

## ON BANACH SPACES OF VECTOR VALUED CONTINUOUS FUNCTIONS

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Let  $K$  be a compact Hausdorff space and let  $E$  be a Banach space. We denote by  $C(K, E)$  the Banach space of all  $E$ -valued continuous functions defined on  $K$ , endowed with the supremum norm.

Recently, Talagrand [*Israel J. Math.* 44 (1983), 317-321] constructed a Banach space  $E$  having the Dunford-Pettis property such that  $C([0, 1], E)$  fails to have the Dunford-Pettis property. So he answered negatively a question which was posed some years ago.

We prove in this paper that for a large class of compacts  $K$  (the scattered compacts),  $C(K, E)$  has either the Dunford-Pettis property, or the reciprocal Dunford-Pettis property, or the Dieudonné property, or property  $V$  if and only if  $E$  has the same property.

Also some properties of the operators defined on  $C(K, E)$  are studied.

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## 1. Introduction

In 1953 Grothendieck [4] axiomatized some relevant properties of  $C(K)$  spaces, introducing among others the so-called Dunford-Pettis, reciprocal Dunford-Pettis and Dieudonné properties. Later Pelczynski [7], in 1962, showed that  $C(K)$  spaces enjoy another property, that he called property  $V$ , which can be defined on analogous terms to the preceding ones. Since then, these properties began to be studied on spaces of vector valued continuous functions, in particular on  $C(K, E)$ . The problem that was posed is the following: if  $E$  has the Dunford-Pettis property, does  $C(K, E)$  have this property too? The same question was asked for the other properties. This problem remained open for some years but, recently, Talagrand [9] constructed a Banach space  $E$  having the Dunford-Pettis property such that  $C([0, 1], E)$  fails to have the Dunford-Pettis property. Talagrand's work shows the interest of looking for conditions on spaces  $K$  and  $E$  to obtain an affirmative answer to the posed problem.

We prove in this paper that for a large class of compacts  $K$  (the scattered compacts), it is enough to take a Banach space  $E$  having one of the mentioned properties to insure that  $C(K, E)$  has the same property.

Since the study of such properties on a space is closely related to the study of the operators defined on it, we devote the first part of our work to show some properties of the operators defined on  $C(K, E)$ .

Throughout the paper  $E$  and  $F$  are Banach spaces,  $K$  is a compact Hausdorff space and  $\Sigma$  is the  $\sigma$ -field of Borel subsets of  $K$ .  $C(K, E)$  is the Banach space of all  $E$ -valued continuous functions on  $K$ , and  $B(\Sigma, E)$  is the Banach space of all functions  $\varphi : K \rightarrow E$  which are the uniform limit of a sequence of  $\Sigma$ -simple functions. Both spaces are endowed with the supremum norm. The term "operator" means a bounded linear operator. We denote by  $L(E, F)$  the space of all operators from  $E$  to  $F$ .

It is well known that an operator  $T : C(K, E) \rightarrow F$  may be represented as an integral with respect to a finitely additive set function  $m : \Sigma \rightarrow L(E, F)$  having finite semivariation on  $K$  ( $\hat{m}(K) < +\infty$ ) and so that  $\|T\| = \hat{m}(K)$  (see, for example, [2], p. 182);  $m$  is called the representing measure of  $T$ .

A compact space  $K$  is scattered if every subset  $A$  of  $K$  has a

point relatively isolated in  $A$ . The class of compact scattered spaces includes all countable compact spaces and all compact ordinals (with the interval topology).

## 2. Some properties of the operators defined on $C(K, E)$

An operator  $T : C(K, E) \rightarrow F$  whose representing measure  $m$  has its values in  $L(E, F)$  determines an extension  $\hat{T} : B(\Sigma, E) \rightarrow F$  given by

$$\hat{T}(\varphi) = \int_K \varphi dm, \quad \varphi \in B(\Sigma, E),$$

with  $\|\hat{T}\| = \|T\|$  (see [1], Theorem 2).

Batt and Berg [1] showed that an operator  $T : C(K, E) \rightarrow F$  is weakly compact if and only if its extension  $\hat{T}$  to  $B(\Sigma, E)$  is weakly compact.

We shall prove that one can obtain analogous results for other properties of  $T$  when  $K$  is metrizable.

