Comparison and Coordination of Time Scales

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Nature and Uses of Time Scales
A time scale may be defined as a system for dating events. A clock is a device for interpolating between observations of the time scale. From antiquity, time scales have been based on the earth’s rotation; even today, the atomic-based time scale Universal Coordinated Time (UTC) is adjusted annually, or when necessary, to keep it in time with the rotating earth. However, the continual search for greater accuracy in time keeping has led to the adoption of time scales based on other phenomena, so that there are now four classes of time scales in use, as summarised in Table I.

Rotational, known as Universal Time (UT), in which the fundamental unit of time is the day, being the interval between successive passages of the ‘fictitious mean sun’ (Mueller, 1969) over the ‘Greenwich’ meridian (see below). In practise, stellar observations of Sidereal Time (ST) are converted to UT by Newcomb’s formula:

$$UT = GST - 6\,38\,45\,836 - 8640184.542t_m - 0.0929t_m^2$$

where $t_m$ is the number of Julian centuries of 36525 days which have elapsed since 1900 January 05 UT, and the prefix G indicates Greenwich. The useful measure of GST is the mean hour angle (corrected for nutation) of the first point of Aries over the zero mean astronomic meridian as defined by the Bureau International de l’Heure (BIH). Astronomical observations are contaminated not only by uncertainties in the nutation and aberration constants, positions, proper motions and parallaxes of the stars, and refraction (most of which are refined from time to time from the observations), but also by polar motion and variations in the earth’s rotational speed. Hence three Universal Time scales are in use:

- UTO:— raw measure of UT, converted from Sidereal Time, as determined at a given observatory;
- UT1:— corrected measure of UT allowing for the effect of polar motion through the formula:

$$UT1 = UTO - (x_p \sin \lambda + y_p \cos \lambda) \tan \phi$$

where $\lambda$, $\phi$ are the geodetic longitude and latitude of the observatory and $x_p$, $y_p$ are the coordinates of the pole of rotation, as determined by the International Polar Motion Service (IPMS) or slightly differently by the BIH, referred to the pole figure (the Conventional International Origin). UT1 is the instantaneous measure of the ‘position angle’ of the earth;
- UT2:— refined measure of UT with seasonal effects removed by the formula (Guinot, 1977):

$$UT2 = UT1 + 0.022 \sin 2\pi t - 0.012 \cos 2\pi t \cdot 0.006 \sin 4\pi t + 0.007 \cos 4\pi t$$

where $t$ is the fraction of the year and the constants have been in use since 1962. UT2 is thus the most uniform rotational time scale available.

Dynamical, based on the motion of the bodies in the solar system. Until recently, Ephemeris Time (ET) was used, in which the fundamental unit of time interval was the ephemeris second, being the fraction 1/31,556,925.9747 of the tropical year at 1900 January 05 ET. The epoch was chosen at the instant near the start of 1900 when the Sun’s geometric mean longitude was 279°41’48.04”. ET is described as ‘the independent time argument of dynamical astronomy’ (see Wilkins 1974), and in fact is determined by inverse interpolation from observations of the moon on the assumption that the lunar and solar ephemerides are perfect. Difficulties with this assumption and with matters of observability have led to the abandonment of ET as the independent argument of dynamics. At the IAU, Grenoble in 1976, a new time scale was recommended (Winkler & van Flandern 1977) possibly to be called Dynamical Time (TD), whose fundamental unit is the day of 86400 SI seconds (see below) and for which 1977 January 01 00 00 00”’ TAI coincides with 1977 January 01 00 03 725 exactly. In reality two such scales are necessary — a ‘proper’ scale (TDP) capable of realisation by atomic clocks on earth, and a ‘coordinate’ scale (TDC) referred to the solar system barycentre, such that there be only periodic variations between the two.

Atomic. From the time of the successful development of the caesium beam frequency standard by Essen and Parry in 1955 it became manifestly evident that atomic time is much less affected by external unmodelled influences than rotational and ephemeris time. Figure 1 is a sketch of a caesium beam. Accordingly, several institutions have set up time scales based on the average of a number of caesium standards, as discussed in the next section. Some, such as National Bureau of Standards, Boulder, U.S.A., National Research Council, Canada, Ottawa, Physikalisch-Technische Bundesanstalt,
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“A” SELECTOR MAGNET

vn

CAESIUM OVEN

CAVITY

MASS SPECTROMETER

AMPLITUDE & PHASE DETECTOR

VCXO

ERROR SIGNAL

DIVIDERS

Figure 1. Caesium beam schematic. 5 MHz from crystal is synthesized to microwave frequency, and is resonantly absorbed by state-selected caesium atoms in the Ramsay cavity. The proportion of atoms undergoing transition is detected. If microwave frequency is exact, 137 Hz modulation is absent; its phase indicates sign of error which is used to correct crystal frequency. Outputs are derived from the crystal.

