Particle acceleration and turbulence transport in heliospheric plasmas

Rami Vainio
Department of Physics, P.O.B. 64, FI-00014 University of Helsinki, Finland
email: rami.vainio@helsinki.fi

Abstract. Plasma turbulence at various length scales affects practically all mechanisms proposed to be responsible for particle acceleration in the heliosphere. In this paper, we concentrate on providing a synthesis of some recent efforts to understand particle acceleration in the solar corona and inner heliosphere. Acceleration at coronal and interplanetary shock waves driven by coronal mass ejections (CMEs) is the most viable mechanism for producing large gradual solar energetic particle (SEP) events, whereas particle acceleration in impulsive flares is assumed to be responsible for the generation of smaller impulsive SEP events. Impulsive events show enhanced abundances of $^3$He and heavy ions over the gradual SEP events. Gradual events often show charge states consistent with acceleration of ions in a dilute plasma at 1–2 MK temperature, while impulsive events have higher charge states. The division of SEP events to gradual and impulsive has been challenged by the discovery of events, which show intensity-vs.-time profiles typical for gradual events but, especially at the highest energies (above 10 MeV/nucleon), abundances and charge states more typical of impulsive events. Although a direct flare component cannot be ruled out, we find that particle acceleration at quasi-perpendicular shocks in the low corona also offer a plausible explanation for the hybrid events. By carefully modeling shock acceleration and coronal turbulence and its modification by the accelerated particles, a consistent picture of gradual events thus emerges from the shock acceleration hypothesis.

Keywords. acceleration of particles, instabilities, shock waves, turbulence, waves, Sun: coronal mass ejections (CMEs), Sun: flares, Sun: particle emission

1. Introduction

Plasma turbulence at various length scales affects practically all particle acceleration mechanisms proposed to be responsible for particle acceleration in the heliosphere. Fluctuating electromagnetic fields interact with charged particles transmitting energy and momentum between different particle populations. The turbulent energy itself can act as the source of energy in the acceleration process, like in the stochastic acceleration mechanism. Alternatively, turbulence can transmit the bulk kinetic energy of the system to the accelerated particles, like in the diffusive shock acceleration (DSA) mechanism. Finally, particle confinement in the acceleration region, whatever the mechanism, will be affected by particle transport in turbulent electromagnetic fields. Thus, understanding the properties and evolution of plasma turbulence is key to development of any particle acceleration models in collisionless plasmas.

According to the present paradigm, coronal mass ejection (CME) -driven shocks are the source of solar energetic particles (SEPs) in large gradual SEP events (e.g., Reames 1999). On the other hand, particle acceleration in impulsive flares is assumed to be responsible for the smaller impulsive SEP events, which show enhanced abundances of $^3$He and heavy ions over the coronal ones (determined from gradual SEP events) (e.g., Reames 1999). Well below MeV/nucleon energies, gradual events show charge states consistent with
acceleration of ions from a pool of seed particles at coronal or solar-wind temperature, while impulsive events have higher charge states indicating a higher electron temperature in the source plasma (e.g., Klecker et al. 2006). At higher energies, both impulsive and gradual events show increasing charge states. This is consistent with higher temperature of the seed population of particles accelerated to the highest energies and/or proton impact ionization (Kocharov et al. 2000) in dense plasma during the particle acceleration process.

The clear-cut division of SEP events to shock-accelerated gradual and flare-accelerated impulsive events has been blurred in recent years by the discovery of hybrid events (e.g., Kocharov & Torsti 2002). It has been found that many events show intensity–time profiles typical for gradual events but, especially at the highest energies (above 10 MeV/nucl), abundances and charge states more typical of impulsive events (Tylka et al. 2005). In addition to the apparent interpretation (e.g., Cane et al. 2006) that these events are superpositions of flare and shock-accelerated populations with flare acceleration extending to higher energies, such events have been explained by shock acceleration of a seed population containing flare material (Tylka et al. 2005; Tylka & Lee 2006). Note that an increase in the ionic charge states as a function of energy is consistent with both scenarios.

In this paper we will concentrate on describing the effects of coronal and heliospheric turbulence on particle acceleration in coronal/interplanetary shocks driven by CMEs. We will show, based on recent empirical and modeling results, that a consistent picture of gradual events emerges from the shock-acceleration hypothesis.

