THE EVOLUTION OF THE NUCLEI OF PLANETARY NEBULAE

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OBSERVATIONAL BASIS

In the past decade, planetary nebulae have assumed considerable importance in elucidating our understanding of the final stages of stellar evolution at low mass. This began with the work of Shklovsky, O'Dell, and Seaton, who showed not only that the nuclei of these nebulae were among the hottest stellar objects, but also that they evolved on a track in the Hertzsprung-Russell diagram (the Harman-Seaton sequence) on a time scale very rapid by stellar evolutionary standards (~ 20,000 years).

A planetary nebula occurs when a star ejects part of its outer envelope to form a gaseous shell. The energetic photons emitted by the stellar remnant ionize the gas, and there is a balancing process of radiative recombination. When such recombination occurs to excited levels of the atoms, the recombination photons and the photons emitted in the subsequent radiative cascade to the ground state, are in the visible region and render the shell observable. The most abundant elements (hydrogen and helium) are represented in the nebular spectrum by such lines. In addition the bath of free electrons in the ionized gas is energetic enough to collisionally excite low-lying metastable levels in certain ions, and the resulting radiative forbidden decays contribute further lines to the spectrum (e.g. [OIII], [NII], [NeIII]). The ejected shell expands into interstellar space becoming too tenuous to detect some tens of thousands of years after ejection.

We summarize below some of the observational properties of these objects:

(i) Occurrence and Mass: It has been estimated that there are 50,000 planetary nebulae in the Galaxy, of which 1036 have been listed in the comprehensive catalogue of Perek and Kohoutek. The galactic plot of these objects shows some concentration to the galactic plane as well as to the galactic centre, and the radial velocities show a dispersion of 30 km sec\(^{-1}\) indicating a disc population with somewhat elliptical orbits. In the direction of the galactic centre, however, the dispersion increases to 90 km sec\(^{-1}\) close to that of population II objects. This fact, and the identification by O'Dell, Peimbert and Kinman of a planetary, K 648, in the globular cluster M15, suggest that planetary nebulae occur in all population types.

The galactic distribution of planeraries in the vicinity of the Sun suggests a population age of 7 \times 10^9 yrs which corresponds to an original mass of 1.2 M_\odot (on the assumption that these objects are nearing the end of their nuclear burning evolution). These figures have a considerable uncertainty corresponding to the observed population spread; they will, however, serve as typical values for the subsequent discussion.

The best current estimate for the rate of formation of planetary nebulae in the solar neighbourhood is 2 \times 10^{-12} per cubic parsec per year, in excellent agreement with the corresponding figure for the birthrate of white dwarfs. In addition Abell and Goldreich have shown that most, if not all, stars of 1.2 M_\odot which are now evolving from the main sequence, must ultimately pass through the planetary stage. Although these rates are subject to considerable uncertainties, they are consistent with the hypothesis that the planetary nebula phenomenon is encountered by all stars of about 1.2 M_\odot before they enter the white dwarf stage.

(ii) Shell Mass: The calculation of shell mass for planetary nebulae is intimately connected with that of distance (see below). Several estimates have been made with 0.2 M_\odot the best current value. We regard this as the typical shell mass with the same reservations as for the total mass.

(iii) Expansion velocity: Radial velocity measurements for planetary shells indicate that they are expanding. Wilson has shown, at least for ellipsoidal nebulae, that the expansion is homologous, the velocity at any point in the shell being proportional to the radial distance to that point. Velocities as high as 100 km sec\(^{-1}\) are observed but in general the expansions are of the order of 20 km sec\(^{-1}\). Planetary shell formation is essentially a slow ejection event; for example it is an order of magnitude slower than that of nova shells. Transverse motions over an interval of 40 years have also been detected for some nebulae by Liller and Liller.

(iv) Shell abundances: Aller and Czyzak have summarized the evidence concerning abundances in planetary shells. Although there are difficulties in deducing total elemental abundances from those of the observed ions, it may be said that (a) galactic planetary nebulae are of moderately uniform composition, and (b) the abundances are in fair agreement with those deduced from stellar atmospheres and the Sun. The one exception is the planetary in M15, for which oxygen is deficient by a factor 60; the helium abundance is normal. It thus appears that planetary shells have abundances which indicate that they are the unprocessed, hydrogen-rich material from which their parent stars were originally formed.

