

MANIFOLDS OF SMOOTH MAPS II: TRANSVERSALITY

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We generalise the standard notion of transversality to maps between manifolds modelled on locally convex spaces and prove that all standard transversal theorems remain valid for this generalised notion.

As first applications, we use our generalised Transversal Density Theorem to prove several simple "generic" results for local smooth maps which parallel the usual ones.

This is a sequel of [5]. In this paper, we restrict our interest to the $B\Gamma$ -manifolds [5] and define a notion of transversality for maps between them. We term it the $B\Gamma$ -transversality which is in fact a generalisation of the standard notion [1], [2], [3]. We then show that all the standard transversal theorems [1], [2], [3] still hold for this generalised notion. Finally, as first applications, we give several simple "generic" results for local smooth maps which parallel the usual ones [2], [3], [4].

The paper consists of three sections. In §1, we define the $B\Gamma$ -transversality and give its standard properties. The next section, §2, is for stating and proving the $B\Gamma$ -transversal theorems and the last section is for the applications.

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1. $B\Gamma$ -transversality

In the following, we shall always assume that F is a Γ -family consisting of sequentially complete spaces. First, in exactly the same way as in the case of Banach manifolds, we can define $B\Gamma$ -submanifolds and $B\Gamma$ -products of $B\Gamma$ -manifolds of class C^r modelled on F . Now, let X, Y be $B\Gamma$ -manifolds of class C^r modelled on $E, F \in F$ respectively ($r \geq 1$), and W be a $B\Gamma$ -submanifold of class C^r of Y . Then a $C^r_{B\Gamma}$ -map [5] $f : X \rightarrow Y$ is said to be $B\Gamma$ -transversal to W at $x \in X$ if either $f(x) \notin W$ or $y = f(x) \in W$ and satisfies the following condition: there exists a $B\Gamma$ -complement Z of $T_y W$ in $T_y Y$ such that, if we denote by $Q : T_y W \oplus Z \rightarrow Z$ the projection, the map $Q \circ T_x f : T_x X \rightarrow Z$ is a $B\Gamma$ -splitting surjection [6], [7]. We say that f is $B\Gamma$ -transversal to W if it is $B\Gamma$ -transversal to W at every $x \in X$.

(1.1) (Local Representative of $B\Gamma$ -transversality [1]). Let X, Y, W and $f : X \rightarrow Y$ be as above and assume that $x \in X$ is such that $y = f(x) \in W$. Then a necessary and sufficient condition for f to be $B\Gamma$ -transversal to W at x is: there exist admissible $B\Gamma$ -charts (U, α) and (V, β) at x and $f(x)$ respectively, $B\Gamma$ -decompositions $E = E_1 \oplus_{B\Gamma} E_2$, $F = F_1 \oplus_{B\Gamma} F_2$ for E and F such that:

$$(a) \quad \alpha(U) = B_1 + B_2 \subseteq E, \quad \beta(V) = D_1 + D_2 \subseteq F, \quad \alpha(x) = 0, \\ \beta(y) = 0, \quad \beta(W \cap V) = D_1, \text{ where } B_i \text{ and } D_i \text{ are open} \\ \text{neighbourhoods of } 0 \text{ in } E_i, F_i \text{ (} i = 1, 2 \text{) respectively;}$$

(b) the local representative $f_{\alpha\beta} : B_1 + B_2 \rightarrow D_1 + D_2$ of f has the form

$$f_{\alpha\beta} = \eta + \phi \circ P_2$$

where $\eta : B_1 + B_2 \subseteq E \rightarrow D_1 \subseteq F_1$ is a $C^r_{B\Gamma}$ -map, ϕ is a $B\Gamma$ -isomorphism of E_2 onto F_2 and $P_2 : E = E_1 \oplus_{B\Gamma} E_2 \rightarrow E_2$ is the second projection.

Proof. Straightforward with the use of the Inverse Mapping Theorem [7, (5.2), p. 45].

(1.2). Let X, Y, W and $f : X \rightarrow Y$ be as above and suppose that f is $B\Gamma$ -transversal to W . Then $f^{-1}(W)$ is either empty or a $B\Gamma$ -submanifold of class C^r of X . In the latter case, we have

(a) for $x \in f^{-1}(W)$ and $y = f(x)$, $(T_x f)^{-1}(T_y W) = T_x(f^{-1}(W))$,

(b) W and $f^{-1}(W)$ have the same codimension.

