LOV, is prepared and the predictions are computed for each of them. The number of LOV points should be selected in such a way that the spacing in the prediction between two consecutive ones is comparable to the field of view of the telescope used for the search. This method has allowed recoveries of especially interesting asteroids, by scanning the LOV trace either on the sky or on archives plates (Boattini *et al.* 2001).

4.2. *Identification*

The quasi-linear algorithm to identify two independent detections of the same asteroid requires the propagation of both orbits with their normal matrices to a common epoch. If the two detections are years apart, the confidence regions are very different from the confidence ellipsoids. Thus the intersections of the confidence regions, where the compromise orbit should be found, cannot be reliably computed as the intersections of the confidence ellipses and the quasi-linear method often fails.

Sampling the LOV of both detections (at epochs near the respective observations) with a number of VA X_1^i and X_2^k, then propagating all to the common epoch and computing the identification penalty $K_{i,k}$ for each "virtual couple" allows to propose many additional identifications with respect to the ones proposed by the quasi-linear algorithm. Moreover, the use of LOV solutions when a nominal solution is not available increases the size of the catalogues of orbits for which an identification can be attempted. The combined effect of these two improvements has increased the number of new confirmed identification, with the same data, by an order of magnitude (Milani *et al.* 2004a, Table 4).

4.3. *Impact monitoring*

The goal of *impact monitoring* is to establish whether the confidence region for a given NEA contains some *Virtual Impactor (VI)*, a small subset of initial conditions leading to a collision with Earth †. The *Impact Probability (IP)* of a VI is roughly proportional to the volume of the VI in the elements space. To find an initial condition belonging to the VI, the *VI representative*, when the IP is minute, would require a very dense sampling.

The current impact monitoring systems, CLOMON2 (Universities of Pisa and Valladolid, at http://newton.dm.unipi.it/neodys and http://unicorn.eis.uva.es/neodys) and Sentry (JPL at http://neo.jpl.nasa.gov/risk/) have solved this problem by using two improvements. First, the confidence region is sampled along the LOV, thus a moderate $(1,000$ to $10,000)$ number of VA allows to reproduce the different dynamical behaviors (Milani *et al.* 1999). Then all the VA orbits are propagated for the next 80 to 100 years and all the close approaches to the Earth are recorded, together with the confidence ellipse on the target plane (according to the quasi-linear algorithm of Section 2). Second, by using the LOV continuous structure: when two VA contiguous in the natural ordering of the LOV have a close approach (at the same epoch) such that there must be a minimum of the approach distance in the segment between the two, this minimum is found by interpolation on the LOV and iterative methods (e.g., *regula falsi*; Milani *et al.* 2004b). If this minimum is less than one Earth radius, a VI representative has been found. If the minimum corresponds to a very deep approach, such that the target plane confidence ellipse touches the Earth, either a VI representative is obtained by another iterative method (Milani *et al.* 2000b) or the local linearity is checked to confirm the VI.

The impact monitoring systems have sometimes to handle more complex behaviors of the LOV trace on the target plane, such as *interrupted returns* (Milani *et al.* 2004b), which can be understood in terms of the theory of *resonant returns* (Valsecchi *et al.* 2003). Anyway the current monitoring systems are very effective in detecting minute VI, at least when the LOV sampling represents well the different possible dynamical conditions. After a VI has been detected, and announced by the impact monitoring systems, the asteroid is reobserved, and this can "destroy" the VI: the confidence region in the space of initial conditions becomes smaller, and the VI may not be a subset anymore.

On average about once per week newly discovered NEA are found to have VI and the case is solved by re-observation. Not only the mathematical but also the public relations problem has been solved: the media and the public opinion now understand that the appearance and then destruction of some VI indicates that the system (the surveys, the orbit computers, the impact monitoring and the follow up observers) is working well and is effectively decreasing the impact risk with respect to the background risk.