(v) Stellar temperature: Temperatures for the central stars of optically thick nebulae may be measured using the Zanstra method, which depends upon all the stellar radiation on the blue side of the Lyman limit being absorbed by the shell and degraded into Balmer and other quanta. In these circumstances, observation of the Balmer radiation from the shell provides a comparison of the ultra-violet light from the nucleus with its visual light, giving the long wavelength base necessary to estimate the temperatures of these extremely hot objects. The method may also be applied to the first and second ionizations of helium giving three such estimates of the Zanstra temperature, and Harman and Seaton have developed criteria for determining which nebulae are sufficiently optically
Having found the temperature of a planetary nucleus, we may obtain its luminosity from the apparent visual magnitude provided its distance can also be estimated. This however is difficult. Direct methods, such as the trigonometric parallax or the spectroscopic parallax of a companion star, can only be used for a handful of nebulae. For the vast bulk of planetaries we must rely on indirect methods involving one or more physical assumptions; the nebular shell is again used as a tool in these estimates.

One such method is that of Seaton\(^8\) who suggested using the electron density derived from the relative intensities of forbidden lines in the nebular spectrum. When this quantity is compared with the measured Balmer-line surface brightness of the nebula we obtain an estimate of its linear dimensions, and hence (comparing with the angular size) of its distance. The method is sensitive to the detailed geometrical arrangement of the emitting material within the shell, and is also limited to those nebulae where sufficient forbidden line intensities have been observed. It is, however, of considerable importance in providing a calibration for the more widely applicable Shklovsky method.

Shklovsky’s method\(^9\) of distance determination rests on the assumption that planetary shells have a typical mass. If this is known, and if the shell is completely ionized, then once again we obtain a relationship between the nebula’s radius and its Balmer-line surface brightness leading to a distance estimate. This method is also sensitive to the geometrical structure of the nebula, as well as the assumed nebular mass (although the latter enters the formula for distance as only the 2/5 power). The calibration of the method involves estimating the typical nebular mass, and this is achieved by using the Seaton distances, those of the few nebulae with direct estimates, and the proper motion statistical mean parallax of planetaries as a group.

The Shklovsky method has been used in several studies of planetary distances and central star luminosities (e.g. O’Dell\(^8\), Seaton\(^5\)). It will be noticed that the requirement that the nebula be fully ionized is contradictory to the Zanstra temperature requirement that it be optically thick. Seaton\(^6\) showed, however, that in many cases it is possible to obtain both Shklovsky distances using the hydrogen radiation, and Zanstra temperatures using the second helium ionization.

Finally, Webster\(^12\) has observed planetary nebulae in the Magellanic Clouds, where the distance is known, and has obtained results in general agreement with the indirect distance scales described above.

Most planetary nuclei have luminosity estimates in the range 1.0 < log \(L/L_\odot\) < 5.0.

The Harman-Seaton track and time scale: O’Dell\(^20\) has combined the temperature and luminosity results for a sizable number of galactic and Magellanic Cloud planetary nuclei into a composite Hertzsprung-Russell diagram. (This is reproduced as part of

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Figure 1. The Hertzsprung-Russell diagram showing the Harman-Seaton sequence for the nuclei of planetary nebulae (points with error bars), the main sequence and the horizontal branch (thin lines), and the white dwarfs (open squares). Superimposed is the calculated sequence for the helium-shell-burning evolution of a 0.8 M_⊙ star (thick line). The observational diagram is that of O’Dell.\(^20\) The gap between the high and low luminosity objects on the Harman-Seaton sequence is an artificial one, due to the difficulty of positioning planetary nuclei in this region of the diagram; similarly the decrease of nuclei at low luminosities is a selection effect caused by the increasing difficulty of identifying planetaries as the surface brightness of the shell decreases.

Figure 1.) Although there is considerable uncertainty for individual results (error bars), the broad sweep of the Harman-Seaton track is abundantly clear. We may again use the nebular shell as a tool in placing a direction of travel and a time scale on this track. Those planetaries with the smallest shells lie at the upper right hand end of the track, and shell size increases as we go to the lower left. We infer, therefore, that this is indeed an evolutionary sequence, along which the planetary nuclei proceed on a time scale equal to that of the shell expansion (∼ 20,000 yrs).