2. Heliospheric turbulence and particle transport

Turbulence in the solar wind plasma and interplanetary magnetic field (IMF) and their relation to heliospheric particle transport has been studied since the sixties (e.g., Jokipii 1966; Coleman 1968; Jokipii & Coleman 1968). The frequency power spectrum of the magnetic fluctuations can be represented as a power law, $P \propto f^{-q}$, over large ranges of frequency. It shows at least three ranges with different values of the spectral index: (i) an energy-containing range at low frequencies (below $\sim 10^{-5}$ Hz at 1 AU), where the power-law spectral index of the fluctuations is about or somewhat below unity; (ii) an inertial range at intermediate frequencies (between $\sim 10^{-5}$ Hz and $\sim 1$ Hz at 1 AU) where the spectral index is consistent with the Kolmogorov value of 5/3; and (iii) a dissipation range at high frequencies (above $\sim 1$ Hz at 1 AU), where the spectral index is $>2$, and variable. For a recent extensive review on solar wind turbulence, see the paper by Bruno & Carbone (2005).

According to the present understanding, the IMF fluctuations can be relatively well described by a two-component model consisting of (i) a two-dimensional (2D) component, in which both the fluctuating magnetic field, $\delta \vec{B}$, and the wave vector, $\vec{k}$, lie in the plane perpendicular to the mean magnetic field; and (ii) a slab component, in which the wave vector is parallel to the mean field (e.g., Matthaeus et al. 1990; Bieber et al. 1996). The solenoidal condition, $\vec{k} \cdot \delta \vec{B} = 0$, is also fulfilled in both cases. According to observations at 1 AU, the 2D component carries about 80% of the power in inertial range (Bieber et al. 1996). Particle transport in such a turbulence has been intensively studied over the last decade (e.g., Bieber et al. 1996; Dröge 2003; Bieber et al. 2004; Shalchi et al. 2004). The main effect of the 2D component is in the perpendicular transport, whereas the slab component seems mainly responsible for the pitch-angle diffusion. In the estimates below, we will simply assume that the slab-mode turbulence is solely responsible for pitch-angle
Particle acceleration and turbulence in heliospheric plasmas

Figure 1. Left: A sketch of the the mean free path of 10-MeV protons, backward extrapolated from 1 AU using slab-mode wave intensity obtained by WKB-transported Alfvén waves (dashed curve) and cascading Alfvén waves (solid curve). Right: A sketch of the unfolding of the cascading Alfvén wave spectrum from 1 AU back to the Sun (solid curves). The dashed curves give the WKB-transported spectra. (At 1 AU, the spectra are assumed to coincide.)

diffusion and, thus, particle diffusion along the mean magnetic field, and that the diffusion coefficient is well approximated by the standard quasi-linear theory at energies of interest (Jokipii 1966; Bieber et al. 1996). In this case, the pitch-angle diffusion coefficient can be approximated by

\[ D_{\mu\mu} = \frac{1}{2} \pi \Omega (1 - \mu^2) \frac{|k_{\text{res}}| I_{\text{slab}}(k_{\text{res}})}{B^2}, \]  

(2.1)

where \( I_{\text{slab}}(k) \) is the intensity of the slab-mode turbulence, \( k_{\text{res}} = \Omega / v \mu \) is the cyclotron resonant wavenumber, \( \Omega \) is the gyrofrequency of the particle, \( v \) the particle speed and \( \mu \) the pitch-angle cosine. The pitch-angle diffusion coefficient gives the scattering mean free path through the well-known expression

\[ \lambda = \frac{3v}{8} \int_{-1}^{+1} \frac{(1 - \mu^2)^2}{D_{\mu\mu}} d\mu. \]  

(2.2)

Observations in the fast solar wind indicate the presence of predominantly outward-propagating Alfvén waves, which seem to be undergoing cascading (Marsch & Tu 1990; see also Bruno & Carbone 2005 and references therein); the spectral index of the waves at low frequencies is about 1, and steepens into about 5/3 at a break point frequency, which moves towards lower values as the distance from the Sun increases. (Note that frequency in spacecraft frame is related to the wavevector approximately as \( f = \vec{k} \cdot \vec{V}_{sw} / 2\pi \), where \( \vec{V}_{sw} \) is the solar-wind velocity.) Assuming that the slab-mode turbulence in the inner heliosphere consists of such cascading Alfvén waves, it is possible to estimate the radial evolution of the scattering mean free path of SEPs from the Sun to 1 AU. Vainio et al. (2003) and Vainio (2006) used a model of WKB-transported Alfvén waves appended with phenomenological cascading to estimate the mean free path between the Sun and 1 AU. With reasonable values of the interplanetary mean free path (~0.1 AU) and an assumed \( f^{-1} \) spectral form of the waves at the Sun, the backwards extrapolated coronal mean free path of 10-MeV protons is of the order of 0.01 solar radii, which is more than four orders of magnitude below the value obtained by backward extrapolation using only WKB transport (see Fig. 1). Note that the amplitude of the adopted solar \( f^{-1} \) spectrum of Alfvén waves is still well below the spectra employed by models of coronal cyclotron heating by high-frequency Alfvén waves (see, e.g., Vainio & Laitinen...
3. Shock acceleration and turbulence

