BULL. AUSTRAL. MATH. SOC. 46G20, 46E50
VOL. 49 (1994) [249–256]

THE STRICT TOPOLOGY ON SPACES OF BOUNDED HOLOMORPHIC FUNCTIONS

JUAN FERRERA AND ANGELES PRIETO

We introduce in this paper the space of bounded holomorphic functions on the open unit ball of a Banach space endowed with the strict topology. Some good properties of this topology are obtained. As applications, we prove some results on approximation by polynomials and a description of the continuous homomorphisms.

0. INTRODUCTION

The aim of this paper is to introduce an appropriate topology on the space of bounded holomorphic functions. It shares nice topological properties with the norm topology and allows better approximation properties than the norm. Our definition of strict topology enjoys good properties when it is restricted to natural subspaces of holomorphic functions: the dual space, the subspaces of continuous polynomials (where the strict topology coincides with the usual norm). This fact will be used to prove that the space of bounded holomorphic functions equals the completion of the space of polynomials. Part of this paper is devoted to the study of $\mathcal{H}^\infty(U)$ with the strict topology as a topological algebra. We apply the density theorem to characterise the strict-continuous algebra-homomorphisms for some Banach spaces. In these cases, some results are obtained, providing a better understanding of the problem than in the analogous case for the norm topology. As an application, we present some results on composition homomorphisms between algebras of type $\mathcal{H}^\infty$.

Throughout this paper, E will denote an arbitrary complex Banach space, E^* will be its dual and U the open unit ball of E. $\mathcal{H}^\infty(U)$ will denote the function space of all homomorphic bounded functions on U. For any non-negative integer n, $\mathcal{P}(^nE)$ will be the space of continuous n-homogeneous polynomials on E. Two important subspaces of $\mathcal{P}(^nE)$ are $\mathcal{P}_f(^nE)$ and $\mathcal{P}_c(^nE)$: $\mathcal{P}_f(^nE)$ is generated by $\{f^n : f \in E^*\}$ and $\mathcal{P}_c(^nE)$ is its completion with respect to the topology of uniform convergence on the unit ball.

For details on the theory of homomorphic functions we refer to [7].

Received 5 April 1993
Research partially supported by DGICYT grant 90-0044, (Spain).

1. Topological properties of the strict topology

Definition 1: Let U be the open unit ball in E. Consider $\mathcal{C}_b^+(U) = \{f\colon U \to [0, \infty),\, f \text{ is continuous, bounded}\}$. We define $K(U) = \{k \in \mathcal{C}_b^+(U)\colon$ for every $\varepsilon > 0$ there exists $r \in (0,1)$ such that $k(z) < \varepsilon$ if $z \in U \setminus rU\}$. Each $k \in K(U)$ defines a seminorm in $\mathcal{H}^\infty(U)$, $p_k(f) = \sup_{z\in U} |k(z)f(z)|$ for every $f \in \mathcal{H}^\infty(U)$. The *strict topology* β is the topology in $\mathcal{H}^\infty(U)$ induced by the family $\{p_k\colon k \in K(U)\}$.

For finite dimensional Banach spaces this definition coincides with the strict topology introduced by Buck in [**6**]. For a more general discussion in the one-dimensional case, see [**18**]. In [**15**], this topology is used for arbitary connected open sets in E such that $\mathcal{H}^\infty(U)$ is infinite-dimensional.

We pay attention to the completeness of $\mathcal{H}^\infty(U)$ endowed with the strict topology.

Theorem 2. *The space $\mathcal{H}^\infty(U)$ with the strict topology is complete.*