O’Dell’s diagram also indicates the main sequence, the region of the horizontal branch, and the white dwarfs (open squares). It is quite apparent from the position and direction of the planetary nuclei sequence that it represents the track traversed by stars entering the white dwarf stage. The proximity of the beginning of the sequence to the blue end of the horizontal branch, on the other hand, is probably not significant. At the start of the track, the planetary nucleus has just lost a substantial fraction of its mass as the shell, and it is extremely likely that its position in the Hertzsprung-Russell diagram will be considerably removed from that of its immediate progenitor.
We may summarize the conclusions of this brief review of the observational basis for the evolution of planetary nuclei as follows: The planetary phenomenon occurs late in the evolution of \( \sim 1.2 \, M_\odot \) stars when \( \sim 0.2 \, M_\odot \) of the original hydrogen-rich material is thrown off as a shell and the remaining nucleus evolves to the white dwarf region on a time scale of \( \sim 20,000 \) yrs. In what follows, we report on calculations relevant to these events, which have been made at Mount Stromlo Observatory over the past few years. In doing so we will build up a plausible description of the processes involved. It should be recognized, however, that space limitations will prevent us from describing other equally plausible mechanisms for some of the processes (e.g. shell ejection). Salpeter\(^{11}\) has recently written an excellent comprehensive review of this subject.

We will describe calculations in three areas (i) the helium core-burning evolution of low mass stars, (ii) their helium shell-burning, and (iii) shell ejection by radiation pressure.

**PRE-EJECTION EVOLUTION**

The evolution of a low mass star is now well understood up to the helium-burning stage. After hydrogen is exhausted in the stellar core, it commences burning in a shell. The core slowly contracts as the shell narrows, and this is mirrored by an expansion of the external layers. The star moves away from the main sequence and up the giant branch with increasing luminosity and decreasing surface temperature. This behaviour is well illustrated by the main sequence—giant branch distribution of stars in the colour magnitude diagrams of both globular clusters and intermediate-age and old galactic clusters. The surface of the star is convective, while continuing contraction causes the electrons in the core to become degenerate. The excursion up the giant branch is terminated when the core contraction finally raises the temperature sufficiently for helium-burning to commence. Since it does so in a region of electron degeneracy the energy release does not give rise to a consequent pressure increase and expansion. Thus the usual processes which control nuclear reaction rates in stars do not immediately come into play, and there is a very rapid increase in helium-burning until the temperature rises sufficiently to remove the degeneracy. This phenomenon is known as the helium flash, and it occurs in a small fraction of the Kelvin time scale. Schwarzschild and Härm\(^{22}\) have computed the evolution through this event, and have concluded that it is not quite violent enough to cause a convective region extending into the hydrogen-burning shell. Following the flash, therefore, the star will have a hydrogen-helium discontinuity at the same interior mass as when it approached the giant branch tip. This is at \( 0.4 \, M_\odot - 0.5 \, M_\odot \) over a wide range of total stellar mass.

The star will now have two nuclear energy sources, the core helium-burning and the shell hydrogen-burning, and models computed for such configurations have given agreement with the observed horizontal branch stars in globular clusters. Thus for population II objects, there is observational evidence for the postulated evolutionary processes as far as the core helium-burning.

Turning now to metal rich objects, the intermediate-age galactic clusters provide evidence for evolution up the giant branch, but there is no extensive feature corresponding to the horizontal branch. Since it has sometimes been argued\(^{11}\) that the most likely progenitors for the planetary nebulae are stars of large radii, such as those at the giant tip, the first possibility we must consider is that planetary nebulae are formed at the helium flash. We could discount this possibility if we could obtain observational and theoretical evidence for population I stars in the core helium-burning stage, similar to that for the horizontal branch stars.

The basic observational evidence for such stars in open clusters has been summarized by Cannon.\(^{23}\)\(^{24}\) Intermediate-age clusters which are sufficiently populous, are found to have a marked concentration of stars near the base of the giant branch. The properties of these clumps are as follows:

(i) The star density in the clump implies a slow evolution lasting about \( 2 \times 10^7 \) yrs.