Braunschweig, FDR, (NBS, NRC and PTB) operate laboratory caesium standards with beam tube cavities typically 3.7 metres long (Glaze et al. 1974), which are considered to produce independently the correct caesium frequency, and are used to provide the basic frequency reference for International Atomic Time (TAI). TAI is calculated by the BIH as the result of these laboratory frequency calibrations coupled with daily comparisons between commercial frequency standards in the Northern Hemisphere; it provides the realization of the SI second which is defined as the interval occupied by 9,192,631,770 cycles of a particular hyperfine transition of the Cs133 atom at sea level. This number was chosen so that the seconds of atomic and ephemeris time at 1957.0 were of equal duration (Markowitz et al. 1962). The origin of TAI is such that TAI and UT2 were in approximate agreement with 0 hours UT2 on 1 January 1958 (IAU, Prague 1967; see Barnes 1974).

Hybrid, of which UTC is the prime example. Since 1972 the rate of UTC has been identical to the rate of TAI and hence, in essence, to the rate of ephemeris time, but it has been subjected to steps in integral multiples of one second so that its epoch, or date, approximates UT1; specifically, CCIR Report 517 for Recommendation 460 at New Delhi, 1970 (see BIH, 1977) requires that the departure of UTC from UT1 should not normally exceed 0.7, and will not exceed 0.9;

Further, UTC is offset from TAI by a whole number of seconds. It is the time scale commonly maintained by national time services, is used in practice if not in law as the basis for civil timekeeping, and is coordinated worldwide by means of standard radio time signals, portable clocks and other methods.

Accuracy Requirements
A summary of some operational requirements in precise time coordination for several types of users is given in Table II. For example, in lunar laser ranging an error of 50 μs in the epoch at which a pulse is sent can introduce a significant error in theoretical range, and it would take something like 6 months for such a timing error to show up in the results, which implies a long-term knowledge of the frequency of the local time scale to 4 parts in 10¹². On the other hand, timing the pulse flight time to 125 pico seconds over 2.5 seconds needs a counter time-base accuracy of 5 parts in 10¹³.

Construction of Atomic Time Scales
Universal and ephemeris time scales are based on naturally recurring phenomena which are expected to last indefinitely. Atomic time scales, however, are derived from man-made

Table II

<table>
<thead>
<tr>
<th>Activity</th>
<th>Time (μs)</th>
<th>Duration</th>
<th>Frequency (x10⁻¹²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Photo Zenith Tube</td>
<td>200</td>
<td>2 years</td>
<td>30</td>
</tr>
<tr>
<td>Lunar Laser Ranging</td>
<td>50</td>
<td>6 months</td>
<td>30</td>
</tr>
<tr>
<td>Pulsar Timing</td>
<td>10</td>
<td>3 years</td>
<td>1</td>
</tr>
<tr>
<td>Deep Space Tracking</td>
<td>1</td>
<td>3 months</td>
<td>1</td>
</tr>
<tr>
<td>VLBI</td>
<td>0.1</td>
<td>10 days</td>
<td>1</td>
</tr>
</tbody>
</table>

Figure 2. Allan variance for a typical caesium standard.

\[
\sigma(\tau) = \frac{\Delta \nu(t+\tau) - \nu(t)^2/2}{\tau}^{1/2}
\]

where \( \nu(t) = \Delta \nu(t)/\nu = [\nu(t + \tau) - \nu(t)]/\tau \).

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devices which have finite lives (typically 5-10 years) and which each have their own systematic and random deviations from ideal performance. Time scales are therefore built up from a number of clocks to achieve the desirable properties of uniformity, stability and durability. The characteristics of all such scales are:
(1) They are the weighted means of as large an ensemble of reliable clocks as possible, usually commercial caesium standards;
(2) They are insensitive to the death or introduction of clocks;
(3) They have facilities for detecting anomalous behaviour of individual clocks, and allowing for them;
(4) They incorporate calibrations against laboratory standards.
Some such scales are described in Table III.