Proof. This follows from (1.1).

2. The $B\Gamma$ -transversal theorems

Let F be a Γ -family which contains E, F, G , and $F \times G$ which is a $B\Gamma$ -product with the projection $\pi : F \times G \rightarrow F$ and $E \subseteq F \times G$. Then E is said to $B\Gamma$ -adapt the projection π if there is a $B\Gamma$ -complement H of E in $F \times G$ such that $F \times G = \pi(E) \oplus \pi(H)$ is a $B\Gamma$ -direct sum when both summands are equipped with the relative calibrations from Γ_F .

(2.1). Let E, F, G be as above and assume that E $B\Gamma$ -adapts π . If $\dim G = n$ and $\text{codim } E = q$ (with respect to $F \times G$) then the restriction of π to E is a $B\Gamma$ -Fredholm map with index $n - q$ [6].

Proof. Let H be the $B\Gamma$ -complement of E in the definition of $B\Gamma$ -adaptation and P, Q be the corresponding projections of the $B\Gamma$ -decomposition $F \times G = E \oplus_{B\Gamma} H$.

Put $L = P\pi(E)$ and $K = E \cap (\{0\} \times G)$. Then it is easily seen that $E = K \oplus_{B\Gamma} L$ and L is $B\Gamma$ -isomorphic to $\pi(E)$ and the proof follows.

Now let A, X, Y be $B\Gamma$ -manifolds modelled respectively on $E, F, G \in F$. Suppose that $E \times F$ is a $B\Gamma$ -product and denote by $A \times X$ the $B\Gamma$ -product of A and X (which is modelled on $E \times F$). Let

$C_{B\Gamma}^r(X, Y)$ be the space of $C_{B\Gamma}^r$ -maps $X \rightarrow Y$. Then, a map

$\rho : A \rightarrow C_{B\Gamma}^r(X, Y)$ is called a $C_{B\Gamma}^r$ -representation [1] if and only if the evaluation map $\text{ev}_\rho : A \times X \rightarrow Y$ defined by $\text{ev}_\rho(a, x) = \rho(a)(x)$ for

$a \in A, x \in X$ is a $C_{B\Gamma}^r$ -map.

(2.2) ($B\Gamma$ -Transversal Density Theorem [1]). Let F be a Γ -family

as at the beginning. Let $E, F, E \times F \in \mathcal{F}$ where $E \times F$ is a $B\Gamma$ -product. Let A be a second countable $B\Gamma$ -manifold modelled on E , X be a second countable $B\Gamma$ -manifold modelled on F with finite dimension n and Y be a $B\Gamma$ -manifold modelled on G , all of class C^r . Let $\rho : A \rightarrow C_{B\Gamma}^r(X, Y)$ be a $C_{B\Gamma}^r$ -representation and W be a $B\Gamma$ -submanifold of class C^r of Y with finite codimension q such that $r > \max(0, n-q)$.

Now assume that ev_ρ is $B\Gamma$ -transversal to W and, for any $(a, x) \in A \times X$ such that $y = \rho(a)(x) \in W$, the subspace $(T_{(a,x)} \text{ev}_\rho)^{-1}(T_y W)$ $B\Gamma$ -adapts the projection $T_a A \times T_x X \rightarrow T_a A$. Then the set of $a \in A$ such that $\rho(a)$ is $B\Gamma$ -transversal to W is residual in A .

Proof. The proof is similar to the proof in [1, Theorem 19.1] with the use of (2.1).

(2.3) (Openness of $B\Gamma$ -transversality [1]). Let \mathcal{F} be a Γ -family, $E, F, G \in \mathcal{F}$ such that $E \times F$ and $F \times G$ are $B\Gamma$ -products. Let A, X, Y be $B\Gamma$ -manifolds of class C^1 modelled on E, F, G respectively. Let W be a closed $B\Gamma$ -submanifold of Y of class C^1 , $K \subseteq X$ a compact subset and $\rho : A \rightarrow C_{B\Gamma}^1(X, Y)$ a $C_{B\Gamma}^1$ -representation. Then the set of $a \in A$ such that $\rho(a)$ is $B\Gamma$ -transversal to W at every $x \in K$ is open in A .