Particle acceleration by CME-driven shocks in turbulent coronal plasma occurs probably via the DSA mechanism (e.g., Bell 1978). In this model, a particle gains energy by repeatedly crossing the shock front and feeling the compression of the flow through its interaction with the small-scale irregularities of the magnetic field. The standard steady-state theory predicts an accelerated particle (species \( s \)) distribution at the shock of a power-law form,

\[
f_{sh}^{(s)}(p) = \frac{\sigma \epsilon_s n_{s1}}{4\pi p_{s0}^3} \left( \frac{p}{p_{s0}} \right)^{-\sigma}, \quad (3.1)
\]

where \( p \) is the particle momentum, \( \sigma = 3X/(X - 1) \) is the spectral index which depends only on the compression ratio \( X = u_{1n}/u_{2n} = \rho_2/\rho_1 \) of the shock, \( n_{s1} \) is the number density of the \( s \)'th species in the upstream region, \( \epsilon_s \) is fraction of particles (of species \( s \)) injected into the acceleration process at the injection momentum \( p_{s0} \), and \( u_{1n}[2n] \) and \( \rho_{1[2]} \) are the shock-frame plasma flow speed in the shock normal direction and mass density in the upstream [downstream] region. In low-Mach number shocks, the compression ratio \( X \) should be replaced by the scattering center compression ratio (Vainio & Schlickeiser 1998, 1999), \( X_{sc} = W_{1n}/W_{2n} \), where the wave speeds \( W_n = u_n + (v_{\phi,n}) \) contain the effect of the average phase speed, \( v_{\phi} \), of the waves with respect to the plasma.

The steady-state assumption is valid only up to energies limited by the available acceleration time. The time scale of particle acceleration in DSA is given by (Drury 1983)

\[
\frac{p}{p_{s0}} = \sigma \left( \frac{\kappa_{n,1}(p)}{u_{1n}^2} + \frac{\kappa_{n,2}(p)}{X u_{2n}^2} \right), \quad (3.2)
\]

where \( \kappa_{n} = \kappa_{||} \cos^2 \theta_n + \kappa_{\perp} \sin^2 \theta_n \) is the spatial diffusion coefficient in the shock normal direction, \( \theta_n \) is the angle between the shock normal and the magnetic field, and \( \kappa_{||[\perp]} \) is the spatial diffusion coefficient parallel [perpendicular] to the mean magnetic field. Since shocks amplify turbulence very efficiently, usually the second term in Eq. (3.2) is neglected. Furthermore, in a weakly turbulent plasma with \( r_L \ll \lambda_{||} \), the perpendicular diffusion coefficient is smaller than the parallel one, so if the shock is oblique with, say, \( \theta_n < 70^\circ \), we can neglect perpendicular transport and obtain \( \kappa_{n} \simeq \kappa_{||} \cos^2 \theta_n \) with \( \kappa_{||} = \frac{1}{3} v \lambda \). Thus,

\[
\frac{p}{p_{s0}} \approx \frac{\sigma v \lambda_1(p) \cos^2 \theta_{n,1}}{3u_{1n}^2}. \quad (3.3)
\]

The cut-off momentum in the spectrum, \( p_c(t) \), can now be estimated by equating the acceleration time scale with the time available for acceleration, i.e.,

\[
\frac{\sigma v_c \lambda_1(p_c) \cos^2 \theta_{n,1}}{3u_{1n}^2} \approx \frac{\Delta s_{||}(t)}{u_{1n}/\cos \theta_{n,1}} \Rightarrow \lambda_1(p_c) = \frac{3u_{1n} \Delta s_{||}(t)}{\sigma v_c \cos \theta_{n,1}}, \quad (3.4)
\]

where \( \Delta s_{||}(t) \) is the distance swept by the shock along a given magnetic field line from the time of first intersection to time \( t \). Here it has been assumed that the shock parameters and \( \lambda \) are not functions of time. If this is not the case, the differential equation (3.3) has to be properly solved, but the form (3.4) still serves as a convenient order-of magnitude
approximation for the mean free path required for particle acceleration up to a given cut-off momentum, if the time-dependent parameters are replaced by typical values.