Proof: Choose an arbitrary β-Cauchy net (f_γ) in $\mathcal{H}^\infty(U)$. Since the space of all holomorphic functions with the compact-open topology, $(\mathcal{H}(U), \tau_0)$, is complete (see, for example [**7**, 16.13]), there exists $f \in \mathcal{H}(U)$ so that (f_γ) converges to f in the topology τ_0. We claim that f is also bounded. Otherwise, for every $n \in \mathbb{N}$ there exists $z_n \in U$ such that $|f(z_n)| > n^2$. Then the set $\{z_n\colon n \geqslant 1\}$ has no adherent points in U and is a closed discrete subset of U. Define $k \in K(U)$ such that $k(z_n) = 1/n$ (such a choice is possible because U is normal). Since $(k \cdot f_\gamma)$ is a uniformly Cauchy net, for $\varepsilon > 0$ there exists γ_0 such that the pointwise limit of the net, $k \cdot f$, satisfies $|k(z)f_{\gamma_0}(z) - k(z)f(z)| \leqslant \varepsilon$ for all $z \in U$. This contradicts the boundedness of k and f_{γ_0} proving that f is bounded. Now we have a β-Cauchy net $(f_\gamma) \subset \mathcal{H}^\infty(U)$ which is τ_0-convergent to $f \in \mathcal{H}^\infty(U)$. If we consider the base of β-closed neighbourhoods of zero $\{f \in \mathcal{H}^\infty(U)\colon p_k(f) \leqslant \varepsilon\}_{\varepsilon>0, k\in K(U)}$, by the Bourbaki Robertson theorem [**13**, p.210], (f_γ) is β-convergent to f, and $(\mathcal{H}^\infty(U), \beta)$ is complete. ∎

Recall that for an arbitrary open set U in E, a subset $V \subset U$ is said to be U-bounded if V is bounded and $\operatorname{dist}(V, E \setminus U) > 0$. It is easy to check that the topology τ_b of uniform convergence on U-bounded sets is weaker on $\mathcal{H}^\infty(U)$ than the strict topology and the latter is weaker than the topology induced by the sup-norm, $\|\cdot\|_\infty$. In fact, it is proved in [**16**] that the strict topology is the mixed topology $\gamma[\tau_b, \|\cdot\|_\infty]$, obtained by mixing the topologies τ_b and the norm. (See [**8**] for an analogous discussion in the one-dimensional case). The general theory on mixed topology provides these important facts:

(i) The strict bounded sets of $\mathcal{H}^\infty(U)$ are the norm bounded sets.

(ii) On norm bounded sets, the strict and τ_b topologies agree.

(iii) If F is a locally convex space and $T\colon \mathcal{H}^\infty(U) \to F$ is linear, then T is β-continuous if and only if $T_{|_B}$ is τ_b-continuous for all B norm-bounded set in $\mathcal{H}^\infty(U)$.

(iv) A sequence in $\mathcal{H}^\infty(U)$ is β-convergent to zero if, and only if, it is norm-bounded and τ_b convergent to zero. (See [**16**, Proposition 4]).

Nevertheless, these three topologies are always different, as the next result shows.

Proposition 3. *The space $\mathcal{H}^\infty(U)$ endowed with the strict topology is not metrisable. Thus, the strict topology and the norm do not agree. Furthermore, the τ_b topology is strictly weaker than the strict topology.*

Proof: Choose $f \in \mathcal{H}^\infty(U)$ such that $\|f\|_\infty = 1$. Now, if we define the set $A = \{f^n + nf^m : n, m \in \mathbb{N}\}$, then 0 is adherent to A, but no sequence in A converges to 0. (See [**18**, Proposition 3.12]; here, the same argument applies, recalling that for a given r in (0.1), there exists s in $(0, 1)$ such that $|f(z)| < s$ if $\|z\| < r$, as [**17**] shows).

Now, if we consider a holomorphic function f, bounded on rU for any r in $(0, 1)$, but unbounded on U, the sequence of partial sums of its Taylor series at the origin is τ_b convergent to f (see [**12**, Proposition 1]). This sequence is not a Cauchy sequence for the strict topology, since $(\mathcal{H}^\infty(U), \beta)$ is complete, but the sequence is not bounded. ∎

Since U is a bounded set, then the dual space E^* and the rest of the spaces of continuous polynomials can be regarded as subspaces of $\mathcal{H}^\infty(U)$.