† The collision subset for a given epoch may be disconnected, when the same collision can be reached through different dynamic routes, e.g., resonant returns in different resonances. In this case a VI is a connected component of the collision subset.

 https://doi.org/10.1017/S1743921304008683

5. Sampling by surfaces: the Admissible Region

A TSA is a set of data such that the positions on the celestial sphere do not significantly deviate from a great circle (the differences being of the order of the expected observation error). This condition typically applies when the available observations only span an arc of $\simeq 1^\circ$ or less on the celestial sphere. Then the data provide only four independent observed quantities, the two angles (α, δ) at some mean time t, and two angular rates $(\dot{\alpha}, \dot{\delta})$, like an arrow on the sky; such a 4-dimensional observable is called an *attributable* (following Milani *et al.* 2001). An average apparent magnitude h may also be available.

Under these conditions, the two coordinates $(r, \dot{r})$ are not constrained at all by the observations. The only way to restrict the set of possible values in the $(r, \dot{r})$ half plane ($r > 0$) is to use a priori information and/or hypotheses independent from the observations. The range and range rate are constrained if we assume that the body belongs to the solar system, but it is not a satellite of the Earth. We can also set a lower limit to the size of the object by an upper bound for the absolute magnitude H: since $H - h$ is a function of r, this condition sets a minimum distance, excluding "shooting stars". The combination of the three conditions above defines a compact subset of the $(r, \dot{r})$ space, the *Admissible Region*; it can have either one or two connected components, the boundary is defined by explicitly computable algebraic curves (Milani *et al.* 2004c).

Thus it is possible to sample the finite Admissible Region with a finite number of points $(r_j, \dot{r}_j),\ j = 1, \ldots, N$. Different sampling strategies can be used†, however sampling with a geometric object has significant advantages. For 1-dim objects the sampling is done by intervals, with optimal metric properties (equal spacing in the LOV parameter). For 2-dim objects such as the Admissible Region the sampling should be done by triangles, with some optimal metric property. The ideal triangulation would include only equilateral triangles, for a complicated shape region this is not possible but we can use the *Delaunay triangulation*, with the largest possible minimum angle among all triangles. Then the nodes of the triangulation are the selected points $(r_j, \dot{r}_j)$ (Milani *et al.* 2004c).

5.1. *Attributable elements*

Given an attributable $A = (\alpha, \delta, \dot{\alpha}, \dot{\delta})$, and given a set of points $(r_j, \dot{r}_j),\ j = 1, \ldots, N$, this defines a set of VA, each one with initial conditions‡ uniquely determined by the six dimensional vector $(\alpha, \delta, \dot{\alpha}, \dot{\delta}, r_j, \dot{r}_j)$. These *attributable elements* can be converted into Cartesian position and velocity (topocentric, then heliocentric), then to any other coordinates, such as keplerian elements.

Such a set of VA can be used, e.g., to compute multiple ephemerides for a planned recovery, as in Figure 1. Each of the *virtual ephemerides* obtained in this way has an associated uncertainty, which is obtained by propagation of the covariance matrix defined by the (linear) least squares fit used to compute the attributable A (Milani et al., in preparation); this uncertainty is represented by ellipses in the plane of Figure 1. The predictions are attributables, the uncertainty is represented in full by ellipsoids in 4-dim space. The union of all the confidence ellipsoids surrounding the alternate ephemerides, from each triangulation node, is an approximation of the confidence region for the ephemerides.

5.2. *Preliminary orbits*

Today the surveys produce large sets of TSA, i.e. of attributables, rather than individual observations. Thus the problem of computing preliminary orbits (to be used as first guess for differential corrections) has to be reformulated. We would like to perform a *linkage*,

† Tholen and Whiteley (private communication) and Marsden (private communication) use a rectangular grid, then discard hyperbolic orbits.

‡ The epoch corresponding to the initial conditions is $t_j = t - r_j/c$, with c the speed of light, to take into account the light travel time.