Proof. First, in the same manner as in the case of Banach manifolds, we can define the $B\Gamma$ -bundle $L_{B\Gamma}(\tau_X, \tau_Y)$ of $B\Gamma$ -continuous linear maps whose fibre over a point $(x, y) \in X \times Y$ (the $B\Gamma$ -product of X and Y) is the space $L_{B\Gamma}(T_x X, T_y Y)$ of $B\Gamma$ -continuous linear maps from $T_x X$ to $T_y Y$. Then we proceed exactly as in [1].

Now let \mathcal{F} be a Γ -family containing E, F, G and assume that $E \times F$ and $F \times G$ are $B\Gamma$ -products. Let A, X, Y be $B\Gamma$ -manifolds of class C^{r+1} ($r \geq 1$) modelled on E, F, G respectively.

Suppose that X is compact (hence F is finite-dimensional), and let

$\rho : A \rightarrow C_{B\Gamma}^{r+1}(X, Y)$ be a $C_{B\Gamma}^{r+1}$ -representation, $W \subseteq Y$ be a closed $B\Gamma$ -submanifold of class C^{r+1} , and $a_0 \in A$ be a point such that $\rho(a_0)$ is $B\Gamma$ -transversal to W . For each $a \in A$, let $W_a = \rho(a)^{-1}(W) \subseteq X$. Then, by (2.3), for all $a \in A$ sufficiently near a_0 , $\rho(a)$ is $B\Gamma$ -transversal to W . Hence, by (1.2), W_a is a $B\Gamma$ -submanifold of class C^{r+1} of X (that is, a C^{r+1} -submanifold of X in the Banach sense since X is a finite-dimensional manifold). Thus one might expect that, for a near a_0 , the submanifolds W_a and W_{a_0} are close. Indeed, we have

(2.4) ($B\Gamma$ -Transversal Isotopy Theorem [1]). *Let F be a Γ -family containing E, F, G such that $E \times F$ and $F \times G$ are $B\Gamma$ -products. Let A, X, Y be $B\Gamma$ -manifolds of class C^{r+1} ($r \geq 1$) modelled on E, F, G with X compact and of class C^{r+3} . Let $\rho : A \rightarrow C_{B\Gamma}^{r+1}(X, Y)$ be a $C_{B\Gamma}^{r+1}$ -representation and $W \subseteq Y$ be a closed $B\Gamma$ -submanifold of class C^{r+1} . Finally, let $a_0 \in A$ be a point such that $\rho(a_0)$ is $B\Gamma$ -transversal to W . Then there is an open neighbourhood N of a_0 in A such that, for all $a \in N$, the submanifold $W_a = \rho(a)^{-1}(W)$ is C^r -isotopic to W_{a_0} .*

Proof. Since X is compact, hence of finite dimension, the proof of [1, Theorem 20.2] can be repeated with suitable modifications.

3. Applications

In this section, as first applications of the $B\Gamma$ -Transversal Density Theorem (2.2), we give some simple "generic" results for local smooth maps which parallel the usual ones [2], [3], [4].

From now on, Ω shall always stand for an open, convex and bounded subset of an Euclidean space \mathbb{R}^n , and for each integer $k \geq 0$, $P^k(n, m)$ shall stand for the space of polynomials of degree less than or equal to k

from \mathbb{R}^n to another Euclidean space \mathbb{R}^m [1, p. 23]. For each $\xi \in C^k(\Omega, \mathbb{R}^m)$ and each $x \in \Omega$, $P^k\xi(x)$ is the point of $P^k(n, m)$ given by [1]:

$$(1) \quad P^k\xi(x) = (\xi(x), D\xi(x), \dots, D^k\xi(x)) .$$

We choose the following norm on $P^k(n, m)$. If

$(a_0, a_1, \dots, a_k) \in P^k(n, m)$ then

$$(2) \quad \|(a_0, a_1, \dots, a_k)\| = \|a_0\| + \|a_1\| + \dots + \|a_k\| ,$$

and, for each $\xi \in C^k(\Omega, \mathbb{R}^m)$, define

$$(3) \quad \|\xi\|_k = \sup\{\|P^k\xi(x)\| : x \in \Omega\} .$$

For each integer $k \geq 0$, let $B^k(\Omega, \mathbb{R}^m)$ denote the space of all $\xi \in C^k(\Omega, \mathbb{R}^m)$ such that $\|\xi\|_k < +\infty$, and put

$$(4) \quad B^\infty(\Omega, \mathbb{R}^m) = \bigcap_{k=0}^\infty B^k(\Omega, \mathbb{R}^m) .$$