Proposition 4. *The strict topology on $\mathcal{P}(^nE)$ coincides with the norm $\|\cdot\|_{\mathcal{P}(^nE)}$ on $\mathcal{P}(^nE)$, when $\mathcal{P}(^nE)$ is regarded as a subspace of $\mathcal{H}^\infty(U)$.*

Proof: It follows from [**5**, Lemma 2.ii] that a net in $\left(\mathcal{P}(^nE), \|\cdot\|_{\mathcal{P}(^nE)}\right)$ converges to zero if and only if there exists an open bounded set V in E such that the net tends to zero uniformly on V. Since we can define k in $K(U)$ with $k_{|rU} \equiv 1$ for a given r in $(0, 1)$, then the strict-topology and the norm are equivalent on $\mathcal{P}(^nE)$. ∎

It follows from Proposition 4 that $(\mathcal{H}^\infty(U), \beta)$ is not reflexive, whenever E is not reflexive. In fact, $(E^*, \|\cdot\|_{E^*})$ is a closed non-reflexive subspace of $(\mathcal{H}^\infty(U), \beta)$.

The convergence condition for sequences of homogeneous polynomials of increasing degree contrasts with the preceding result for polynomials of fixed degree.

Proposition 5. *Let U be the open unit ball of E. For each $n \in \mathbb{N}$, consider $P_n \in \mathcal{P}(^nE)$. Then, the sequence $(P_n)_{n=1}^\infty \subset \mathcal{H}^\infty(U)$ is strict convergent to zero if and only if $\{\|P_n\|_\infty : n \geqslant 1\}$ is bounded.*

Proof: The forward implication follows easily from the remark (i) before Proposition 3. Conversely, suppose $\{P_n : n \geqslant 1\}$ is $\|\cdot\|_\infty$-bounded. It is enough to show (see remark (iii) above) that, for a fixed $r_0 < 1$, the sequence (P_n) converges uniformly to zero on r_0U. And this follows from the boundedness of (P_n), as the degree increases. ∎

From the Cauchy inequalities and Proposition 5 we obtain the following Corollary.

Corollary 6. *Let U be the open unit ball of E and $f \in \mathcal{H}^\infty(U)$. Then the sequence $\left(\left(\widehat{d}^n f(0)\right)/(n!)\right)_{n\geqslant 0}$ is strict convergent to zero.*

Proposition 7. *Let U be the open unit ball of E. For each $n \in \mathbb{N}$, the space $\left(\mathcal{P}(^nE), \|\cdot\|_{\mathcal{P}(^nE)}\right)$ is a closed complemented subspace of $(\mathcal{H}^\infty(U), \beta)$.*

Proof: It is clear that the mapping $\Pi_n\colon (\mathcal{H}^\infty(U), \beta) \to \left(\mathcal{P}(^nE), \beta_{|\mathcal{P}(^nE)}\right)$ defined by $\Pi_n(f) = \left(\widehat{d}^n f(0)\right)/(n!)$ is a projection onto $\mathcal{P}(^nE)$. In order to show that Π_n is continuous, we choose a net $(f_\alpha)_\alpha$ β-convergent to zero in $\mathcal{H}^\infty(U)$. For $r \in (0, 1)$, the net $\left(f_{\alpha|rU}\right)_\alpha$ is uniformly convergent to zero. The Cauchy inequalities and Proposition 4 prove that Π_n is continuous and, hence, $\left(\mathcal{P}(^nE), \|\cdot\|_{\mathcal{P}(^nE)}\right)$ is a closed complemented subspace. ∎

We give a characterisation of the functions in $\mathcal{H}^\infty(U)$ having convergent Taylor series for the strict topology.

Proposition 8. *Let U be the open unit ball of E. Given $f \in \mathcal{H}^\infty(U)$, for each $n \in \mathbb{N}$, let $f_n(z) = \sum_{m=0}^{n} \left(\widehat{d}^m f(0)\right)/(m!)$. Then the sequence $(f_n)_{n\geqslant 0}$ is β-convergent to f if and only if $(f_n)_{n\geqslant 0}$ is uniformly bounded.*

Proof: The forward implication is obvious. Conversely, if (f_n) is uniformly bounded, we need only show that (f_n) is τ_b-convergent to f (comment (iv) above). It is clear, by [**12**, Proposition 1]. ∎

The next example proves the existence of functions in $\mathcal{H}^\infty(U)$ having non-convergent Taylor series.