**THEOREM 1.** *Let  $K$  be metrizable. Then an operator  $T : C(K, E) \rightarrow F$  is unconditionally converging if and only if its extension  $\hat{T}$  to  $B(\Sigma, E)$  is unconditionally converging.*

**Proof.** Let  $T : C(K, E) \rightarrow F$  be an unconditionally converging operator. Then, by Theorem 3 and Lemma 2 of [3], its representing measure  $m$  has its values in  $L(E, F)$  and there is a finite non negative measure  $\lambda$  on  $\Sigma$  so that

$$(1) \quad \lim_{\lambda(A) \rightarrow 0} \hat{m}(A) = 0.$$

If we suppose that  $\hat{T} : B(\Sigma, E) \rightarrow F$  is not unconditionally converging, then there exist  $\varepsilon > 0$  and a weakly unconditionally convergent series  $\sum \varphi_n$  in  $B(\Sigma, E)$  such that

$$(2) \quad \|\hat{T}(\varphi_n)\| > \varepsilon \quad \text{for all } n \in \mathbf{N}.$$

It is well known that a series  $\sum x_n$  in a Banach space is weakly unconditionally convergent if and only if the set

$$\left\{ \sum_{n \in \sigma} x_n : \sigma \subset \mathbf{N} \text{ is finite} \right\}$$

is bounded. Therefore there is  $M > 0$  verifying

$$(3) \quad \left\| \sum_{n \in \sigma} \varphi_n \right\| < M \text{ for all finite subsets } \sigma \text{ of } \mathbb{N} .$$

By (1) we can choose  $\delta > 0$ ,  $\delta < \lambda(K)$ , such that

$$(4) \quad \widehat{m}(A) < \frac{\varepsilon}{4M} \text{ for } A \in \Sigma \text{ with } \lambda(A) < \delta .$$

According to Lusin's theorem, for each  $n \in \mathbb{N}$ , there exists a compact  $K_n \subset K$  such that  $\lambda(K \setminus K_n) < \delta/2^n$  and  $\varphi_n|_{K_n}$  (the restriction of  $\varphi_n$  to  $K_n$ ) is continuous. Put  $K_0 = \bigcap_{n=1}^{\infty} K_n$ . Then  $\lambda(K \setminus K_0) < \delta$ , and  $K_0 \neq \emptyset$  because  $\delta < \lambda(K)$ . Let us denote  $\Phi_n = \varphi_n|_{K_0}$  for  $n \in \mathbb{N}$ . By (3) the series  $\sum \Phi_n$  is weakly unconditionally convergent in  $C(K_0, E)$ .

Now, by the Borsuk-Dugundji theorem (see 21.1.4 of [8]), there is an operator  $S : C(K_0, E) \rightarrow C(K, E)$ , with  $\|S\| = 1$ , so that  $S(\Phi)|_{K_0} = \Phi$  for every  $\Phi \in C(K_0, E)$ . The operator  $TS : C(K_0, E) \rightarrow F$  is unconditionally converging. However, the series  $\sum TS(\Phi_n)$  does not converge in  $F$  because, by (2), (3) and (4), for each  $n \in \mathbb{N}$ ,

$$\begin{aligned} \|TS(\Phi_n)\| &= \left\| \int_K S(\Phi_n) dm \right\| \geq \left\| \int_{K_0} \varphi_n dm \right\| - \left\| \int_{K \setminus K_0} S(\Phi_n) dm \right\| \\ &\geq \left\| \int_K \varphi_n dm \right\| - \left\| \int_{K \setminus K_0} \varphi_n dm \right\| - \|S(\Phi_n)\| \widehat{m}(K \setminus K_0) \\ &\geq \|\widehat{T}(\varphi_n)\| - 2M\widehat{m}(K \setminus K_0) > \varepsilon - \frac{\varepsilon}{2} = \frac{\varepsilon}{2} . \end{aligned}$$

This contradiction shows that if  $T : C(K, E) \rightarrow F$  is unconditionally converging then  $\widehat{T}$  is also. The converse is obvious.