Plugging in the numbers as $\Delta s_\parallel = 1 R_\odot$, $u_{1,n} = 1000 \text{ km s}^{-1}$, $\sigma = 4$ and $\theta_{n,1} = 70^\circ$, and using the coronal mean free path deduced by backward extrapolation from 1 AU produces proton cut-off energies of the order of $E_c \sim 10 \text{ MeV}$ (Vainio 2006). Some tens of MeVs may be obtained by increasing the shock speed and/or the shock normal angle. One should, however, bear in mind, that the neglect of perpendicular transport in the estimates above forbids their use at nearly perpendicular shocks.

If, instead, we take the shock normal angle to be close to $90^\circ$, we might use $\kappa_{nn} \simeq \kappa_\perp$. As an increase in the level of turbulence typically increases perpendicular diffusion, we should now neglect the first term in Eq. (3.2) rather than the second. Assuming that the downstream region is very turbulent, we may use the Bohm limit of the diffusion coefficient, $\kappa_\perp \simeq \kappa_B = \frac{1}{3} \gamma v^2 / \Omega_0$ with $\Omega_0$ being the non-relativistic gyrofrequency and $\gamma$ the Lorentz factor of the particle, and obtain

$$\frac{p}{\dot{p}} \simeq \frac{\sigma \gamma v^2}{3 X u_2^2 \Omega_{0,2}} = \frac{\sigma \gamma v^2}{3 u_1^2 \Omega_{0,1}} \tag{3.5}$$

where $\Omega_{0,1(0,2)}$ is the non-relativistic gyrofrequency in the upstream [downstream] region of the shock, and $\Omega_{0,2} = X \Omega_{0,1}$ for a perpendicular shock. Thus,

$$dE = v dp \simeq (3/\sigma) m u_1^2 \Omega_{0,1} \tau_\perp \tag{3.6}$$

and integrating this from $t = 0$ to the available acceleration time $\tau_\perp$ in the perpendicular shock gives

$$E_c \simeq (3/\sigma) m u_1^2 \Omega_{0,1} \tau_\perp \tag{3.7}$$

Using $\tau_\perp = 100 \text{ s}$, $u_{1,n} = 1000 \text{ km s}^{-1}$, $\sigma = 6$ (i.e., $X = 2$), and $\Omega_{0,1} = 10^4 \text{ s}^{-1}$ (a proton in a 1-gauss field), one obtains $E_c \simeq 5 \text{ GeV}$ for protons. Thus, in a turbulent perpendicular coronal shock particles may be accelerated up to relativistic energies in tens of seconds. Note that the maximum energy per nucleon in this estimate scales like $E_c/A \propto Q/A$, where $Q$ and $A$ are the ionic charge state and the mass number of the particle.

If perpendicular shocks can easily accelerate particles to relativistic energies in the corona, then why do we observe such high energies from the Sun very rarely, during cosmic-ray Ground Level Enhancements (GLEs) only? Of course, the estimate above assumes that a coronal shock may exist in a region of relatively strong perpendicular field for a sufficient amount of time, which is not necessarily easily satisfied in the case of CME-driven shocks. In addition, particles may be able to escape the system along the field lines, if the size of the perpendicular region of the shock is very limited. Our estimate also assumes that particle motion is well approximated by diffusion, at least in the direction perpendicular to the magnetic field. This, however, is not automatically satisfied at all energies of interest. The parallel and perpendicular transport are coupled to each other, and all turbulence scenarios do not lead to efficient diffusion. This is important especially at the lowest energies, where inefficient diffusion transverse to the field may lead to an injection problem: low-energy ions incident on the shock from the upstream region may not be able to return to the shock after being transmitted to the downstream side. Thus, the amount of accelerated particles remains very small unless particles with high initial speeds are available for the shock in the upstream region. Furthermore, while the compression of the perpendicular fields at the shock dramatically decreases the diffusion coefficient in a parallel shock (Vainio & Schlickeiser 1998, 1999), this is not so obvious in a perpendicular shock: there, by compressing the perpendicular field components,
the shock compresses the mean field, the possible compressional turbulence components (with $\delta \vec{B} \parallel \vec{B}$), and the transverse turbulent field component in the shock plane, but the field component in the shock normal direction remains uncompressed. This means that at least the field-line random walk in the shock normal direction is actually suppressed. The turbulent component in the shock plane may, however, be strong enough to scatter the particle from one field line to another.