Example 9: Let D be the open unit disk in $\mathbb{C}$. For each $n \in \mathbb{N}$, consider $L_n\colon \mathcal{H}^\infty(D) \to \mathbb{C}$ defined by $L_n(f) = \sum_{i=0}^{n} a_i$, when $f(z) = \sum_{i=0}^{\infty} a_n z^n$, $z \in U$. Then L_n is a linear, continuous mapping, for all n. Nevertheless, taking the polynomial

$$g_m(z) = \left(\frac{z}{m} + \frac{z^2}{m-1} + \ldots + \frac{z^m}{1} - \frac{z^{m+1}}{1} - \ldots \frac{z^{2m}}{m}\right),$$

it is shown (see [**14**, p.49]) that (g_m) is uniformly bounded, but $L_m(g_m) > \log m$. Thus, $(\|L_m\|_\infty)_m$ tends to infinity. Then, using the Uniform Boundedness Principle, there exists $f \in \mathcal{H}^\infty(U)$ with $\sup_{n\in\mathbb{N}} |L_n(f)| = \infty$. It follows that the partial sums of the Taylor series of f are not bounded. ∎

This example can be transferred to the unit ball of any Banach space.

We study now the problem of density of the family of polynomials. For the classical spaces of holomorphic functions (such as the space $\mathcal{H}(U)$ of all holomorphic functions, or the space $\mathcal{H}_b(U)$ of all holomorphic functions which are bounded over U-bounded sets) it is shown that the space of polynomials is dense with respect to their natural topologies (compact-open and τ_b, respectively). On the other hand, the completion of the class of all the polynomials with respect to the uniform topology (which is the classical topology for $\mathcal{H}^\infty(U)$) is just the space of all bounded holomorphic functions which can be extended continuously to the closed ball. Our last aim in this section will be to describe the space of bounded holomorphic functions as the completion of the space of all the polynomials with respect to the strict topology.

Lemma 10. *Let U be the open unit ball of E. Given $f \in \mathcal{H}^\infty(U)$ and $\lambda \in (0,1)$, let $f_\lambda(z) = f(\lambda z)$. Then the net (f_λ) converges to f in the strict topology, when λ tends to 1.*

Proof: For every $\lambda \in (0,1)$, $\|f_\lambda\|_\infty \leqslant \|f\|_\infty$. Thus, it is enough to show that the net (f_λ) converges to f uniformly on rU, for all $r \in (0,1)$ (see the comment (ii) preceding). And this follows from the Cauchy inequalities and a standard dilation argument. ∎

Theorem 11. *Let U be the open unit ball of E. The space of polynomials on E, $\mathcal{P}(E)$, is dense in $\mathcal{H}^\infty(U)$ with respect to the strict topology β. Hence, $\mathcal{H}^\infty(U)$ is the completion of $\mathcal{P}(E)$ with respect to the strict topology.*

Proof: Fix $f \in \mathcal{H}^\infty(U)$. The function f_λ defined by $f_\lambda(z) = f(\lambda z)$ is holomorphic and bounded in $U_\lambda = (1/\lambda)U$, for all $\lambda \in (0,1)$. The radius of boundedness of f_λ at 0 is $1/\lambda$ (see [**7**, p.219]) and the Taylor series of f_λ at 0 converges uniformly to f_λ on U. Now, fixing $\varepsilon > 0$ and $k \in K(U)$, there exists $\lambda_0 \in (0,1)$ such that $p_k(f - f_{\lambda_0}) < \varepsilon/2$. Given λ_0, there exists $n_0 \in \mathbb{N}$ such that $\left| f_{\lambda_0}(z) - \left(\sum_{k=0}^{n_0} \left(\widehat{d}^k f_{\lambda_0}(0)\right)/(k!)\right)(z)\right| < \varepsilon/2 \sup_{z\in U} k(z)$. Therefore, the polynomial $P(z) = \left(\sum_{k=0}^{n_0} \left(\widehat{d}^k f_{\lambda_0}(0)\right)/(k!)\right)(z)$ satisfies $p_k(f-P) < \varepsilon$. This completes the proof. ∎

2. The algebra $(\mathcal{H}^\infty(U), \beta)$

If the vector space $\mathcal{H}^\infty(U)$ is endowed with the pointwise product, it turns into an algebra. We obtain this basic result:

Proposition 12. *The space $\mathcal{H}^\infty(U)$ endowed with the strict topology is a commutative topological algebra with unity. Furthermore, its maximal closed ideals are the kernels of the complex valued strict continuous homomorphisms.*

