Note on Professor Allardice's Paper "On the Locus of the Foci of a System of Similar Conics through three Points." [Proceedings, Vol. XXVII.]

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The fundamental equation in this paper,

\[ a_1^2 + b_1^2 + c_1^2 - 2b_1c_1\cos A - 2c_1a_1\cos B - 2a_1b_1\cos C = \varepsilon^2 I^2, \]

can be derived, easily and without using trilinears, from the following property of conics.

Let \( F \) be one focus of a conic circumscribed to the triangle \( ABC \); draw \( AL \) perpendicular to the corresponding directrix from \( A(x_1y_1) \), \( BM \) from \( B(x_2y_2) \), \( CN \) from \( C(x_3y_3) \). Then

\[ \Sigma a^2 \cdot FA^2 - 2\Sigma FB \cdot FC \cdot bccos \Delta \]

\[ = \Sigma a^2(FA - FB)(FA - FC) = \varepsilon^2 \Sigma a^2(AL - BM)(AL - CN) \]

\[ = \varepsilon^2 \Sigma[(x_2 - x_1)^2 + (y_2 - y_1)^2](x_1 - x_2)(x_1 - x_3). \]

The expression \( \Sigma(x_2 - x_3)^2(x_1 - x_2)(x_1 - x_3) \) clearly vanishes; and the expression \( \Sigma(y_2 - y_3)^2(x_1 - x_2)(x_1 - x_3) \) is found to be the square of \( \Sigma(x_1y_2 - x_2y_1) \). Thus \( \Sigma a^2 \cdot FA^2 - 2\Sigma FB \cdot FC \cdot bccos \Delta = 4\varepsilon^2 \Delta^2. \)

As \( a/2\sin A = b/2\sin B = c/2\sin C = \Delta/I = R \), we get

\[ \Sigma FA^2\sin^2 A - 2\Sigma FB\sin B \cdot FC\sin C \cdot \cos \Delta = \varepsilon^2 I^2. \]
Now $FA^2 \sin^2 A$ is the square of the line joining the feet of the perpendicularly from $F$ on $AB$ and $AC$, and is thus equal to $\beta^2 + \gamma^2 + 2\beta \gamma \cos A$, i.e. to $a_1^2$. Hence we deduce the result above referred to.

It would appear that the property was obtained simultaneously by Professor M. T. Naraniengar: see the solution of his Question 16835 in the *Educational Times*, October 1910.