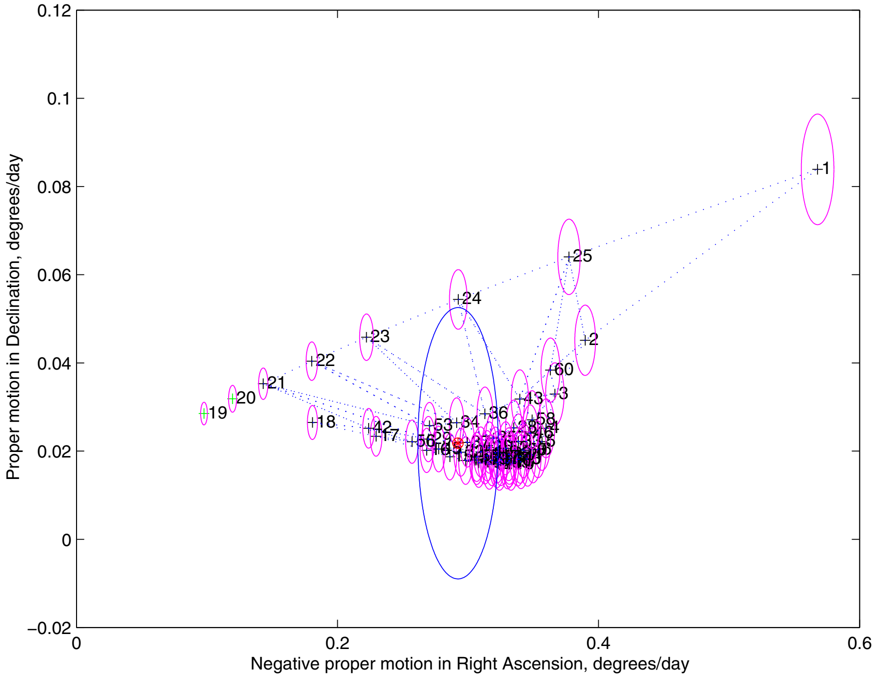

Figure 1. By using only the observations of the discovery night of the asteroid 2003 BH_{84}, recovery observations have been predicted for 11 days later in the plane of the proper motion (angular rates); for each VA the prediction is marked with the corresponding integer index, the sides of the triangles are dotted. The small ellipses give the uncertainty of the prediction, the larger ellipse gives the uncertainty of the attributable computed with the recovery data.

a special form of identification: to compute some orbit (either nominal solution or LOV) fitting together the data from two TSA, that is two attributables. The method of Gauss for a preliminary orbit requires 3 well separated observations, thus it is not applicable.

Given an attributable A_0 at time t_0 and the triangulation of the corresponding Admissible Region, for each VA we can compute for the time t_2 of another attributable A_2 the predictions $A_1(j)$ with uncertainty described by the normal matrix $C_1(j)$. As shown in Figure 1, the second attributable comes with its own uncertainty, represented by a normal matrix C_2. The same algorithm used for identification of orbits in a 6-dim space can be used to identify attributables in a 4-dim space: the formulas are the same of Section 2, only the size of the vectors and matrices are different. In this way we define a *attributable identification penalty* $K(j)$ for each VA, and we can select the values of the index j for which the penalty is low (if any). Moreover, for each one of these we can use the "compromise attributable", together with the values of $(r_j, \dot{r}_j)$, to define an orbit fitting both attributables with a moderate cost function: this is the new type of preliminary orbit (Milani at al., in preparation).

5.3. *Identifications*

If from the two attributables A_0 and A_2 it is possible to generate some of these preliminary orbits then they can be used as first guess in a differential corrections procedure, which may converge to a nominal solution or at least to a number of LOV solutions. Figure 2 shows this procedure: from the nodes with low values of $K(j)$, a number of LOV solutions have been computed. The LOV is very close to a straight line in the $(r, \dot{r})$ plane, the LOV solutions are marked with the index j. The "best" label indicates the nominal solution, the $+$ sign indicates the position of the real asteroid (as determined a posteriori).