(ii) The position of the clump is typically \( M_v = 1, \ B - V = 1.0, \ \text{corresponding to} \ log \ L/L_\odot = 1.6, \ log \ T_{\text{eff}} = 3.70. \)

(iii) Individual clumps show no measurable spread in temperature (\( \Delta \log \ T_{\text{eff}} < 0.02 \)) but a small spread in luminosity \( \Delta \log \ L/L_\odot = 0.25. \)

(iv) These three properties apply irrespective of the cluster age in the range \( 3 \times 10^8 - 10^{10} \) yrs, which corresponds to red giant masses from 2.25 to 1.0 \( M_\odot \).

Helium-burning models which reproduce these features have been calculated by Cannon and the author.\(^{25}\) The evolution has been commenced following the helium flash by taking combinations of total stellar mass (1.5 \( M_\odot \) and 2.0 \( M_\odot \)) and core mass (0.4 \( M_\odot \) and 0.5 \( M_\odot \)). The hydrogen shell-source is still the dominant energy source in all cases, contributing 80-90 per cent of the star's luminosity. A Hertzsprung-Russell diagram of the resulting evolution and its comparison with the observations is given elsewhere in this volume.\(^{10}\) The models for all mass combinations fall close together in the diagram; their position is almost independent of total mass (c.f. (iv) above) but the luminosity increases slightly with increasing core mass. The models with 0.4 \( M_\odot \) cores are in good agreement with the base of the observed clump.

The evolution of the (1.5 \( M_\odot \), 0.4 \( M_\odot \)) initial clump model was followed through the entire core helium-burning stage till near helium-exhaustion. The treatment included a new type of semi-convection\(^{29}, \, \, 27\) outside the convective core, which has the effect of increasing the amount of helium participating in the core-burning phase, prolonging its life to \( 1.75 \times 10^8 \) yrs. During this time the star increases in luminosity by \( \Delta \log \ L/L_\odot = 0.3. \) Thus the models reproduce the observed properties of the giant clumps in all respects, and we may reasonably conclude that these features represent the Population I equivalent of the horizontal branch. This precludes the formation of planetary nebulae at the tip of the first excursion up the giant branch and we must look to subsequent evolutionary phases for their origin.

Following the exhaustion of helium in the core, it too will commence burning in a shell and the star will begin a second evolution up the giant branch as a two shell-source object. The start of this ascent is already evident as helium is exhausted in the core.\(^{26}\) It was at this stage of evolution for population II stars, that Schwarzschild and Härm\(^{28}, \, \, 29\) discovered the thermal instability phenomenon. Helium-burning reactions are very temperature sensitive, and in a sufficiently thin shell-source, the pressure changes during expansion become too small to fulfil their usual function in...
thermostating the nuclear reactions. The burning then increases rapidly giving a thermal pulse of $\sim 100$ yrs rise time, and the star is found to undergo relaxation oscillations with a period between pulses of several thousand years. This phenomenon has been amply confirmed for double shell-source models,40-42 and its existence makes the computation of this evolutionary phase complex and rather uncertain. In particular, Paczynski39 has pointed out that unless an adequate representation of the shell profile is maintained, even the existence of thermal pulses may be incorrectly determined in some cases.

In summary, the double shell-source phase of the evolution of low mass stars is characterized by a second ascent of the giant branch, superimposed upon which are rapid thermal pulses. Much work is still needed to clarify this phenomenon has been amply confirmed for double shell-source models,30-32 and its existence makes the computation of this evolutionary phase complex and rather uncertain. In particular, Paczynski39 has pointed out that unless an adequate representation of the shell profile is maintained, even the existence of thermal pulses may be incorrectly determined in some cases.

In summary, the double shell-source phase of the evolution of low mass stars is characterized by a second ascent of the giant branch, superimposed upon which are rapid thermal pulses. Much work is still needed to clarify this phase, and its relationship to planetary formation, and it seems desirable to terminate the discussion at this point and jump discontinuously to the evolution of the remnant star after the ejection of the shell.