Endow $B^\infty(\Omega, \mathbb{R}^m)$ with the calibration consisting of the following sequence of increasing norms:

$$(5) \quad \Gamma = \{\|\cdot\|_k : k \geq 0\}$$

where each $\|\cdot\|_k$ is defined by (3).

(3.1). *The space $B^\infty(\Omega, \mathbb{R}^m)$ equipped with the sequence of increasing norms in (5) is a separable Fréchet space.*

Proof. We need only prove separability. For each $k \geq 0$, define the space

$$UB^k(\Omega, \mathbb{R}^m) = \{f : \Omega \rightarrow \mathbb{R}^m / f \in B^k(\Omega, \mathbb{R}^m) \text{ and } f, Df, \dots, D^k f \text{ are uniformly continuous on } \Omega\} .$$

Then, for every $k \geq 0$, $UB^k(\Omega, \mathbb{R}^m)$ is a Banach space. Furthermore, from the fact that $UB^0(\Omega, \mathbb{R}^m)$ is separable, using the map

$$P^k : UB^k(\Omega, \mathbb{R}^m) \rightarrow UB^0(\Omega, P^k(n, m)) : f \rightarrow P^k f ,$$

it is seen that $UB^k(\Omega, \mathbb{R}^m)$ is separable for all $k \geq 0$. From this, $B^\infty(\Omega, \mathbb{R}^m)$ is separable.

Now note that each $\|\cdot\|_j$ on $B^\infty(\Omega, \mathbb{R}^m)$ induces a norm p_j on $B^\infty(\Omega, \mathbb{R}^m) \times \mathbb{R}^n$ defined by

$$(6) \quad p_j(\xi, x) = \|\xi\|_j + \|x\| \quad \text{for } (\xi, x) \in B^\infty(\Omega, \mathbb{R}^m) \times \mathbb{R}^n .$$

(3.2). Let Ω be as above. Let r be an integer greater than or equal to 1 and k be an integer such that $0 \leq k \leq r$. Then we can always find an integer i_0 such that the following assertions are true [6], [7]:

- (a) the map $ev_k : B^\infty(\Omega, \mathbb{R}^m) \times \Omega \rightarrow \Omega \times P^k(n, m)$ defined by $ev_k(\xi, x) = (x, P^k \xi(x))$ for $x \in \Omega$, $\xi \in B^\infty(\Omega, \mathbb{R}^m)$ is $C^r_{B\Gamma}$ with respect to the calibration

$$\Gamma = \{ (p_{r+k+i}, \|\cdot\|) : i \geq i_0 \}$$

for the pair $(B^\infty(\Omega, \mathbb{R}^m) \times \mathbb{R}^n, \mathbb{R}^n \times P^k(n, m))$, where $\|\cdot\|$ is the standard norm of $\mathbb{R}^n \times P^k(n, m)$;

- (b) for each $(\xi, x) \in B^\infty(\Omega, \mathbb{R}^m) \times \Omega$, the $B\Gamma$ -derivative $Dev_k(\xi, x) : B^\infty(\Omega, \mathbb{R}^m) \times \mathbb{R}^n \rightarrow \mathbb{R}^n \times P^k(n, m)$ is onto and has its kernel $B\Gamma$ -splitting in $B^\infty(\Omega, \mathbb{R}^m) \times \mathbb{R}^n$;