Let \( x_i \) be the time displayed by clock \( i \) at a particular instant, \( p_i \) be a weight associated with that clock, and \( n \) be the number of clocks in the ensemble. The simplest time scale is then the weighted mean, say \( X(0) \):

\[
X(0) = \frac{\sum p_i x_i}{\sum p_i}
\]

or

\[
\sum p_i (X(0) - x_i) = 0.
\]

Clearly, if a clock dies or misbehaves, or if a new one is introduced, a change will occur both in the mean time and in its rate. The jump in time can be overcome by introducing a constant \( a_i \), for each clock, so that the next level of time scale \( X(1) \) is

\[
X(1) = \frac{\sum p_i x_i + a_i}{\sum p_i}
\]

or

\[
\sum p_i (X(1) - x_i) = \sum p_i a_i.
\]

By choosing:

\[
a_i = E[X(1) - x_i]
\]

at the instant before the change occurs, continuity is preserved although continuity of rate is not. The same values of \( a_i \) can be used until the next change. This was the method of construction of UTC (AUS) until March 1979, (Luck and Woodger 1976); changes and anomalous behaviours were detected visually and corrected manually.

If a further constant \( A \), the same for each clock, be added:

\[
a_i - a_i = A + x_i
\]

then the time scale can be adjusted to coincide with another time scale such as TAI at a given instant, for example at the time of a portable clock visit if we choose

\[
A + x_i = TAI - x_i
\]

(In practice here, UTC(USNO) rather than TAI is used.)

### TABLE III

<table>
<thead>
<tr>
<th>Scale</th>
<th>Approx No. of Contributors</th>
<th>Comparison methods</th>
</tr>
</thead>
<tbody>
<tr>
<td>TAI</td>
<td>90 Cs, 3-5 lab Cs</td>
<td>TV, Loran-C</td>
</tr>
<tr>
<td>UTC (USNO)</td>
<td>20 Cs</td>
<td>Local</td>
</tr>
<tr>
<td>UTC (USNO)</td>
<td>8 Cs, 1-3 lab Cs</td>
<td>TV, Local</td>
</tr>
<tr>
<td>UTC (USNM)</td>
<td>10 Cs</td>
<td>TV, Local</td>
</tr>
<tr>
<td>UTC (DNM)</td>
<td>4 Cs</td>
<td>Local</td>
</tr>
</tbody>
</table>

Comparison methods include Long Range Navigation system (LORAN-C), flying clocks (FC), Timepiece/Navigation Technology satellites, television (TV) and intran laboratory (LOCAL).

The defining relationship for this time scale \( X(2) \) is thus:

\[
X(2) = \frac{\sum p_i (x_i + A + a_i)}{\sum p_i}
\]

that is, the condition:

\[
\sum p_i (X(2) - x_i) = \sum p_i (A + a_i)
\]

is imposed on the ensemble for its duration.

An obvious extension of this condition which will guarantee continuity and uniformity if the ensemble clocks behave perfectly linearly except for discrete measurable jumps and rate changes, is:

\[
\sum p_i (X(3) - x_i) = \sum p_i (A + a_i + b_i(t - t_i))
\]

which implies, for this time scale \( X(3) \) at time \( t \):

\[
X(3) = \frac{\sum p_i (x_i + a_i + A + b_i(t - t_i))/\sum p_i}{1}
\]

where \( t_i \) is the time at which \( A \) was measured and

\[
b_i = E[d/dt (X(3) - x_i)]
\]

is the rate of each clock as determined by prior calibration against \( X(3) \). Again \( b_i \) and be replaced by \( b_i + B \) where \( B \) is a calibration

\[
B = E[TAI - X(3)]
\]

of \( X(3) \) against TAI. Deterministically, \( X(3) \) is the basis of all atomic time scales, for caesium standards by and large do behave linearly (Barnes 1974).
Now consider the effects of random noise $\epsilon_i$, superimposed on the linear clock performances. From (2) it can be seen that

$$TAI = X(3) = \sum p \epsilon / \sum p_i$$

If it can always be assumed that $\sum p \epsilon_i = 0$, then $X(3)$ becomes an accurate predictor of TAI after the initial calibrations of $A$ and $B$ have been completed. To approximate this condition in $X(3) = UTC(DNM)$ at any hour in a given day, the coefficients $a_i$ and $b_i$ are estimated by a least squares fit through the hourly results $X(3) - x_i$ of the 24 valid sets up to 0 UT on this day and weights assigned according to the inverse of the variances $s_i^2$ of the fits, subject to a maximum weight. The residuals $\epsilon_i$ for all clocks at a particular hour are calculated on the first iteration; if the largest residual fails the test:

$$|\epsilon_i| < 2.5 \sqrt{(n/\sum (1/s_i^2))}$$

that clock is rejected and the process repeated until all residuals pass the test or until only one clock is left, in which case UTC (DNM) is not calculated for that hour. UTC(AUS) is to be operated similarly, except that measurements are made daily, weights, $a_i$ and $b_i$ are calculated monthly including the current month through a preliminary iteration, and the minimum ensemble is four. The BIH practise of calculating weights on the inverse variance of the rates of the previous six and the current months is being tested for UTC(AUS).