**THEOREM 2.** *Let  $K$  be metrizable. Then an operator  $T : C(K, E) \rightarrow F$  transforms weakly Cauchy sequences into weakly convergent ones if and only if its extension  $\widehat{T}$  to  $B(\Sigma, E)$  transforms weakly Cauchy sequences into weakly convergent ones.*

Proof. Let  $T : C(K, E) \rightarrow F$  be an operator which maps weakly Cauchy sequences into weakly convergent sequences. Then  $T$  is unconditionally converging. Let  $m$  and  $\lambda$  be as in the preceding proof. Let  $(\varphi_n)$  be a weakly Cauchy sequence in  $B(\Sigma, E)$  and let  $y'' \in F''$  so that  $(\widehat{T}(\varphi_n))$  is  $\sigma(F'', F')$ -convergent to  $y''$ . If we suppose that  $(\widehat{T}(\varphi_n))$  is not weakly convergent in  $F$  then  $y'' \notin F$ . By using Grothendieck's completeness theorem (see 3.11.4 of [5]) it follows that there exist  $\varepsilon > 0$  and a net  $(y'_i)_{i \in I}$  in the unit ball of  $F'$  which is  $\sigma(F', F)$ -convergent to zero such that

$$(5) \quad |\langle y'_i, y'' \rangle| > \varepsilon \text{ for all } i \in I.$$

Choose  $\delta > 0$ ,  $\delta < \lambda(K)$ , verifying

$$\widehat{m}(A) < \frac{\varepsilon}{\delta \sup \|\varphi_n\|} \text{ for } A \in \Sigma \text{ with } \lambda(A) < \delta.$$

Similarly as in the preceding proof we can take a non empty compact  $K_0 \subset K$  so that  $\lambda(K \setminus K_0) < \delta$  and  $\Phi_n = \varphi_n|_{K_0}$  is continuous for  $n \in \mathbb{N}$ , and an operator  $S : C(K_0, E) \rightarrow C(K, E)$ , with  $\|S\| = 1$ , such that  $S(\Phi)|_{K_0} = \Phi$  for  $\Phi \in C(K_0, E)$ . For each  $t \in K_0$  the sequence  $(\Phi_n(t))$  is weakly Cauchy in  $E$ , therefore, according to Theorem 9 of [3],  $(\Phi_n)$  is weakly Cauchy in  $C(K_0, E)$ . Then  $(TS(\Phi_n))$  is weakly convergent to an element  $y \in F$ . Since  $(y'_i)_{i \in I}$  is  $\sigma(F', F)$ -convergent to zero there exists  $i_0 \in I$  so that

$$|\langle y, y'_i \rangle| < \varepsilon/6 \text{ for all } i \geq i_0.$$

Let  $i \geq i_0$ ; then there is  $n \in \mathbb{N}$  verifying

$$|\langle \widehat{T}(\varphi_n) - y'', y'_i \rangle| < \varepsilon/6 \text{ and } |\langle TS(\Phi_n) - y, y'_i \rangle| < \varepsilon/6.$$

Thus we have

$$\begin{aligned}
 |\langle y'', y'_i \rangle| &\leq |\langle y'' - \hat{T}(\varphi_n), y'_i \rangle| + |\langle \hat{T}(\varphi_n) - TS(\Phi_n), y'_i \rangle| \\
 &\quad + |\langle TS(\Phi_n) - y, y'_i \rangle| + |\langle y, y'_i \rangle| \\
 &\leq \frac{\varepsilon}{2} + \|y'_i\| \|\hat{T}(\varphi_n) - TS(\Phi_n)\| \\
 &\leq \frac{\varepsilon}{2} + \left\| \int_{K \setminus K_0} (\varphi_n - S(\Phi_n)) dm \right\| \\
 &\leq \frac{\varepsilon}{2} + 2\|\varphi_n\| \hat{m}(K \setminus K_0) < \frac{3}{4} \varepsilon .
 \end{aligned}$$

But this contradicts (5).

The converse is clear.