- (c) for any $(B\Gamma$ -splitting) subspace F_1 of $\mathbb{R}^n \times P^k(n, m)$, the inverse image $E_1 = Dev_k(\xi, x)^{-1}(F_1)$ is $B\Gamma$ -splitting in $B^\infty(\Omega, \mathbb{R}^m) \times \mathbb{R}^n$ and has a $B\Gamma$ -complement E_2 such that the restriction of $Dev_k(\xi, x)$ to E_2 is a $B\Gamma$ -isomorphism

onto a $B\Gamma$ -complement F_2 of F_1 in $\mathbb{R}^n \times F^k(n, m)$;

(d) the subspace $E_1 = \text{Dev}_k(\xi, x)^{-1}(F_1)$ $B\Gamma$ -adapts the first projection $B^\infty(\Omega, \mathbb{R}^m) \times \mathbb{R}^n \rightarrow B^\infty(\Omega, \mathbb{R}^m)$.

Proof. (a) follows from the remark in [7, p. 26].

(b) The surjectivity of $\text{Dev}_k(\xi, x)$ is seen as in the proof of [1, Theorem 10.4, p. 27]. Now, for a fixed $(\xi, x) \in B^\infty(\Omega, \mathbb{R}^m) \times \Omega$, defined [1],

$$K_1 = \{ \zeta \in B^\infty(\Omega, \mathbb{R}^m) / D^i \zeta(x) = 0 \text{ for } i = 0, 1, \dots, k \} ,$$

$$K_2 = \{ \zeta \in B^\infty(\Omega, \mathbb{R}^m) / D^i \zeta \equiv 0 \text{ for } i \geq k+1 \} .$$

Then it is not hard to see that $K_2 = F^k(n, m)$, $B^\infty(\Omega, \mathbb{R}^m) = K_1 \oplus_{B\Gamma} K_2$ and $B^\infty(\Omega, \mathbb{R}^m) \times \mathbb{R}^n = (K_1 \times \{0\}) \oplus_{B\Gamma} (K_2 \times \mathbb{R}^n)$. From this we have (b).

(c) Let F_1 be a subspace of $\mathbb{R}^n \times F^k(n, m)$; then we have

$K_1 \times \{0\} = \text{Ker } \text{Dev}_k(\xi, x) \subseteq E_1 = \text{Dev}_k(\xi, x)^{-1}(F_1)$. From this it follows quickly that, if $L = E_1 \cap (K_2 \times \mathbb{R}^n)$, we have

$$E_1 = L \oplus_{B\Gamma} (K_1 \times \{0\}) .$$

Since $\text{codim } L < +\infty$ and $K_2 \times \mathbb{R}^n$ is a normed space with norm $\|(\zeta, h)\| = \|\zeta\|_k + \|h\|$, we can find a complement E_2 of L in $K_2 \times \mathbb{R}^n$ such that

$$\pi(L) \oplus_{B\Gamma} \pi(E_2) = K_2 \quad (\text{see [1]})$$

where $\pi : B^\infty(\Omega, \mathbb{R}^m) \times \mathbb{R}^n \rightarrow B^\infty(\Omega, \mathbb{R}^m)$ is the first projection.

Then, by a simple argument, we can prove (c).

(d) It is not hard to verify that

$$\pi(E_1) \oplus_{B\Gamma} \pi(E_2) = B^\infty(\Omega, \mathbb{R}^m)$$

which proves that E_1 $B\Gamma$ -adapts π .

Let Ω be as above and let $B^\infty(\Omega, \mathbb{R})$ be the space of all C^∞ functions $\Omega \rightarrow \mathbb{R}$ with all derivatives bounded on Ω . Let us denote by $M^\infty(\Omega, \mathbb{R})$ the subset of *Morse functions* [2], [3] in $B^\infty(\Omega, \mathbb{R})$.

(3.3). $M^\infty(\Omega, \mathbb{R})$ is dense in $B^\infty(\Omega, \mathbb{R})$. In other words, every function in $B^\infty(\Omega, \mathbb{R})$ can be approximated by Morse functions.