In the following, $\mathcal{H}_E^\infty$ will denote the space $\mathcal{H}^\infty(U)$, where U is the open unit ball of E. In the next results, we shall use the extension of functions in $\mathcal{H}_E^\infty$ to the unit ball of the bidual, given by [**9**], and the extension to the bidual of polynomials in E

(see [4]). For $f \in \mathcal{H}_E^\infty$ and $P \in \mathcal{P}(E)$, we shall denote by $\widehat{f}$ and $\widehat{P}$ the corresponding extensions to $B_{E^{**}}(0, 1)$ and E^{**}. Here is a class of strict-continuous homomorphisms.

PROPOSITION 13. *Let E be a Banach space. For every z in $B_E{}^{**}(0, 1)$, the homomorphism defined by $\phi_z(f) = \widehat{f}(z)$, for all $f \in \mathcal{H}_E^\infty$, is strict continuous.*

PROOF: Let $(f_\alpha) \subset \mathcal{H}_E^\infty$ be a net strict-convergent to zero. Choose $r < 1$ such that $z \in B_{E^{**}}(0, r)$. By [**9**, Theorem 2] there exists a net (x_γ) in $B_E(0, r)$ which converges to z in the polynomial-star topology. The fact that $\left(f_{\alpha|B_E(0,r)}\right)$ is uniformly convergent to zero together with the fact that for a fixed α, $\left(f_\alpha(x_\gamma)\right)_\gamma$ converges to $\widehat{f}_\alpha(z)$ [**9**, Lemma], yields that ϕ_z is strict-continuous. ∎

It is obvious that the ideal $M_z = \{f \in \mathcal{H}_E^\infty : \widehat{f}(z) = 0\}$ is a strict closed maximal ideal, for any $z \in B_{E^{**}}(0, 1)$. We show the converse for a certain class of Banach spaces.

PROPOSITION 14. *Let E be a Banach space. For each algebra-homomorphism $\phi\colon \mathcal{H}_E^\infty \to \mathbb{C}$ there exists a unique z in the closed unit ball of E^{**} such that $\phi(P) = \widehat{P}(z)$, for all $P \in \mathcal{P}_c(E)$. Furthermore, if ϕ is strict-continuous, then $\|z\| < 1$.*

PROOF: Since $(\mathcal{H}_E^\infty, \|\cdot\|_\infty)$ is a Banach algebra, any homomorphism $\phi\colon \mathcal{H}_E^\infty \to \mathbb{C}$ is $\|\cdot\|_\infty$-continuous. Hence, $\phi_{|E^*}$ is $\|\cdot\|_{E^*}$-continuous. Thus, there exists a unique $z \in E^{**}$ such that $\phi_{|E^*} = z$; but $|\phi(f)| \leqslant \|f\|_{E^*}$, for all $f \in E^*$. This shows that $\|z\| \leqslant 1$. Now we choose $P \in \mathcal{P}_f({}^nE)$, $P = \sum_{i=1}^k f_i^n$ with $f_1, \dots, f_k \in E^*$; then $\phi(P) = \sum_{i=1}^k \phi(f_i^n) = \sum_{i=1}^k z(f_i^n) = \widehat{P}(z)$. From the norm continuity of ϕ, it follows that $\phi(P) = \widehat{P}(z)$, for all $P \in \mathcal{P}_c(E)$. Finally, if ϕ is strict-continuous and $\|z\| = 1$, for each $n \in \mathbb{N}$, we can choose $f_n \in B_{E^*}(0, 1)$ with $z(f_n) > 1 - 1/n$. The sequence $(f_n^n)_{n \geqslant 1}$ contradicts the continuity of ϕ and proves that $\|z\| < 1$. ∎

The next result follows from Proposition 14 and the strict density of $\mathcal{P}(E)$.