POST-EJECTION EVOLUTION

Before attempting evolutionary calculations for the nuclei of planetary nebulae, we must make some assumptions about the stellar configuration after shell ejection. We have noted that planetary shells have abundances typical of hydrogen-rich, unprocessed stellar material. On the other hand, as the nuclei evolve to the white dwarf stage, almost all their mass reaches temperatures high enough for hydrogen to burn. The time integral of luminosity up to this stage, however, indicates that there can have been very little hydrogen left in the nucleus after shell ejection (Osterbrock34). We are forced to the conclusion, therefore, that the separation of the planetary shell takes place almost exactly at the hydrogen-helium discontinuity in the star. Thus a helium shell-burning star with a carbon and oxygen core but with no hydrogen envelope, seems to be an appropriate configuration for the study of post-ejection evolution.

Two reservations must be made:

(i) In view of the uncertainty surrounding the final stages of evolution before ejection, we do not know the appropriate mass fraction for the helium shell-source. We proceed by computing the evolution from an early stage, bearing in mind that only the last parts of our calculations will be physically realistic.

(ii) The presence of even a small percentage of residual hydrogen on the surface of a pure helium star produces a considerable decrease in effective temperature (but little change in luminosity). We must also be prepared, therefore, for our models to be somewhat too hot should the separation of the planetary shell leave even a little hydrogen-rich material behind.

The evolution of helium shell-burning at 0.8 $M_\odot$ has been reported elsewhere in this volume,36 and similar calculations at other masses are also referenced there. The calculated evolutionary track is superimposed on the Harman-Seaton sequence in Figure 1. It will be seen that there is general agreement, especially if one allows for some temperature decrease due to residual hydrogen. The direction of evolution is also in agreement with that observed. As we have predicted, however, it is only the downward part of this track which has relevance to the post-ejection evolution of planetary nuclei. The time spent on the rising branch is $\sim 10^6$ yrs, during which the helium shell burns outwards to a mass fraction of 0.96. This is much longer than the total planetary nuclei evolution time ($\sim 20,000$ yrs).

If neutrino losses are neglected in the 0.8 $M_\odot$ evolution the shell ceases to burn at a mass fraction of 0.98 when carbon-burning commences in the core.36 With these losses included, however, carbon burning is suppressed and the helium shell remains active until, at 0.991, a thermal instability consisting of a single thermal pulse appears.35 Following this the shell source becomes extinct as the star evolves to the white dwarf region.

Figure 2 illustrates the time scale for luminosity changes during the latter stages of this evolution. The pulse has a rise time of $\sim 200$ yrs and a subsequent decay of some thousands of years. The neutrino luminosity is also shown; only in the last stages of contraction to the white dwarf region, does it exceed the radiative luminosity. The time scale of the observed luminosity decrease for planetary nuclei (assuming a shell expansion of 20 km sec$^{-1}$) is shown in Figure 2 for comparison. We return to a discussion of these time scales in the concluding section.

SHELL EJECTION BY RADIATION PRESSURE

Many mechanisms have been suggested for the ejection of planetary nebula shells, including instabilities originating in the hydrogen-ionization zone37-39, or in the region of the helium flash40; centrifugal force41; corpuscular radiation42,43 and dynamical instability caused by thermal pulses in the helium shell-source.44 The mechanism favoured by the present author is ejection by radiation pressure.45-48

If, in a star, we consider a spherical shell at interior mass $M_\tau$, through which the energy flow is $L_\tau$, then it is well known that the radiation pressure gradient exerted against the opacity, $\kappa$, of the stellar material will exceed the gravitational attraction provided

$$L_\tau \geq 4 \pi c G M_\tau/\kappa$$

(1)

Inserting 1 $M_\odot$ for $M_\tau$ and 0.34, the electron scattering opacity of Population I material, for $\kappa$, we find that a luminosity of $3.8 \times 10^4 L_\odot$ is required to balance gravity in
the envelope of a typical planetary progenitor; should the luminosity exceed this value, acceleration of the envelope material by radiation pressure will result. Figure 1 shows that planetary nuclei are observed to have luminosities of this order shortly after formation lending support to the radiation pressure hypothesis for shell ejection.

