I was pleased to see that the author does not perpetuate the myth that cycling in LP can be ignored. Indeed he is quite clear in pointing out that degeneracy is a common feature in many practical LP models. Unfortunately the cure that he describes and justifies, that is, the least index rule, is certainly not a practical method in floating point. As far as I could see, there is no description of techniques such as DEVEX and EXPAND which are used in practical codes. In general, although the author gives some feeling for the attention to detail needed to write a good LP code, I do not think that this book is a good source for finding out about this detail.

The book contains a description in Chapter 8 of various interior point methods, although it comes after a heavy dose of projective geometry. This follows a very detailed chapter on the properties of polyhedra. The presentation hereabouts in the book will appeal to those of a pure mathematical bent. There is probably quite a bit of material here that cannot easily be found in other texts and, for example, I found the description of the exact arithmetic (division free) Gaussian elimination algorithm to be of particular interest.

I was surprised that combinatorial optimization occupies a relatively small part of the book, particularly in view of the author's research interests in this area. The reader will find the discussion of the *branch and cut* method particularly interesting. The book ends with three appendices, each describing a large scale application of LP. Added to some other non-trivial examples in the introduction, these form a very useful feature.

R. FLETCHER

ARNOLD, V. I. *Topological invariants of plane curves and caustics* (University Lecture Series, Vol. 5, American Mathematical Society, Providence, RI, 1995), 60pp., paperback, 0 8218 0308 5, £17.50.

The book contains lectures on a range of topics from the author's most recent investigations. It is devoted to Vassiliev type invariants of plane curves and problems of low-dimensional symplectic and contact geometry.

The singularity theory approach to topological classification of objects of any nature follows a standard strategy going back to Poincaré. One considers the infinite-dimensional space of objects of interest, including both generic and degenerate objects. In this space degenerate objects form a discriminant hypersurface. An invariant is a locally constant function on its complement. The difference of its values on two neighbouring connected components can be related to the degeneracy corresponding to the piece of the discriminant which separates them.

This general approach was used with great success by Vassiliev in his work on knot invariants. In this book Arnold is taking the first step in constructing a similar theory for immersed plane curves and plane fronts (cooriented curves with cusps). Arnold represents three basic first order invariants which are dual to the three bifurcations: triple points on regular curves and two types of self-tangencies (with coinciding and opposite coorientations of the branches). He studies their properties and gives a series of geometrical interpretations. For example, the coinciding self-tangency invariant is basically the Bennequin-Tabachnikov number of Legendrian knots in the solid torus with the standard contact structure of spherisation of cotangent bundle of the 2-plane.

As a whole the theory of invariants of plane curves promises to be a sort of non-commutative version of knot theory.

Arnold's introduction of Vassiliev's technique to the study of plane curves was a by-product of investigations on a different topic (the story of Vassiliev invariants repeats!). The major aim (not yet achieved in the maximal desired generality) of Arnold's programme was to prove what might be called the Last Geometrical Theorem of Jacobi. Jacobi observed that the caustic of a point (that is, the set of intersection points of infinitesimally close geodesics starting at this point) on a closed convex surface should have a cusp. In fact, a point generates an infinite sequence of caustics. Jacobi's *Vorlesungen über Dynamik*, which was published after his death, contains an unfinished manuscript in which he announces (giving no proof) that for the surface of an ellipsoid the number of cusps on a caustic is equal to four.

In this book Arnold proves that in the sequence of caustics of a point on a convex surface close to a sphere each curve has at least 4 cusps. The proof requires an increase of closeness as the number of the caustic increases. For the first caustic this is the classical result of calculus of variations, which does not require any smallness condition at all. The author conjectures that such a condition is also unnecessary in the general setting.

Arnold discusses various generalisations and versions of the main problem he is considering. These are elegant statements of symplectic and contact geometry. Many of them concern the magic number 4. Some of them he proves: for example, the existence of at least four inflection points on a curve dividing the sphere into two parts of equal area. Some of them are left as conjectures: for example, the estimate of the number of bifurcations in a generic homotopy reversing a circular front on the plane. Although easy to formulate, the latter are apparently not so easy to prove.

The book is an excellent introduction to the area of low-dimensional geometry in which a mathematician of any level from an MSc student to a skilled professor would be able to find a source of interesting problems to solve. As in many previous cases the author opens up a new subject and encourages the reader to make his or her own contributions. Arnold suggests that investigations around the Jacobi Theorem constitute an area which is waiting for revolutionary methods like those which were used to solve another famous problem of his, the estimation of the number of fixed points of a symplectomorphism, and led, for example, to the Floer homology construction.

The book is extremely readable and contains many illustrations.

V. V. GORYUNOV

PETROV, V. V. *Limit theorems of probability theory: sequences of independent random variables* (Oxford Studies in Probability, Vol. 4, Clarendon Press, Oxford, 1995), x+292pp., 0 19 853499 X, £50.

The results of limit theory for sums of independent random variables play a central role in probability theory, mathematical statistics and their applications. Much attention is paid to works of the Russian probabilistic school headed by A. N. Kolmogorov (Moscow) and Yu. V. Linnik (St Petersburg). In 1954 Gnedenko and Kolmogorov wrote a classic book on this subject. The present work is in the same spirit and contains many recent as well as classical limit theorems and probability inequalities for sums of independent random variables. A useful feature of the book is that every chapter has a section of addenda with precise formulations of many (mostly recent) results which are usually inaccessible in book form. It is a well-written scholarly work based on 487 references. Whilst reading the book one is amply rewarded by theory, but occasionally one feels a lack of concrete applications.

The first chapter contains a summary of some basic concepts and theorems of probability theory that are needed for the later discussion. In Chapter 2 useful inequalities for the maximum, for moments and for the concentration functions of sums of independent random variables are given. Besides, a brief treatment of exponential bounds is presented. The third chapter starts with the condition of infinite smallness and after infinitely divisible distributions have been characterized as limit laws—the fundamental result due to Khintchine (1937)—various weak limit theorems within the structure of such distributions are given. The next two chapters treat the central limit theorems and the weak law of large numbers with their rates of convergence in the sense of weak limit concept. The last two chapters deal with strong limit theorems, the strong law of large numbers and the law of the iterated logarithm.

There is a wealth of useful material in Chapters 3–7. But this is not reflected in the given subject index. A finer and more detailed indexing of these results would have enhanced the utility of this book both for researchers and students. On the other hand, the author index at the end is a useful guide and so are the bibliographical notes and addenda in each chapter. The St