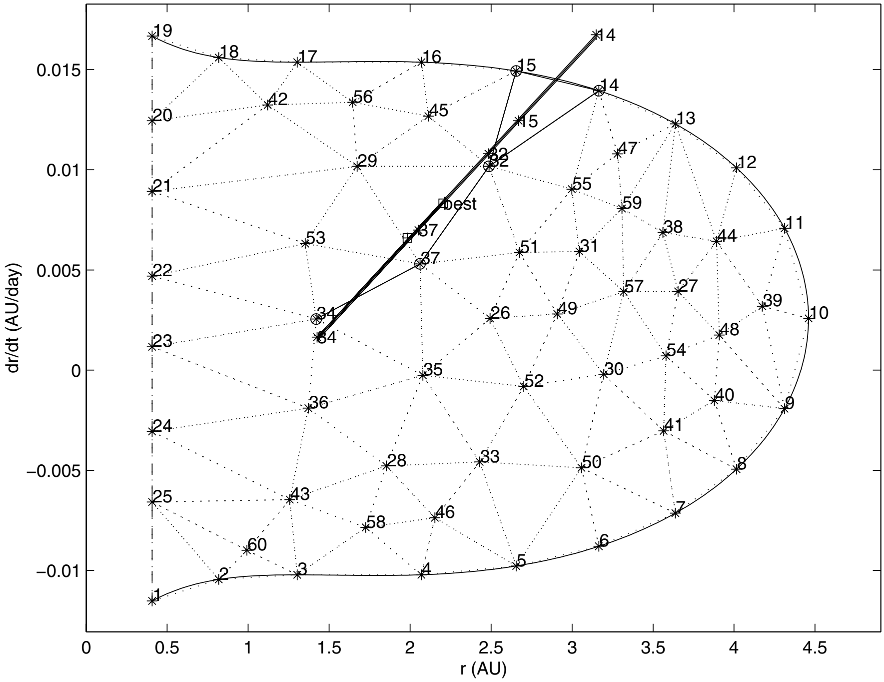

Figure 2. The Admissible region in the $(r, \dot{r})$ plane for the attributable from the discovery night of 2003 BH_{84} with its the Delaunay triangulation. The sides marked with continuous lines join the nodes with (normalized) square root identification penalty $\sqrt{K(j)} < 9$ for identification with the recovery attributable of 11 days later.

If this procedure is successful, starting from a set of VA sampling a 2-dim set it provides another set of VA sampling a 1-dim set. Indeed, the orbit fitting two independent detections is "less undetermined", but still quasi-linear conditions do not apply. Then the procedure is repeated to try and find a third attributable. Each one of the second generation VA is propagated to the time of a third attributable candidate for *attribution*, the kind of identification in which an attributable is identified with an object for which an orbit is already available (Milani *et al.* 2001, Milani *et al.* 2004a). If the identification penalty is low, differential corrections can be attempted and they provide a nominal orbit fitting three attributables. Then these orbits are used again to search for an identification with a fourth attributable, and so on: the procedure is recursive, that is the algorithms and even the software are the same to fit together 3,4,5,... attributables.

These new algorithms have so far been tested only on simulations of future surveys. The tests on real data will be more demanding: the problem is to organize a computationally efficient procedure to handle the very large data sets expected from the next generation of asteroid surveys. However, the simulations results are very encouraging: they indicate that the algorithms described above can be used to solve the identification problem for large observational data sets, given enough computing power.

5.4. *Impact monitoring*

When a NEA has been observed over a very short arc, like 1°, the impact monitoring techniques based upon LOV sampling may not be effective. The confidence region being like a flat disk, the LOV definition is strongly dependent upon the coordinates and the units used (see Milani *et al.* 2004a, Figures 1 and 2). The LOV is just one chord of a such disk, thus LOV sampling is not representative of the entire confidence region. The use of geometric 2-dim sampling should solve this problem; this is the subject of current research (see Tommei, these proceedings).