The author has calculated such ejection using, as the planetary progenitor, a $1.0 M_\odot$ core surrounded by a $0.2 M_\odot$ hydrogen-rich envelope. The energy production in the core is assumed to be such that the inner boundary of the envelope is subject to a linearly increasing luminosity

$$L_t = L_0 \left(1 + \frac{t}{K}\right)$$

where the zero of time is chosen when the luminosity in the shell is just sufficient to balance gravity, $L_0$. The time scale of luminosity rise is set by $K$. The acceleration of the shell is calculated in the simplest possible way, by describing all processes in terms of a single representative point situated midway in the shell, at $1.1 M_\odot$. The luminosity at this representative point will rise less rapidly than that at the base of the shell, since, as soon as the shell material starts to move, its expansion will cause a negative value of the gravitational-internal energy production, and consequently an absorption of energy.

On the assumption that the shell expansion is homologous, the acceleration of the representative point may be described by an equation of the form

$$\ddot r = \text{const}_1 \frac{t}{Kr^2} - \text{const}_2 \frac{r}{r^4},$$

where the first term gives the acceleration due to luminosity increase, and the second the diminution of this due to shell expansion. The initial conditions are

$$\dot r = r = 0, \quad r = r_0 \text{ at } t = 0,$$

where $r_0$ represents the initial radius of the shell representative point.

We thus have an initial value problem with two parameters $r_0$ and $K$. This was solved for $r_0$ values of 1 and 10 $R_\odot$ (which are typical of the radial distance of the 1.1 $M_\odot$ point in a 1.2 $M_\odot$ star during its nuclear burning evolution), and for $K$ values of $10^4$ and $10^8$ yrs which are typical time scales for luminosity rises in the quiescent helium-shell-burning, and in the thermal pulses, respectively.

The results of this investigation are illustrated in Figure 3 which plots time and velocity as a function of the radial distance to the shell representative point, $r$. It is apparent that the radiation pressure gradient associated with a rising luminosity is capable of ejecting a planetary shell, and that the expansion velocity which the shell attains by the time it has reached typical dimensions of planetary nebulae ($5 \times 10^{16} - 2 \times 10^{18}$ cm) is of the order of the observed expansion velocities ($\sim 20$ km sec$^{-1}$).

The dash-dot curve of Figure 3 gives the escape velocity from a 1.0 $M_\odot$ star as a function of distance. When the computed expansion velocity exceeds this value, the luminosity at the base of the shell could fall to zero and the nebula would still escape. If we note the time of this effective escape from the upper plots, we find that, in the extreme case, it occurs when the luminosity has risen to just 1.6 of its initial value. It is for this reason that the exact form of the luminosity rise considered is unimportant. A linear increase with time will differ little from, say, an exponential increase, over a factor of only 1.6. The total energy requirements for shell ejection by this mechanism have been shown to be reasonable, being supplied with the consumption of only 0.005 $M_\odot$ in helium-burning.

We see from Figure 3 that the form of solution and the final expansion velocity are moderately insensitive to the radius, $r_0$, and time scale $K$. It is therefore likely that the ejection of a planetary shell is a general phenomenon, and not dependent on very restrictive circumstances which might limit the number of stars passing through this phase. This is in keeping with the conclusion that most stars of 1.2 $M_\odot$ will become planetaries (see (i) above).

One of the most pleasing features of the radiation pressure ejection mechanism is the absence of high expansion velocities (see (iii) above). It is very difficult to account for the low observed velocities using any mechanism which involves giving the shell a ballistic push. The energy of escape required to remove a 0.2 $M_\odot$ shell from a 1.0 $M_\odot$ star to infinity is

$$E_1 \approx 7.6 \times 10^{44} \text{ ergs.}$$

where $R_1$ the radius originally separating the shell and the star, is expressed in solar units. On the other hand, the kinetic energy of a 0.2 $M_\odot$ shell moving at 20 km sec$^{-1}$ is

$$E_2 = 8.0 \times 10^{44} \text{ ergs.}$$

Thus we see that if such a mechanism operates at solar dimensions, it must impart exactly the escape energy, to a precision of one part in a thousand, for the final expansion velocity to be as low as observed. This is exceedingly implausible, and only by proposing that ejection takes place at very large radii ($\sim 1000 R_\odot$) can the difficulty be over-
come. The low velocities of the radiation pressure mechanism stem from the fact that the accelerations due to radiation pressure gradient, and to gravitational attraction are both proportional to $r^{-2}$. Thus this mechanism imparts energy to the shell throughout the entire expansion, and high escape velocities are nowhere required.

