# MAKING A SEISMIC SOLAR MODEL AND AN ESTIMATE OF THE NEUTRINO FLUXES

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**Abstract.** We present a method of making a solar model based on the helioseismic data. We first invert the observed eigenfrequencies to determine the sound speed profile, and then solve the basic equations governing the stellar structure with the imposition of the determined sound-speed profile. This approach is different from that of the standard solar model in the sense that the 'seismic' solar model is a snapshot model of the sun constructed without any assumption about the history of the sun. We invert the data obtained at the South Pole by the Bartol/NSO/NASA group along with BISON, HLH, and LOWL data. Finally we estimate the neutrino fluxes of the seismic model.

## 1. Introduction

One of the scientific goals of helioseismology is to discriminate between the possible solutions of the solar neutrino problem; a defect in the modeling of the sun or one in particle physics. It will be helpful for this purpose to examine quantitatively whether the helioseismic data and the neutrino flux measurements are consistent each other. If the predicted neutrino fluxes of a model based on the helioseismic data are consistent with the neutrino flux measurements, evolutionary solar models are the likely source of the neutrino problem. It is thus desirable to determine the solar-interior structure from the helioseismic data, and to compare the expected neutrino fluxes based on such a model with the detected neutrino fluxes. In our previous work we have deduced the solar-interior structure subject to the constraint that the sound-speed profile is that determined by helioseismic data of Libbrecht et al. (1990) (Shibahashi and Takata 1996). In that work, however, we adopted the sound-speed profile from a previous inversion of the data by



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Vorontsov and Shibahashi (1991). In order to estimate the error level more precisely, we should have performed an inversion to obtain the sound-speed profile itself as a part of the work to reconstruct a seismic solar model. In this paper, we perform inversion of the observed eigenfrequencies of the sun to determine the sound speed profile, and, then we deduce the density, pressure, temperature, and hydrogen profiles in the solar interior by solving the basic equations governing the stellar structure constrained by the sound-speed profile. The error levels are estimated by a Monte-Carlo simulation using Gaussian noise added to the frequency data. We invert the data obtained by the Bartol/NSO/NASA group along with data from BISON, HLH, and LOWL, and estimate the neutrino fluxes.

## 2. Methodology of Making a Seismic Solar Model

The standard solar models are based on assumptions about the evolutionary history of the sun (cf. Provost, in these proceedings). Although the standard theory of stellar evolution has succeeded in explaining many observational properties of stars, its success has been in treating stars as a statistical group. There is no guarantee that a specific star, the sun in this case, follows this theory precisely. In the following we depart from the standard construction of a solar model, and try to reconstruct a solar model by using only experimentally measured quantities. These quantities are the mass $M_\odot$, radius $R_\odot$, photon luminosity $L_\odot$, and the sound-speed distribution $c(r)$ obtained from helioseismology (Shibahashi 1993, Shibahashi and Takata 1996). We assume that the sun is in hydrostatic equilibrium. Whether the sun is in thermal balance is uncertain. In this paper, however, we assume that the sun is in thermal balance. The model is spherically symmetric and we ignore the effects of rotation and the magnetic field. We want to emphasize that we do not need any assumptions concerning the history of the sun, and that the seismic solar model constructed in this way is a snapshot model of the present day sun.

The basic equations for constructing a model with the above assumptions are the same as those used in theory of stellar structure:

$$dM_r/dr = 4\pi r^2 \rho, \tag{1}$$

$$dP/dr = -GM_r \rho / r^2, \tag{2}$$

$$dL_r/dr = 4\pi r^2 \rho \varepsilon, \tag{3}$$

$$dT/dr = \begin{cases} -\frac{3}{4ac}\frac{\kappa\rho}{T^3}\frac{L_r}{4\pi r^2} & \text{if radiative} \\ (dT/dr)_{conv} & \text{if convective} \end{cases}. \tag{4}$$