COROLLARY 15. *Let E be a Banach space such that $\mathcal{P}({}^nE) = \mathcal{P}_c({}^nE)$, for all $n \in \mathbb{N}$. Then for every strict-continuous homomorphism $\phi\colon \mathcal{H}_E^\infty \to \mathbb{C}$ there exists $z \in B_{E^{**}}(0, 1)$ such that $\phi(f) = \widehat{f}(z)$, for all $f \in \mathcal{H}_E^\infty$.*

The condition in Corollary 15 is satisfied by every finite-dimensional space; in particular, the strict-continuous homomorphisms on $\mathcal{H}^\infty(D(0, 1))$ are exactly the evaluations at points of $D(0, 1)$. Recall that the analogous result for the norm topology involves the well known Corona problem (see [**11**] for a discussion).

Consider the cpace c_0 of all complex sequences which tend to zero. It is known (see [**3**]) that $\mathcal{P}({}^nc_0) = \mathcal{P}_c({}^nc_0)$ for all $n \in \mathbb{N}$. Hence, every strict continuous homomorphism on $\mathcal{H}_{c_0}^\infty$ is the evaluation at a point of $B_{\ell^\infty}(0, 1)$ of the canonical extension given in [**9**].

Let T^* be the original Tsirelson space. This space was introduced by Tsirelson in 1973 [**19**] and provides an important example of a reflexive Banach space not containing any copy of ℓ_p nor c_0. In [**2**, Theorem 6], it is proved that $\mathcal{P}(^nT^*)$ is reflexive, for all $n \in \mathbb{N}$. Now, by [**1**, Theorem 7], it follows that $\mathcal{P}(^nT^*) = \mathcal{P}_c(^nT^*)$, for all $n \in \mathbb{N}$ and the result can be applied. Since T^* is reflexive, the strict-continuous homomorphisms on $\mathcal{H}^\infty_{T^*}$ are exactly the evaluations at points of $B_{T^*}(0, 1)$.

After considering the scalar-valued homomorphisms, the natural question arises of describing the strict continuous homomorphisms between algebras of bounded holomorphic functions. We claim that these homomorphisms, for certain classes of Banach spaces, are associated with holomorphic mappings between the underlying unit balls. We shall say that the algebra homomorphism $\phi\colon \mathcal{H}^\infty_F \to \mathcal{H}^\infty_E$ is a *composition homomorphism* if there exists a holomorphic mapping $\Phi\colon B_E \to B_{F^{**}}$ such that $\phi(f) = \widehat{f} \circ \Phi$, for all f in $\mathcal{H}^\infty_F$, where $\widehat{f}$ is its canonical extension to $B_{F^{**}}$.

Proposition 16. *The composition homomorphism $\phi\colon \mathcal{H}^\infty_F \to \mathcal{H}^\infty_E$ associated with a holomorphic mapping $\Phi\colon B_E \to B_{F^{**}}$ is continuous for the respective strict topologies.*

Proof: Since the strict topology in $\mathcal{H}^\infty_F$ is the mixed topology $\gamma[\tau_b, \|\cdot\|_\infty]$, it is enough to show that a bounded net (f_α) in $\mathcal{H}^\infty_F$ convergent to f_0 satisfies that the net $(\phi(f_\alpha))$ in $\mathcal{H}^\infty_E$ converges to $\phi(f_0)$ uniformly on every $B_E(0, r)$ $(0 < r < 1)$ (see (iii) in Section 1). This follows from the fact that $\Phi(B_E(0, r))$ is contained in $B_{F^{**}}(0, s)$, for some s in $(0, 1)$, by [**17**], and $(\widehat{f}_\alpha)$ converges to f_0 uniformly on this ball. ∎

We finish with a converse for certain Banach spaces.

Theorem 17. *Let E be a Banach space and let F a Banach space such that each polynomial on F is in $\mathcal{P}_c(^nE)$. Then every strict-continuous homomorphism from $\mathcal{H}^\infty_F$ to $\mathcal{H}^\infty_E$ is a composition homomorphism.*