Finally, the radiation pressure mechanism provides an explanation for shell separation at the hydrogen-helium discontinuity. The electron scattering opacity for helium-rich material is less than that for material with the shell composition, by a factor of 1.7. Thus helium-rich material will be accelerated only when the luminosity has risen to 1.7 times its value for hydrogen acceleration (Equation (1)). We have noted, however, that by this time the shell has effectively escaped from the nucleus. The 1.2 $M_\odot$ star has become a 1.0 $M_\odot$ star due to the loss of the shell, and the entire evolution will have been changed. Thus, in all probability, the star never acquires a luminosity high enough for helium-rich material to be accelerated.

Recently Finzi and Wolf have presented time-independent models for radiation pressure ejection, in which the time required for a given element of matter to traverse the envelope is small compared with the total time for shell ejection. This approach is essentially complementary to that just considered (which may be described as completely time dependent since we have treated the entire shell as our escaping element); it is encouraging that in both cases shell expansion velocities are found in substantial agreement with those observed.

CONCLUSIONS

We have just described a mechanism for shell ejection which depends upon the luminosity of the progenitor star reaching a sufficiently high value. In the evolutionary calculations which we have considered two types of luminosity rise suggest themselves—(i) the slow rise up the giant branch associated with the quiescent aspects of the double shell-source evolution, and (ii) the very rapid rises associated with thermal pulses. Is it possible to distinguish which of these is responsible for planetary shell ejection? On the basis of the ejection calculations alone we can say nothing, since they are found to be very insensitive to time scale of luminosity rise. Nevertheless, there are two lines of evidence which suggest that it may be the rapid thermal pulses which are responsible.

Firstly, we may compare the observed time scale of the luminosity decrease along the Harman-Seaton track, with the calculated decreases for both these possibilities. This comparison is made in Figure 2. Although the decrease from the quiescent luminosity peak (model 3) is considerably more rapid with neutrino processes included, it is still several times slower than the observed decrease. The decay of the thermal pulse, on the other hand, shows much better agreement. The particular pulse in Figure 2 was calculated for a 0.8 $M_\odot$ star without a hydrogen envelope, but it seems entirely plausible that a corresponding pulse during the progenitor evolution could give rise to the shell ejection. The very rapid rise time of the pulse event may also explain the observed variability of some planetary nuclei.

Secondly, since the thermal instability has frequently been found to give rise to a series of thermal pulses, we must examine the possibility of repeated shell ejection. Many planetaries do appear to have an extended outer shell indicative of an earlier ejection (see the Perek-Kohoutek Catalogue). In addition, for the variable star FG Sagittae, there is evidence for a second shell ejection currently taking place. This peculiar star has been increasing in brightness since the turn of the century, and has been moving to later spectral types since observations began. Herbig and Boyarchuk have presented convincing evidence that these changes are due to a deep expanding atmosphere overlying a hot, subluminous star. We may place the time scale for this expansion at ~ 100 yrs. On the other hand, FG Sge is already surrounded by a nebulous envelope, which appears to be a very advanced planetary shell ejected some thousands or tens of thousands of years ago. These time scales are in excellent agreement with the computed rise times and intervals for the pulses in the thermal instability phenomenon.

The probability of a repeated shell ejection will depend upon the effectiveness with which all the hydrogen-rich material was removed during the first. If removal is complete, the resulting helium nucleus will be much less susceptible to further thermal pulses than was the progenitor star. Even in this case, however, pulses are possible, although it is not yet clear whether they can become sufficiently active to remove a helium shell.

In summary, then, it appears that the planetary nebula phenomenon takes place during a second ejection up the giant branch at the double shell-source evolutionary stage. The thermal pulses that occur at this time may give a sufficient luminosity increase to cause shell ejection by the radiation pressure mechanism, leaving a stellar remnant that is almost entirely hydrogen-free. This nucleus can still exhibit thermal pulses if of sufficiently low mass ($< 0.8 M_\odot$) but the helium shell source soon becomes extinct, and it evolves directly into the white dwarf stage.

We thank Dr C. R. O'Dell for permission to reproduce his composite Hertzsprung-Russell diagram for the nuclei of planetary nebulae.

32 Faulkner, D. J. and Wood, P. R., Proc. ASA, 2, 205 (1972).
78 INVITED PAPERS Proc. ASA 2 (2) March 1972