The above expressions differ slightly from the usual ones in that the gravitational energy release is ignored in equation (3): $\varepsilon_g = 0$. A more important difference is the treatment of the auxiliary equations. Since the sound speed, which we regard as a known function of $r$, is a thermodynamically determined quantity, it is a function of two other thermodynamical quantities such as $\rho$ and $P$ along with the chemical composition $X_i$. We can express the pressure as a function of $c$, $\rho$ and $X_i$;

$$P = P[\rho, X_i, c(r)]. \tag{5}$$

Similarly, $T = T[\rho, X_i, c(r)]$, $\kappa = \kappa(\rho, T, X_i) = \kappa[\rho, X_i, c(r)]$, and $\varepsilon = \varepsilon(\rho, T, X_i) = \varepsilon[\rho, X_i, c(r)]$. Then the basic four equations (1)-(4) are a set of equations for $M_r$, $\rho$, $L_r$, and $X_i$ for the given $c(r)$.

We assume that the abundance ratios of the various heavy elements in the solar interior are the same as those observed spectroscopically near to the solar surface. We adopt $Z/X = 0.0277$ (Grevesse 1984). To fix $X$ and $Z$, we fix the helium abundance $Y = 0.23$, which is consistent with the helioseimologically determined value of $Y$ (e.g., Basu and Antia 1995). The convection zone is assumed to be chemically homogeneous, and then, $X_i$ is fixed in the convection zone. The extent of the convection zone is helioseismologically determined from the kink of $c(r)$.

Equations (1)–(4) form a boundary value problem with the following boundary conditions: $M_r = 0$ and $L_r = 0$ at $r = 0$, $M_r = M_\odot$ at $r = R_\odot$ and

$$X_i = \textit{ the given value } \quad \text{at } r = R_\odot. \tag{6}$$

One of the boundary conditions (6) is used instead of $L = L_\odot$ at $r = R_\odot$, which is obviously required as a solar model. It should be remembered that the luminosity and radius should be determined as eigenvalues in solving the equations for the stellar structure. Hence, there is not always a solution which satisfies $L = L_\odot$ at $r = R_\odot$. If there is no such solution, this means (i) the inverted sound speed is incorrect due to defect of the inversion method, or (ii) the observed frequencies involve large errors, or (iii) our knowledge about micro-physics (the nuclear reaction rates, the opacity, and the equation of state) is poor, or (iv) the sun is not in thermal balance. We have not considered the possibility (iv), and we take account of errors involved in the inversion method and those in the frequency data and in micro-physics. Our policy at the moment is to accept the reasonable micro-physics as it is, while we estimate the error levels due to inversion or the frequency data by performing a Monte-Carlo simulation with Gaussian noise on the frequency data. Among solutions of (1)-(4) obtained in this way, we admit only the solution satisfying $L = L_\odot$ as a seismic solar model.

## 3. Method of Inversion of the Frequency Data to the Sound Speed Profile

Let us now turn to the problem of inversion of the eigenfrequencies to the sound speed profile. There are various methods (cf. Basu, in these proceedings), and we adopt here the asymptotic method developed by Vorontsov (1990). According to this theory, the function $\mathcal{T}$ defined by $\mathcal{T} \equiv (n + 1/2)\pi/\omega$ is decomposed into a combination of a function of $\tilde{w} \equiv (\ell + 1/2)/\omega$ and a function of $\omega$, and the cross term, which is separated into a function of $\tilde{w}$ over $\omega^2$; —that is,

$$\mathcal{T}(\tilde{w}, \omega) = F(\tilde{w}) + G(\omega) + \Phi(\tilde{w})/\omega^2, \tag{7}$$

where $n$ is the radial order, $\ell$ is the degree of the mode, and $\omega$ denotes the eigenfrequency. The frequency is considered to be continuous function of the continuous variables $n$ and $\ell$, and then the function $\mathcal{T}$ is treated as a continuous function of $\omega$ and $\tilde{w}$. The function of $\tilde{w}$, $F(\tilde{w})$, is given by