**THEOREM 3.** *Let  $K$  be metrizable. Then an operator  $T : C(K, E) \rightarrow F$  maps weakly convergent sequences into norm convergent sequences if and only if its extension  $\hat{T}$  to  $B(\Sigma, E)$  maps weakly convergent sequences into norm convergent ones.*

**Proof.** Let  $T : C(K, E) \rightarrow F$  be an operator which maps weakly convergent sequences into norm convergent ones. Then  $T$  is unconditionally converging. Let  $m$  and  $\lambda$  be as in the proof of Theorem 1. Let  $(\varphi_n)$  be a sequence in  $B(\Sigma, E)$  which is weakly convergent to zero. Suppose that there exist  $\varepsilon > 0$  and a subsequence of  $(\varphi_n)$  (which we still denote by  $(\varphi_n)$ ) so that

$$(6) \quad \|\hat{T}(\varphi_n)\| > \varepsilon \text{ for all } n \in \mathbb{N} .$$

Choose  $\delta > 0$ ,  $\delta < \lambda(K)$ , verifying

$$\hat{m}(A) < \frac{\varepsilon}{6 \sup \|\varphi_n\|} \text{ for } A \in \Sigma \text{ with } \lambda(A) < \delta .$$

Reasoning as in the proof of Theorem 1, there exist a non empty compact  $K_0 \subset K$ , with  $\lambda(K \setminus K_0) < \delta$ , such that  $\Phi_n = \varphi_n|_{K_0}$  is continuous for all  $n \in \mathbb{N}$ , and an operator  $S : C(K_0, E) \rightarrow C(K, E)$ , with  $\|S\| = 1$ , so that  $S(\Phi)|_{K_0} = \Phi$  for  $\Phi \in C(K_0, E)$ . According to Theorem 9 of [3],  $(\Phi_n)$  is weakly convergent to zero in  $C(K_0, E)$ . Then  $(TS(\Phi_n))$  is norm convergent to zero and there exists  $n_0 \in \mathbb{N}$  such that

$$\|TS(\Phi_n)\| < \varepsilon/3 \quad \text{for all } n \geq n_0 .$$

Thus if  $n \geq n_0$  one has

$$\begin{aligned} \|\hat{T}(\varphi_n)\| &\leq \|\hat{T}(\varphi_n) - TS(\Phi_n)\| + \|TS(\Phi_n)\| \\ &< \left\| \int_{K \setminus K_0} (\varphi_n - S(\Phi_n)) dm \right\| + \frac{\varepsilon}{3} \\ &\leq 2\|\varphi_n\| \hat{m}(K \setminus K_0) + \frac{\varepsilon}{3} < \frac{2}{3} \varepsilon . \end{aligned}$$

But this contradicts (6).

The converse is obvious.

### 3. Some properties on $C(K, E)$

**THEOREM 4.** *If  $K$  is scattered then  $C(K, E)$  has the Dunford-Pettis property if and only if  $E$  has.*

**Proof.** It is clear that if  $C(K, E)$  has the Dunford-Pettis property then  $E$  has it too.

Suppose that  $E$  has the Dunford-Pettis property.

(A) If  $K$  is metrizable then, by 8.5.5 of [8],  $K$  is countable. Now the proof of Theorem 13 (a) of [3] works the same here.

(B) For a general  $K$ , let  $T : C(K, E) \rightarrow F$  be a weakly compact operator and let  $(\Phi_n)$  be a sequence in  $C(K, E)$  weakly convergent to zero. Similarly as in [1], page 236, we can construct a metrizable quotient space  $\bar{K}$  of  $K$  and a sequence  $(\bar{\Phi}_n) \subset C(\bar{K}, E)$  such that  $\bar{\Phi}_n(\pi(t)) = \Phi_n(t)$  for all  $t \in K$  and  $n \in \mathbb{N}$ , where  $\pi : K \rightarrow \bar{K}$  is the canonical mapping. By 8.5.3 of [8],  $\bar{K}$  is scattered, and Theorem 9 of [3] implies that  $(\bar{\Phi}_n)$  is weakly convergent to zero in  $C(\bar{K}, E)$ . If we consider the operator  $\bar{T} : C(\bar{K}, E) \rightarrow F$  defined by  $\bar{T}(\bar{\Phi}) = T(\bar{\Phi} \cdot \pi)$  for  $\bar{\Phi} \in C(\bar{K}, E)$ , it follows from (A) that  $\lim_n \|\bar{T}(\bar{\Phi}_n)\| = 0$ . Since  $T(\Phi_n) = \bar{T}(\bar{\Phi}_n)$  for all  $n \in \mathbb{N}$ , we conclude that  $C(K, E)$  has the Dunford-Pettis property.

