

From this formula it may readily be verified that l is 6046 ft. in latitude 0° and 6108 ft. in latitude 90° .

The U.K. nautical mile, which is the standard unit of distance used by British navigators, is 6080 ft. (1853.18 m.). This is about 0.06 per cent greater than the international nautical mile which is 1852 m. exactly. The United States nautical mile, the length of which is 6080.22 ft., is defined as the length of a minute arc of a great circle of a sphere having an area equivalent to that of the Clarke 1866 spheroid, the compression of which is $1/295$. An illuminating account of these and related matters is to be found in a paper by Moody.²

Whatever disadvantages the standard nautical mile may have as a navigational unit of distance, it does not come into that category of units mentioned in the second paragraph of Ronald Turner's paper.¹ On the contrary, it is a unique unit of distance systematically related to, which blends harmoniously with, the units of the sexagesimal system of angular measurement.

There is no doubt that advantages are to be gained by adopting an international unit for heights and depths shown on charts, and by replacing the foot and fathom by the international metre. But whether or not the mariner's unit and scale of distance, which correspond respectively to the unit of d. lat. and the scale of latitude on a Mercator chart, and whether or not the standard nautical mile is obsolescent, are matters which demand close attention.

If the kilometre is to be adopted as the standard unit of distance for navigational purposes, then it would not be unreasonable for navigators to demand a new system of angular measure in which the desirable harmony between the respective units of distance and angle is preserved. Such a system of angular measure could well be based on the decimal system, proposed by French philosophers of the last century, in which a right angle is divided into 100 equal parts or *grades*, each grade being sub-divided into 100 *centigrades*. The important factor is that the new unit should bear the same ratio to the minute of the sexagesimal system as the kilometre bears to the standard nautical mile. In this event, the kilometre would be the logical unit for use in navigation, and the present relationships between rhumb-line course, rhumb-line distance, d. lat., departure, and d. long., would be preserved. But the problems involved in changing to such a system of angular measure would be so considerable that the result would not be worth the effort.

REFERENCES

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Line of Position from a Horizontal Angle

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A horizontal angle between two charted objects provides a line of position and two angles between three or four objects will provide a fix or running fix except in the unique case of a 'swinger' or 'revolver'. The method is faster than the conventional three-arm protractor.

INTRODUCTION. A pelorus is frequently used to obtain two or more bearings for a position fix with generally sufficient accuracy. In rough weather the fix may not be particularly accurate, especially if the charted objects are a long distance away, such as lighthouses. If greater accuracy is desired, one solution is to use a sextant held horizontally to measure the two angles between three charted objects, plotting the fix by means of a three-arm protractor. The angles must be less than the 120-degree capability of the sextant and must be taken essentially simultaneously unless the ship's speed is slow. In some cases, it may be found that two objects lie to starboard and two to port, but the angles between the pairs exceed 120 degrees. The three-arm protractor cannot be used in this case. The method to be described will provide a fix or running fix without using a three-arm protractor.

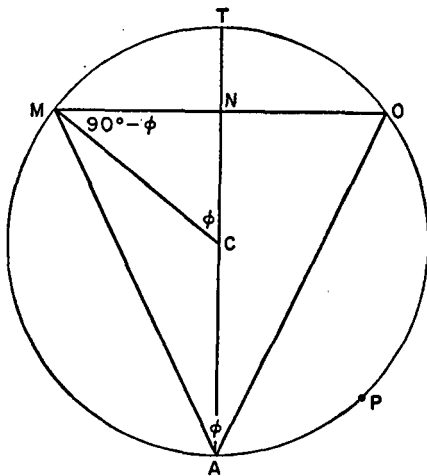


FIG. 1. The geometry

THEORY. In Fig. 1, P represents the ship's position where a horizontal angle ϕ is measured between two objects at M and O. If a circle is drawn through M, O and P, a line of position MAPO is obtained. The angle ϕ would be measured at any point on this position line, while all angles measured along the arc MTO would be the supplement of ϕ . The point A was selected where the position line intersects the perpendicular bisector of the base-line MO. The centre of the circle at C lies on this bisector and it is obvious that the angle MCN is also ϕ and that the angle NMC is the complement of ϕ . Thus, if the perpendicular bisector is drawn on the chart, the centre of the circle can be easily

located by using a protractor at M or O and laying off the angle $90-\phi$. A quicker method is to use a Weems Paralane plotter, with the angle ϕ on the meridian scale placed over the bisector line. Moving the plotter to M or O provides the centre point C. The radius of the position line is CM or CO. Two position lines will provide a fix. If one line must be advanced for a running fix, the centre C is advanced the proper amount before drawing the circle.

The accuracy can be improved somewhat, especially if either ϕ or the base-line length is small, by noting that $CN = \frac{1}{2}MO \cot \phi$. A slide rule is usually sufficient for the calculation after looking up the $\cot \phi$ in a table. This formula has been used¹ in accurate surveying with specially prepared large-scale charts even though in some cases the charted objects were off the chart used. The circle radius was calculated from the relation $R = \frac{1}{2}MO \operatorname{cosec} \phi$. Grids of position lines were drawn on plastic overlays for each base-line pair using different coloured inks.

Fig. 2 shows a fix plotted from sextant angles of $30\frac{1}{2}$ and $63\frac{1}{2}$ degrees. The complete position line was drawn in one case for illustrative purposes. The bisectors have been marked with a scale showing the centre locations for various values of ϕ . These scales can be drawn on plastic overlays if preferred. For

sextant angles larger than 90 degrees, the centre of the position line is on the opposite side of the baseline from the ship. By using such scales, a fix can be plotted in 30 seconds. If a sextant angle is smaller than any on the scale, the centre can be located by a simple procedure. The sextant angle is doubled and an arc of this position line is drawn across the bisector. The intersection is the centre of the desired position line. This technique is also useful for checking the scale for accuracy.

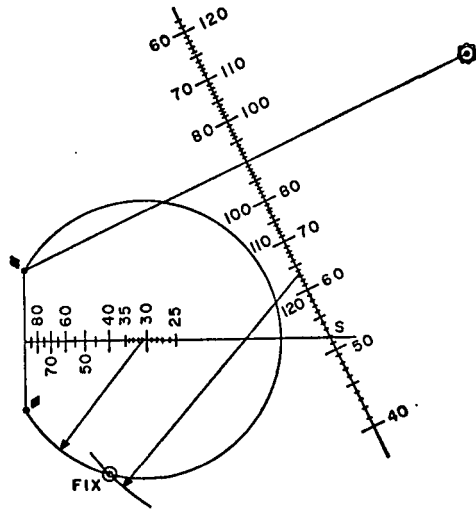


FIG. 2. Use of scales for a quick fix

If the centre of one of the position lines comes close to the point S, the intersection of the two perpendicular bisectors, the centre of the other position line will likewise come close, causing the intersecting arcs at the fix to cross at a small angle which is detrimental to accuracy. In the limit, both centres at S results in a 'swinger' or 'revolver'.² In this case, a fix may be obtained by taking a bearing on the closest object if another charted object is not available.

CONCLUSIONS. The method provides a position line for each horizontal angle measured and can provide a fix faster than with a three-arm protractor. The method also indicates the proximity of the condition of a 'swinger'.

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