$$F(\tilde{w}) \equiv \int_{r_1}^{R_\odot} \left(c^{-2} - \tilde{w}^2 r^{-2}\right)^{1/2} dr, \tag{8}$$

and, hence, once the function $F(\tilde{w})$ is discriminated, the sound-speed profile $c(r)$ is obtained by solving an Abel-type integral equation, which is led by differentiation of (8) with respect to $\tilde{w}$. Therefore, a key process is decomposition of $(n+1/2)\pi/\omega$. We adopt the following method. The function of $\omega$ alone, $G(\omega)$, is first eliminated by taking the partial derivative of $\mathcal{T}$ with respect to $\tilde{w}$, and then, the functions $F(\tilde{w})$ and $\Phi(\tilde{w})$ are obtained by minimizing

$$\chi^2 \equiv \int\int \left(\mathcal{T}_{\tilde{w}} - F_{\tilde{w}} - \Phi_{\tilde{w}}/\omega^2\right)^2 d\tilde{w}d\omega, \tag{9}$$

where the subscript $\tilde{w}$ means the derivative with respect to $\tilde{w}$. Taking variation of $\chi^2$ associated with a slight change of $F_{\tilde{w}}$ and $\Phi_{\tilde{w}}$ and requiring $\delta\chi^2 = 0$ for any $\delta F_{\tilde{w}}$ and $\delta\Phi_{\tilde{w}}$, we get two equations, by which $F_{\tilde{w}}$ and $\Phi_{\tilde{w}}$ are separated from $\mathcal{T}_{\tilde{w}}$. Here the derivative $\mathcal{T}_{\tilde{w}}$ is evaluated by $\mathcal{T}_{\tilde{w}} = -\pi(\partial\omega/\partial\ell)/(\partial\omega/\partial n)$ . Since the asymptotic inversion is a nonlinear inversion, it may produce undesirable, spurious features. To eliminate the possibility of spurious results, we invert the frequencies of a theoretical solar model to get the sound speed and calibrate the inverted result of the observed frequencies by seeing how well the sound speed of the model is reproduced from the theoretical frequencies.

Vorontsov's asymptotic formula (7) has been used by Vorontsov and Shibahashi (1991) and the sound speed thus obtained was used in our previous work (Shibahashi and Takata 1996). However, decomposition of

$\mathcal{T}(\omega, \tilde{w})$ has been carried out by using cubic B-spline, which does not necessarily lead a unique solution. The present method of determining $F(\tilde{w})$ is essentially a nonlinear least square method and it is more objective. The adoption of a new decomposition method is a major difference from our previous work.

## 4. Actual Inversion Using the Observational Data

The data obtained at the South Pole by Jefferies, Pomerantz, Harvey, and Duvall in 1987, 1988, and 1990 cover a wide range of $\ell$ ($1 \leq \ell \leq 700$), and are suitable for our purpose (cf. Jefferies et al. 1990, Duvall 1995). (Hereafter we call these data SP87, SP88, and SP90, respectively.) We use only the frequencies $2.2\text{mHz} \leq \nu \leq 4.8\text{mHz}$. The higher degree modes data are supplemented by the HLH data taken in 1993 at Kitt Peak (Bachmann et al. 1995). From the HLH data of $100 \leq \ell \leq 1200$, we selected only the modes which are not present in the South Pole data and $\ell \leq 750$. The low degree modes are important for determining the structure of the nuclear reacting core. We combine the data with the low-degree frequency data obtained by the BISON group (Elsworth et al. 1994), since their error estimates are lower than those of the low-degree mode of the South Pole data. It is known that the p-mode frequencies change with solar activity, and the BISON group presented a frequency data set which was corrected to the minimal level of radio flux. We adopt this corrected frequency data set.

