Maximal functions and harmonic analysis

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The aim of this thesis is to present a study of various maximal functions, starting from the Hardy-Littlewood maximal function on $\mathbb{R}^n$ to more general ones on compact Lie groups. Our main interest here will be the $L^p$-inequality

$$\|Mf\|_p \leq C_p \|f\|_p, \quad f \in L^p(X),$$

where $M$ denotes a maximal operator on $L^p(X)$, $1 < p \leq \infty$.

We discuss various approaches to this subject, including covering lemmas, interpolation theorems, the Fourier transform method, the $g$-function argument, the Mellin transform technique and Lie group and representation theoretic arguments. We obtain, in particular, some results for maximal functions associated with distributions on $\mathbb{R}^n$ and a generalisation of those on compact semisimple Lie groups.