Proof. Let $J^1(\Omega, \mathbb{R})$ be the space of 1-jets $\Omega \rightarrow \mathbb{R}$ and let

$$S_1 = \{ \sigma \in J^1(\Omega, \mathbb{R}) : \text{corank } \sigma = 1 \}$$

be the submanifold of $J^1(\Omega, \mathbb{R})$ defined in [2, p. 60]; then

$$M^\infty(\Omega, \mathbb{R}) = \left\{ f \in B^\infty(\Omega, \mathbb{R}) : j^1 f \pitchfork S_1 \right\}.$$

From this we can apply (2.2) with $A = B^\infty(\Omega, \mathbb{R})$ considered as a $B\Gamma$ -manifold, $X = \Omega \subseteq \mathbb{R}^n$, $Y = J^1(\Omega, \mathbb{R})$, $W = S_1$ and

$\rho : A \rightarrow C^1_{B\Gamma}(\Omega, J^1(\Omega, \mathbb{R}))$ defined by $\rho(f) = j^1 f$, where $f \in A$ and $j^1 f : \Omega \rightarrow J^1(\Omega, \mathbb{R})$ is the 1-jet prolongation of f [3].

Now consider the tangent bundle $T\Omega = \Omega \times \mathbb{R}^n$ and the space of all C^∞ sections of $T\Omega$. Each element of the latter space is a vector field $\xi : \Omega \rightarrow T\Omega$ defined by

$$(7) \quad \xi(x) = (x, \tilde{\xi}(x)) \quad \text{for all } x \in \Omega$$

where $\tilde{\xi} : \Omega \rightarrow \mathbb{R}^n$ is C^∞ .

Denote by $S^\infty(T\Omega)$ the space of all such ξ with $\tilde{\xi} \in B^\infty(\Omega, \mathbb{R}^n)$. Consider the usual topology on $B^\infty(\Omega, \mathbb{R}^n)$ defined by (5), and for each $k \geq 0$, defined

$$(8) \quad \|\xi\|_k = \|\tilde{\xi}\|_k \quad \text{for all } \xi \in S^\infty(T\Omega).$$

Then $S^\infty(T\Omega)$ equipped with $\{\|\cdot\|_k : k \geq 0\}$ is a separable Fréchet space. We say that $\xi \in S^\infty(T\Omega)$ is a *0-transversal vector field* on Ω [1, p. 62] if and only if every critical point of ξ is non-degenerate.

(3.4). The set $G_0 \subseteq S^\infty(T\Omega)$ of 0-transversal vector fields on Ω is dense in $S^\infty(T\Omega)$.

Proof. Similar to the one of (3.3).

REMARK. It is clear that each $\xi \in G_0$ has isolated critical points. Thus (3.4) also shows that there is a dense subset $G_0 \subseteq S^\infty(T\Omega)$ such that every $\xi \in G_0$ has only isolated critical points. Indeed, we can prove it directly as follows.

(3.5). There is a dense subset $G \subseteq S^\infty(T\Omega)$ such that every $\xi \in G$ has only isolated zeros.

Proof. Apply the Infinite Codimension Lemma [4] to find an integer k so large that

$$(9) \quad q = \text{codim}(W^k \text{ in } J^k(n, n)) > n$$

where W^k is the algebraic subset constructed in [4, p. 140].

Define $\tilde{W} = \Omega \times W^k \subseteq \Omega \times J^k(n, n) = J^k(T\Omega)$ and write \tilde{W} as a finite union of submanifolds of $J^k(T\Omega)$ of codimension greater than n . Then apply (2.2) to each member of the union.

Let $\Omega \subseteq \mathbb{R}^n$ be as before and let $f \in B^\infty(\Omega, \mathbb{R}^n)$. We say that f is *locally finite-to-one* [4] if and only if every point $x \in \Omega$ has a neighbourhood $U \subseteq \Omega$ such that $f^{-1}(y) \cap U$ is finite for all $y \in \mathbb{R}^n$.

(3.6). There is a dense subset $G \subseteq B^\infty(\Omega, \mathbb{R}^n)$ such that each $f \in G$ has the property that $f : \Omega \rightarrow \mathbb{R}^n$ is locally finite-to-one.

Proof. Similar to the one of (3.5).

Our last application is concerned with fixed points of C^∞ maps $f \in B^\infty(\Omega, \mathbb{R}^n)$.

(3.7). There is a dense subset $G \subseteq B^\infty(\Omega, \mathbb{R}^n)$ such that every $f \in G$ has only isolated fixed points.

Proof. As the one of (3.6).

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