The LOWL provides us another uniform data set of frequencies for $0 \leq \ell \leq 99$ (Tomczyk et al. 1995). The data adopted here is the weighted average frequencies obtained in the period 2/26/94 - 2/25/96 and computed by Schou and Tomczyk (1995). The higher degree mode data are again supplemented by the HLH data. We also constructed data sets by combining the LOWL data with the SP data and the HLH data.

Figure 1 shows the sound-speed profile obtained by using these data sets. The lines show the profiles for the most likely values of the frequencies along with the 1-$\sigma$ level error bars estimated by a Monte-Carlo simulation. In solving the basic equations (1)-(4) with the imposition of the sound-speed profile, we adopted the OPAL opacity library (Rogers and Iglesias 1992) and a subroutine written by Bahcall et al. (1995) to provide the opacity and the nuclear reaction rates, respectively. We treated $^3$He distribution as being in equilibrium in the deep interior, and assumed that the distribution in the outside follows the accumulation of $^3$He due to the D(p,$\gamma$)$^3$He reaction without destruction. The CNO cycle is ignored. We adopted a simple perfect gas law as the equation of state. It should be remembered that there is not always a solution which satisfies $L = L_\odot$ at $r = R_\odot$ and that our

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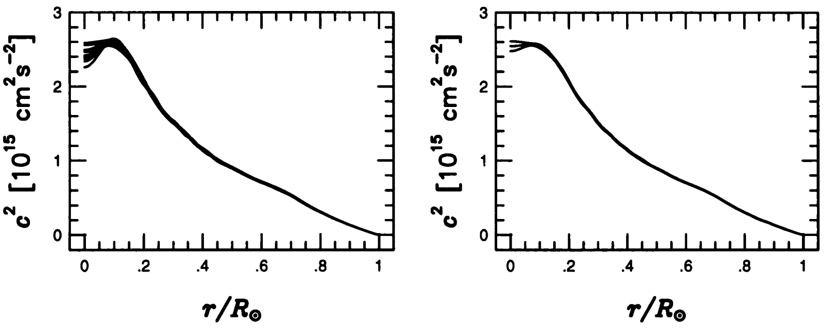


*Figure 1.* Figure 1. Squared sound-speed profile inverted from the data of various combinations of SP/HLH, BISON, and LOWL (left) and LOWL/HLH (right).

present policy is to accept the reasonable micro-physics as it is, while we perform a Monte-Carlo simulation with Gaussian noise on the frequency data. Among solutions of (1)-(4) obtained in this way, we admit only the solution satisfying $L = (1 \pm 0.01)L_\odot$ as a seismic solar model. The density profile and and pressure profile obtained as solutions are shown in figure 2. By using these profiles, we confirmed that the core is convectively stable. Figures 3 shows our estimates for the temperature and the hydrogen abundance profiles. The latter is fairly constant outside the nuclear reacting core though there remains a wiggly feature. It should be emphasized that such a constancy is not assumed in making a seismic model as in the case of a standard solar model. This means that roughly speaking the OPAL opacity is correct. The slight decrease of $X$ with depth from the base of the convection zone might be due to diffusion. The neutrino fluxes at one

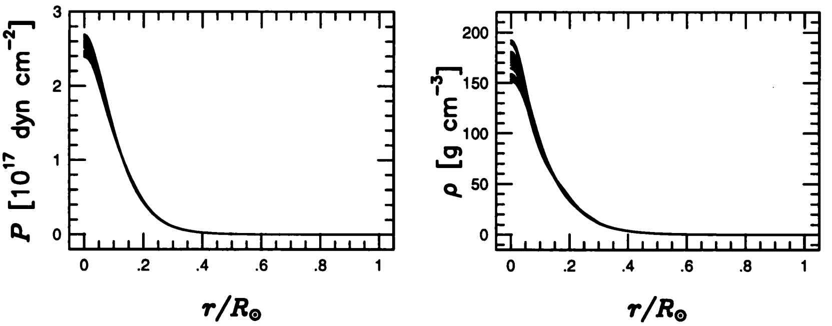


*Figure 2.* Pressure profile (left) and density profile (right) obtained from the various combinations of SP/HLH/BISON/LOWL.