Note that if  $C(K, E)$  has the Dunford-Pettis property when  $K$  is

metrizable then, as in (B) of the preceding proof, it follows that  $C(K, E)$  has the Dunford-Pettis property for every compact  $K$ . Therefore an immediate consequence of 8.5.7, 21.5.10 and 21.5.1 of [8], and Theorem 4 is the following:

**COROLLARY 5.**  $C(K, E)$  has the Dunford-Pettis property for every compact  $K$  if and only if  $C([0, 1], E)$  has the Dunford-Pettis property.

Recall that if  $m$  is the representing measure of an operator  $T : C(K, E) \rightarrow F$ , it is said that the semivariation  $\hat{m}$  of  $m$  is continuous on  $\Sigma$  if for every decreasing sequence  $(A_n)$  in  $\Sigma$ , with  $\bigcap_n A_n = \emptyset$ , there is  $\lim_n \hat{m}(A_n) = 0$ .

**LEMMA 6.** Let  $K$  be a metrizable scattered compact space and let  $T : C(K, E) \rightarrow F$  be an operator whose representing measure  $m$  verifies

- (i)  $m(\Sigma) \subset L(E, F)$ ,
- (ii)  $m(A) : E \rightarrow F$  is weakly compact for each  $A \in \Sigma$ ,
- (iii)  $\hat{m}$  is continuous on  $\Sigma$ .

Then  $T$  is weakly compact.

*Proof.* By 8.5.5 of [8],  $K$  is countable. Put  $K = \{t_i : i \in \mathbb{N}\}$ . Let  $(\Phi_n)$  be a bounded sequence in  $C(K, E)$ . For each  $n \in \mathbb{N}$  we can take a  $\Sigma$ -simple function  $\varphi_n \in B(\Sigma, E)$  so that  $\|\varphi_n - \Phi_n\| < 1/n$ .

According to condition (ii), for every  $i \in \mathbb{N}$  the sequence  $(m(\{t_i\})(\varphi_n(t_i)))_n$  has a weakly convergent subsequence. This fact enables us to use Cantor's diagonal argument to extract a subsequence of  $(\varphi_n)$  (which we still denote by  $(\varphi_n)$ ) such that  $(m(\{t_i\})(\varphi_n(t_i)))_n$  is weakly convergent in  $F$  for all  $i \in \mathbb{N}$ . For each  $i \in \mathbb{N}$  let  $y_i$  be the  $\sigma(F, F')$ -limit of  $(m(\{t_i\})(\varphi_n(t_i)))_n$ . The series  $\sum y_i$  converges in  $F$ . To prove this suppose that there exist  $\varepsilon > 0$  and a sequence  $(\sigma_j)$  of finite subsets of  $\mathbb{N}$ , with  $\max \sigma_j < \min \sigma_{j+1}$  for  $j \in \mathbb{N}$ , such that

$$\left\| \sum_{i \in \sigma_j} y_i \right\| > \varepsilon \text{ for all } j \in \mathbb{N}.$$



Hence for every  $j \in \mathbb{N}$  we can choose  $y'_j$  in the unit ball of  $F'$  verifying

$$\left| \left\langle \sum_{i \in \sigma_j} y_i, y'_j \right\rangle \right| > \varepsilon .$$

Thus it follows from the choice of  $(y_i)$  that there is an increasing sequence  $(n_j) \subset \mathbb{N}$  such that

$$\left| \left\langle \sum_{i \in \sigma_j} m(\{t_i\}) (\varphi_{n_j}(t_i)), y'_j \right\rangle \right| > \varepsilon \text{ for } j \in \mathbb{N} .$$

We set  $A_j = \bigcup_{k=j}^{\infty} \{t_i : i \in \sigma_k\}$ ,  $j \in \mathbb{N}$ . Then one has

$$\begin{aligned} \hat{m}(A_j) &\geq \hat{m}(\{t_i : i \in \sigma_j\}) \\ &\geq \frac{1}{\sup \|\varphi_n\|} \left\| \sum_{i \in \sigma_j} m(\{t_i\}) (\varphi_{n_j}(t_i)) \right\| \\ &> \frac{\varepsilon}{\sup \|\varphi_n\|} . \end{aligned}$$

This contradicts condition (iii) since  $(A_j)$  is a decreasing sequence in  $\Sigma$  with  $\bigcap_j A_j = \emptyset$ . Therefore  $\sum y_i$  converges in  $F$ . Let  $y = \sum y_i$ .