Further refinement is obtained by taking advantage of the stochastic nature of the noise processes in atomic clocks. The autocorrelation function is customarily represented by the two-sample Allan variance of fractional frequency:

$$\sigma^2(\tau) = \langle (y(t + \tau) - y(t))^2 / \rangle$$

where $y(t)$ is an estimate of the fractional frequency $\Delta \nu / \nu$ at time $t$, and $\tau$ is the lag (Allan et al. 1974; Winkler 1977). A typical plot is shown at Figure 2, where the known processes are identified. The one-sided spectral density $S_f(y(t)$ is instructive, since $S_f(y(t)$ is proportional to $f^\alpha$ where $f$ is Fourier frequency and $\alpha = 2, 1, 0, -1, -2$ for white phase, flicker phase, white frequency or random walk phase, flicker frequency, and random walk frequency modulation noise processes respectively. Once the constants of proportionality have been determined, optimal filters can be designed to estimate the $\epsilon_i$ for each clock in the current calculation, using the past few residuals as input. For example, for white noise
Figure 5. UTC(AUS) - CLOCK from test run using real data Nov 78 - Apr 79. Every seventh day is shown and overall trends have been removed. UTC(AUS) was based on 10 caesiums, including those shown, in Eastern Australia. DNM1109 was deliberately jumped 300 µs on 43843, and suffered a spontaneous rate change two weeks later.

frequency modulation (α = 0) the optimal prediction at time t + Δt is given by

\[ \epsilon(t + \Delta t) = \epsilon(t) + \Delta t[\epsilon(t) - \epsilon(t - \Delta t)] / \tau_e \]

where \( \tau_e \) is a calibration period, long compared with Δt.

Variations of this approach are used by the BIH with a Yoshimura filter (Guinot 1974), US Naval Observatory with an integrated moving average autoregressive filter (Percival 1978) and at Turin with a Kalman filter (Angeli 1974).

Figure 3 shows the results of passing simulated clocks with several of these processes through the program that calculates UTC(DNM) which does not yet have optimal filters built in. The seventh clock was given very small white phase noise and was not used in the ensemble; its result therefore reflects the accuracy and stability of the UTC(DNM) computation. Figure 4 shows the results of real data processed in real time at the Lunar Laser Ranger (LLR) Observatory operated by the Division of National Mapping at Orroral ACT; it is believed to be a fairly accurate representation of true clock performance. Figure 5 shows some results from the new UTC(AUS) programme, and confirms our impression that NML201 and ORR197 are very good clocks.

Physical Realization of UTC(DNM)

The atomic time scales described above are ‘paper clocks’ or ‘fictitious mean clocks’ because they are merely the results of calculation. In practice, the quantities \( \epsilon_d \) are not the times displayed by individual clocks but comparisons of the time interval between a reference clock (REF) and the contributing clocks; the measurements are thus (REF - \( x_i \)). At the LLR Observatory (NATMAP) the comparisons are automatically scheduled each hour by the HP21MX minicomputer, read by an Eldorado time-interval counter to one nanosecond resolution, and used to calculate, following (2) with \( \lambda(3) \) identified as UTC(DNM)

\[ (UTC(DNM) - REF) = \sum p_i [(a_i + A) + (b_i + B) (t - t_i) - (REF - x_i)] / \sum p_i \]

where \( N \leq n \) is the number of clocks of the total ensemble accepted that hour according to criterion (3), and stored on a disc file. The relationship of any clock to UTC(DNM) is then readily calculable as:

\[ (UTC(DNM) - x_i) = (UTC(DNM) - REF) + (REF - x_i) \]

A device has been built called station clock (STN) which tracks UTC(DNM) with an accuracy of a few nanoseconds. It is shown schematically in Figure 6. In essence, it converts the 5MHz sinusoidal output from REF (which may be any stable clock and is currently a rubidium) to a 5MHz pulse train, and creates a new 5MHz pulse train in which each pulse is offset from its corresponding input pulse by an amount equal to the

![Figure 6. Station clock system. 1 pps which tracks UTC(DNM) is generated from any clock.](https://www.cambridge.org/core/terms).
calculated rate of \( (\text{UTC(DNM} - \text{REF}) \) evaluated for a 200 nanosecond interval. The latter is accomplished by a D/A converter, and requires the insertion or deletion of one pulse each time the offset reaches 200 nanosecond. The calculated rate is updated hourly in the UTC(DNM) algorithm, and may be modified automatically to drive STN back to UTC(DNM) in the event that an unusual offset has occurred. The 5MHz pulse train is divided down to 1pps, which can be synchronised to and offset from any real clock to provide the initial time reference.