astronomical unit can be estimated along with a calculation of the nuclear

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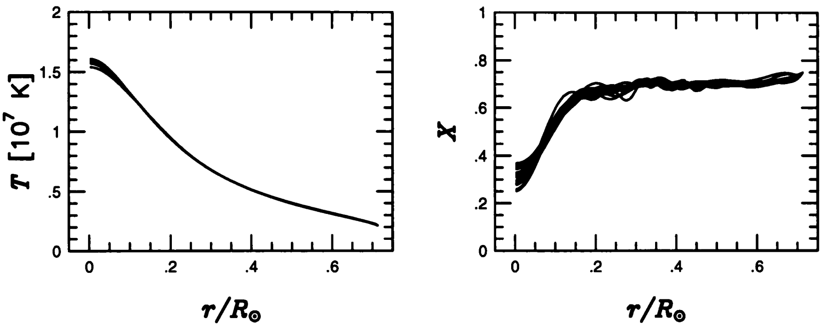


*Figure 3.* Temperature profile (left) and hydrogen abundance profile (right) obtained from the various combinations of SP/HLH/BISON/LOWL.

TABLE 1. Capture rates predicted by the seismic solar models

| Helioseismic Data | Kamiokande | Cl (SNU) | Ga (SNU) |
|---|---|---|---|
| SP87/HLH/LOWL | 6.0-7.4$\times10^6$ cm$^{-2}$s$^{-1}$ | 8.1-9.9 | 127.-135. |
| SP88/HLH/LOWL | 6.7-9.4 | 9.0-12.3 | 130.-142. |
| SP90/HLH | 4.0-6.6 | 5.7-8.6 | 116.-130. |
| SP87-90/HLH/BISON | 6.1-7.2 | 8.2-9.5 | 127.-132. |
| LOWL/HLH | 7.9-8.6 | 10.3-11.2 | 134.-139. |

reaction rates from the estimated temperature and chemical composition distributions. Table 1 summarizes the estimated neutrino fluxes based on the present seismic solar models.

## 5. Discussion

We found that, as far as we adopt the most likely values of observed frequencies and the most likely micro-physics, the resulting luminosity of the model is smaller than $1L_\odot$. This was noted in our previous work (Shibahashi and Takata 1996) as well as by Roxburgh (1996) and Antia and Chitre (1996). A seismic solar model satisfying $L = L_\odot$ can be obtained taking account of uncertainties of either the seismic data or micro-physics. Indeed, Roxburgh (1996) and Antia and Chitre (1996) reproduced the solar luminosity by modifying the nuclear reaction rates. Our policy is to take account of the uncertainties of both the seismic data and micro-physics and to find the model satisfying $L = L_\odot$ with the least deviation from the most likely values in the multi-dimensional space of uncertainties. The present work is the first step of our attempt: we adopted the most likely micro-physics (but for the equation of state) and took account of only the observational errors. We want to stress that we did this so that we could construct a snapshot solar model from the helioseismic data and could quantitatively estimate the neutrino fluxes without the help of the so-called standard solar models. There is a discrepancy between the present inverted results and our previous one (Shibahashi and Takata 1996) based on the data compiled by Libbrecht et al. (1990) even though the input physics is the same. This is mainly due to the difference in decomposition of $\mathcal{T}(\tilde{w}, \omega)$ into $F(\tilde{w})$ rather than the difference in the observational data.

From the present result, we can say that the helioseismic data and the measured neutrino fluxes are inconsistent if we accept the most likely micro-physics. It may seem that this supports the particle physics solution of the solar neutrino problem. However, we should further examine the neutrino fluxes of seismic models, taking account of uncertainties in micro-physics, before reaching any conclusions. Such approach is now in progress.

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