We claim that  $(T(\Phi_n))$  is weakly convergent to  $y$ . Let  $\varepsilon > 0$  and let  $y' \in F'$  with  $\|y'\| \leq 1$ , then there exist  $n_0 \in \mathbb{N}$  and  $k \in \mathbb{N}$  such that

$$\left\| \sum_{i > n_0} y_i \right\| < \varepsilon/4, \quad \hat{m}(\{t_i : i > n_0\}) < \frac{\varepsilon}{4 \sup \|\varphi_n\| + 1}, \quad \frac{1}{k} < \frac{\varepsilon}{4 \|T\| + 1},$$

and

$$\left| \left\langle \sum_{i=1}^{n_0} m(\{t_i\}) (\varphi_n(t_i)) - \sum_{i=1}^{n_0} y_i, y' \right\rangle \right| < \varepsilon/4 \text{ for all } n \geq k .$$

If  $\hat{T}$  is the extension of  $T$  to  $B(\Sigma, E)$  and we put  $B = \{t_i : i > n_0\}$  then for each  $n \geq k$  one has

$$\begin{aligned}
 |\langle T(\Phi_n)_{-y}, y' \rangle| &\leq |\langle \hat{T}(\Phi_n - \varphi_n), y' \rangle| + |\langle \hat{T}(\varphi_n)_{-y}, y' \rangle| \\
 &< \frac{\varepsilon}{4} + \left| \left\langle \int_B \varphi_n dm, y' \right\rangle \right| + \left| \left\langle \left( \int_{K \setminus B} \varphi_n dm \right)_{-y}, y' \right\rangle \right| \\
 &\leq \frac{\varepsilon}{4} + \|\varphi_n\| \hat{m}(B) + \left| \left\langle \sum_{i>n_0} y_i, y' \right\rangle \right| \\
 &\quad + \left| \left\langle \sum_{i=1}^{n_0} m(\{t_i\}) (\varphi_n(t_i)) - \sum_{i=1}^{n_0} y_i, y' \right\rangle \right| < \varepsilon .
 \end{aligned}$$

Thus we conclude that  $T$  is weakly compact.

REMARK. Note that conditions (i), (ii) and (iii) of Lemma 6 are necessary for an operator  $T : C(K, E) \rightarrow F$  to be weakly compact but, in general, they are not sufficient.

THEOREM 7. *Suppose that  $K$  is scattered. Then  $C(K, E)$  has either the reciprocal Dunford-Pettis property, or the Dieudonné property, or property  $V$  if and only if  $E$  has the same property.*

Proof. We only consider the case of the Dieudonné property. The rest can be proved in the same way.

If  $C(K, E)$  has the Dieudonné property it is clear that  $E$  has it too.

Assume that  $E$  has the Dieudonné property.

(A) Let us first suppose that  $K$  is metrizable. Let  $T : C(K, E) \rightarrow F$  be an operator which maps weakly Cauchy sequences into weakly convergent ones. Then  $T$  is unconditionally converging and, by Theorem 3 of [3], its representing measure  $m$  verifies conditions (i) and (iii) of Lemma 6. For each  $A \in \Sigma$  the map  $\tau_A : E \rightarrow B(\Sigma, E)$  defined by  $\tau_A(x) = x\chi_A$  is a bounded linear map. So it follows from Theorem 2 that the operator  $m(A) = \hat{T}\tau_A : E \rightarrow F$  transforms weakly Cauchy sequences into weakly convergent sequences. Since  $E$  has the Dieudonné property  $m(A) : E \rightarrow F$  is weakly compact. Therefore, according to Lemma 6,  $T$  is weakly compact.

(B) For a general  $K$  the same method used in [1], page 236, and the fact that a metrizable quotient space of a scattered space is scattered (see 8.5.3 of [8]), proves that  $C(K, E)$  has the Dieudonné property.

The next result is an immediate consequence of 8.5.7, 21.5.1 and 21.5.10 of [8] and Theorem 7, by means of the standard reduction to the case  $K$  metrizable.

**COROLLARY 8.**  $C(K, E)$  has either the reciprocal Dunford–Pettis property, or the Dieudonné property, or property  $V$  for every compact  $K$  if and only if  $C([0, 1], E)$  has the same property.

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