UTC(AUS) is calculated from daily TV readings, monthly in arrear and off-line. No physical realization is planned at the moment.

Comparisons of Time Scales
Implicit in the foregoing discussion is the concept that each laboratory has its own local time scale, even if derived from only one frequency standard; that groups of institutions combine to form, say a national time scale; and that TAI and its derivative UTC(BIH) are the results of comparisons of clocks in many countries. Methods of comparison in regular use are:

- **Intra-laboratory.** 50-ohm cable connections from 1pps outputs to a time-interval counter. Accuracy is limited by counter resolution.
- **HF Radio.** Most countries broadcast standard time signals on high frequency radio, with time of emission within 0.2 millisecond of UTC, e.g. in Australia VNG broadcasts on 4.5, 7.5 and 12 MHz. Accuracy of reception is about one millisecond.
- **VLF Radio.** Primarily a navigation tool, VLF is a continuous transmission which can, with care, yield relative accuracy of 2-3 microsecond worldwide over several months. The introduction of modulation shift keying (MSK) has degraded its time-keeping usefulness, however. GBR, England, transmits on 16 KHz but is weak in Australia and tends to get confused by the arrival of both short- and long-path signals of comparable strength.
- **Loran-C.** Again a navigation system, it consists of chains of stations which transmit groups of pulses on 100 KHz which repeat at a characteristic interval, e.g. 59600 microseconds, depending on the chain. Every so often, e.g. 149 whole seconds in this example, the start of a pulse coincides with the start of a UTC second (Blair 1974) to within a few microseconds. USNO publishes in advance tables of such Times of Coincidence. Accuracy of timing the TOC pulses received via groundwave propagation is achieved to 0.2 - 0.4 microseconds. Loran-C is the method used to incorporate North American and European clocks in to TAI.
- **TV.** The basics of time comparison by TV have been described by Miller (1970). Since then the Australian network has grown
to include Darwin, Brisbane, Sydney, Parkes, ACT, Melbourne, and Adelaide, as shown in Figure 7, while the introduction of rubidium-stabilised colour TV has resulted in comparison standard errors of less than 0.1 µs (which is attributed to propagation variation) for many participating clocks in fits over 1-3 months. All measure the time of arrival of the first synchronising pulse transmitted after 1301 EAST during the networked ABC news program originating in Sydney, and need only a TV set, a simple pulse selection circuit and a time interval counter. Many establishments make the measurements automatically. TV is the principal comparison method in UTC(AUS), is used widely in USA and Europe, and connects European clocks contributing to TAI. Lavanceau and Shepard (1978) and Kovacevic (1978) describe some innovations in time dissemination via TV.

Portable Clock. Portable caesium standards have been used since the early 1960s to synchronise clocks at widely separated locations, and are now used routinely to provide calibrations between time scales and to measure propagation delays in TV and Loran-C systems. In Australia, NATMAP's portable clock DN590 does a week long calibration trip every six months on average, with closing errors on the order of 0.1 µs. Both time scales UTC(DNM) and UTC(AUS) rely heavily on visits by flying clocks from the US Navy to provide reference to UTC(USNO) and hence to TAI, since other routine methods have proved inadequate in our isolation.

Satellites. Navigation Technology Satellites (NTS), erstwhile known as Timation, have demonstrated capacity to transfer time worldwide with precision of less than 0.1 µs (Luck and Morgan 1975; Buisson et al. 1977). Each receiving station measures the time of reception of signals transmitted from the satellite under control of an on-board crystal or atomic clock, as shown in Figure 8, corrects for satellite-station propagation delay and refraction, and obtains the relationship of its clock to the satellite clock. Assumptions of stable orbit and regular on-board clock performance then allow time transfer between the stations to be accomplished. Some recent results are presented in Figure 9. Over the last twelve months, our NTS receiver system and the satellites have suffered hardware malfunctions which seem to have been resolved now, so it is hoped soon to incorporate the results UTC(USNO) - UTC (DNM) via NTS in UTC(AUS), at least in a reporting mode if not as a participating member of the ensemble.

I would like to thank John Woodger and Col Cochran for great technical and numerical assistance, and Peter Morgan for useful discussions. US Naval Research Labs have been instrumental in running the NTS systems.