6. Future applications

As computer technology progresses, the balance between the cost of new data and the cost of computations will tip more and more in favor of computationally intensive methods. This does not imply that brute force methods will dominate: the need for sophisticated analytical tools from celestial mechanics will remain. Thanks to the computing power available and to the new methods such as geometric sampling, we will tackle more and more complex problems such as orbit determination for large populations.

Among the next targets of research in this field, the detection of VI for asteroids without a least squares orbit (TSA), the algorithms to process the data of the next generation surveys (expected to discover $\simeq 100$ times more asteroids that the current ones), a catalog of space debris complete to very small sizes, and more yet to be invented.

Acknowledgements

In this paper I have also reported on my own research, for which I would like to acknowledge the essential contribution of many coworkers, in alphabetical order: O. Arratia, A. Boattini, M. Carpino, S.R. Chesley, P. W. Chodas, M. de' Michieli Vitturi, G. Gronchi, Z. Knežević, A. La Spina, M.E. Sansaturio, G. Tommei, G.B. Valsecchi. I also thank J. Virtanen for her referee report, containing useful suggestions.

References

Boattini, A., D'Abramo, G., Forti, G. and Gal, R. 2001, *Astron. Astrophys.* 375, 293

Carpino, M., Milani, A. and Chesley, S.R. 2003, *Icarus*, 166, 248

Chodas, P. W. and Yeomans, D. K. 1996, in: *IAU Colloq. 156: The Collision of Comet Shoemaker-Levy 9 and Jupiter*, p. 1

Chodas, P. W. and Yeomans, D. K. 1999, Paper AAS 99-462, AAS/AIAA Astrodynamics Specialists Conference, Girdwood, Alaska.

Dickinson, E. 1859, Poem 128.

Granvik, M., Virtanen, J., Muinonen, K., Bowell, E., Koehn, B. and Tancredi, G. 2003, *Earth, Moon, Planets*, 92, 73

Le Verrier, U.-J. 1844, *Compt. Rend. Acad. Sci.* 19, 982

Milani, A. 1999. *Icarus*, 137, 269

Milani, A. and Valsecchi, G.B. 1999, *Icarus* 140, 408

Milani, A., Chesley, S.R. and Valsecchi, G.B. 1999, *Astron. Astrophys.* 346, L65

Milani, A., La Spina, A., Sansaturio, M. E. and Chesley, S. R. 2000a, *Icarus*, 144, 39

Milani, A., Chesley, S.R., Boattini, A. and Valsecchi, G.B. 2000b, *Icarus* 145, 12

Milani, A., Sansaturio, M.E. and Chesley, S.R. 2001, *Icarus* 151, 150

Milani, A., Chesley, S.R., Chodas, P.W. and Valsecchi, G.B. 2002, in: W.F. Bottke Jr. et al. (eds.), *Asteroids III*, Univ. Arizona Press, p. 55

Milani, A., Sansaturio, M.E., Tommei, G., Arratia, O. and Chesley, S.R. 2004a, *Astron. Astrophys.*, in press

Milani, A., Chesley, S.R., Sansaturio, M.E., Tommei, G. and Valsecchi, G. 2004b, *Icarus*, in press

Milani, A., Gronchi, G.F., de' Michieli Vitturi, M. and Knežević, Z. 2004c, *Cel. Mech. Dyn. Astron.*, in press

Muinonen, K., Virtanen, J. and Bowell, E. 2001, *Cel. Mech. Dyn. Astron.*, 81, 93

Valsecchi, G.B., Milani, A., Gronchi, G.-F. and Chesley, S.R. 2003, *Astron. Astrophys.* 408, 1179

Virtanen, J., Muinonen, K. and Bowell, E. 2001, *Icarus*, 154, 412

Virtanen, J., Muinonen, K., Tancredi, G. and Bowell, E. 2003, *Icarus*, 161, 419