Proof: Consider $\phi\colon \mathcal{H}^\infty_F \to \mathcal{H}^\infty_E$ a strict-continuous homomorphism. For every x in B_E the evaluation homomorphism δ_x, given by $\delta_x(g) = g(x)$ for all g in $\mathcal{H}^\infty_E$, is strict continuous. Hence, Corollary 15 assures that for any x in B_E there exists a unique z_x in $B_{F^{**}}$ so that $\delta_x \circ \phi(f) = \widehat{f}(z_x)$, for all f in $\mathcal{H}^\infty_F$. Define $\Phi\colon B_E \to B_{F^{**}}$ by $\Phi(x) = z_x$. Since $F^* \subset \mathcal{H}^\infty_F$, Φ is well-defined. In order to prove that Φ is holomorphic, choose v in F^*; its canonical extension $\widehat{v}$ to $B_{F^{**}}$ coincides with its inclusion in F^{***}, as B_F is $\sigma(F^{**}, F^*)$ dense in $B_{F^{**}}$. Then $\langle \Phi(x), v\rangle = \widehat{v}(\Phi(x)) = \phi(v)(x)$ for every x in B_E and v in F^*, that is, $\widehat{v} \circ \Phi$ is holomorphic in B_E for all v in F^*. Since F^* is norming in F^{***}, the Graves-Taylor theorem (see, for example, [**7**, 14.9]) together with the Dunford theorem (see [**10**, Theorem 76]), yields that Φ is holomorphic. ∎

References

[1] R. Alencar, 'On the reflexivity and basis for $\mathcal{P}(^mE)$', *Proc. Roy. Irish Acad.* **85 A** (1985), 131–138.

[2] R. Alencar, R. Aron and S. Dineen, 'A reflexive space of holomorphic functions in infinitely many variables', *Proc. Amer. Math. Soc.* **90** (1984), 407–411.

[3] R. Aron, 'Compact polynomials and compact differentiable mappings between Banach spaces', in *Sém. Pierre Lelong*, Lecture Notes in Maths 529 (Springer-Verlag, Heidelberg, Berlin, New York, 1976), pp. 213–222.

[4] R. Aron and P. Berner, 'A Hahn-Banach extension theorem for analytic mappings', *Bull. Soc. Math. France* **106** (1978), 3–24.

[5] J. Bochnak and J. Siciak, 'Polynomials and multilinear mappings', in *Algebraic properties of classes of analytic functions Sem. Analytic Functions II*, (R.C. Buck, Editor), 1957, pp. 175–188.

[6] R.C. Buck, 'Bounded continuous functions on locally compact spaces', *Michigan Math. J.* **5** (1958), 95–104.

[7] S.B. Chae, *Holomorphy and calculus in Banach spaces* (Marcel Dekker, New York, 1985).

[8] J.B. Cooper, 'The strict topology and spaces with mixed topology', *Proc. Amer. Math. Soc.* **30** (1971), 583–592.

[9] A. Davie and T. Gamelin, 'A theorem on polynomial-star topology', *Proc. Amer. Math. Soc.* **106** (1989), 351–356.

[10] N. Dunford, 'Uniformity in linear spaces', *Trans. Amer. Math. Soc.* **44** (1938), 305–356.

[11] J. Garnett, *Bounded analytic functions* (Academic Press, New York, 1981).

[12] J. Isidro, 'Topological duality on the functional space $(\mathcal{H}_b(U;F), \tau_b)$', *Proc. Roy. Irish Acad.* **79A** (1979), 115–130.

[13] G. Köthe, *Topological vector spaces*, Grundlehren Math. Wiss. **159** (Springer-Verlag, Berlin, Heidelberg, New York, 1969).

[14] E. Landau, 'Abschätzung der Koeffizientensumme einer Potenzreihe', *Arch. Math. Phys.* **21** (1913), 42–50.

[15] A. Prieto, *La topología estricta en espacios de funciones holomorfas*, Thesis (Universidad Complutense de Madrid, 1989).

[16] A. Prieto, 'Strict and mixed topologies on function spaces', *Math. Nachr.* **155** (1992), 289–293.

[17] A. Prieto, 'Sur le lemme de Schwarz en dimension infinite', *C.R. Acad. Sci. Paris Ser. I* **314** (1992), 741–742.

[18] L. Rubel and A. Shields, 'The space of bounded analytic functions on a region', *Ann. Inst. Fourier (Grenoble)* **16** (1966), 235–277.

[19] B. Tsirelson, 'Not every Banach space contains an imbedding of ℓ_p or c_0', *Funct. Anal. Appl.* **8** (1974), 138–141.

Departamento de Analisis Matematico
Universidad Complutense de Madrid
28040 Madrid
Spain