In fact, numerical simulations of particle acceleration in quasi-perpendicular shocks (Giacalone 2005) show that if the upstream region of the shock is turbulent enough, there is no injection problem and perpendicular shocks accelerate particles rapidly into high energies. For an upstream fluctuation amplitude of $(\Delta B/B)^2_1 = 0.1$, Giacalone (2005) obtains a cutoff energy of about $E_c \sim (5 \cdot 10^5) m u^2_{\text{in}}$ for a simulation run with $X = 4$ and $\tau_\perp = 5 \cdot 10^4 \Omega^{-1}$, which is consistent with $\kappa_\perp \sim 0.1 \kappa_B$ rather than Bohm diffusion. Thus, even with a relatively high value of the upstream turbulence amplitude (compared to the cases we are considering for corona) and further compression of the transverse fields at the shock, the transport in the downstream region is not governed by Bohm diffusion. The injection efficiency in Giacalone’s (2005) simulation with $(\Delta B/B)^2_1 = 0.1$ is suppressed by an order of magnitude relative to the case with $(\Delta B/B)^2_1 = 1$. Thus, with coronal field fluctuations obtained from our turbulence transport model, i.e., $(\Delta B/B)^2_1 \lesssim 0.001$, the injection problem in quasi-perpendicular shocks may well be a serious one in perpendicular coronal shocks without a preaccelerated seed particle population. Thus, we may infer that particle acceleration at coronal shocks to energies in the GeV range should be limited to rare occasions, as observed.

4. Self-consistent modeling of large gradual SEP events

The geometry of the CMEs implies that most of the time during their propagation through the outer corona they drive oblique or quasi-parallel shocks. However, during large gradual SEP events, shocks seem to be efficiently accelerating particles up to hundreds of MeVs at heights above the low corona. Thus, the turbulence responsible for particle acceleration in these events is not likely to be the rather weak ambient turbulence in the corona.

Already Bell (1978) pointed out that DSA does not have to rely on the ambient turbulence to scatter the particles around the shock. The accelerated particles streaming away from the shock in the upstream plasma frame drive the outward-propagating Alfvén waves unstable. These waves bootstrap the diffusive acceleration process and lead to rapid acceleration at the shock. The idea was applied to traveling interplanetary shocks by Lee (1983), and his steady-state theory has survived, at least in a semi-quantitative sense, the test against observations at 1 AU (Kennel et al. 1986). Lee’s steady-state model, appended by the assumption that the acceleration time scale is equal to the dynamic scale of shock propagation, was applied to coronal/interplanetary shock acceleration by Zank et al. (2000) and Rice et al. (2003). Using this acceleration model and an assumption that an ad-hoc fraction of particle flux at the shock will escape from the shock complex towards the upstream region, and following the propagation of the escaping particles in the ambient medium, Li et al. (2003) computed the time-intensity profiles of gradual events at 1 AU. In the coronal shock acceleration problem, however, a self-consistent model of particle escape from the self-generated turbulent trap is needed, before the model can be considered fully adequate. The modeling of these authors, however, convincingly demonstrated that CME-driven shocks can accelerate ions up to hundreds of MeVs and beyond.
The first analytical model to quantitatively address the escape from self-generated waves was the one by Vainio (2003). He considered time dependent excitation of waves and concluded that a relatively large fluence of energetic protons can actually escape from the corona before the waves have grown to substantial amplitudes. In Vainio’s (2003) model, small gradual events at MeV energies, and practically all events at relativistic energies have fluences that do not meet the threshold for efficient wave generation. However, Vainio (2003) used an artificially sharpened resonance condition, \( k_{\text{res}} = \frac{\Omega}{v} \), in his calculations, which meant that high-rigidity particles could not resonate with waves generated by lower-rigidity ones, which is possible if the full quasi-linear resonance condition is employed. Another analytical model that treats the escape from the coronal/interplanetary shock in a consistent manner was developed by Lee (2005). His model is quasi-stationary, but it includes adiabatic focusing in the upstream region, which is able to drive the particles away from the shock allowing them to escape.

More recently, two numerical simulation models combine the effects of wave growth, diffusive acceleration at the shock and focused particle transport in self-generated turbulence. Vainio & Laitinen (2007, 2008) developed a Monte Carlo simulation model tracing individual particles in the upstream region of a shock under the influence of turbulent fields amplified by the particles themselves. The particles were being accelerated by a propagating parallel coronal shock, treated as a boundary condition in the simulation. This is equivalent with the assumption that particle scattering in the downstream region is very efficient, so that the contribution from the residence time in the downstream region to the acceleration time scale can be neglected. The particle scattering rates and wave growth rates in the upstream region were taken from quasi-linear theory but using the sharpened resonance condition, \( k_{\text{res}} = \frac{\Omega}{v} \). The results indicated that coronal shocks would have no problem in accelerating particles up to hundreds of MeVs in a few minutes even if the injection efficiency of the shock was taken to be rather low to keep the upstream wave intensities in the linear regime. The simulation model agreed with the predictions of the steady-state theory of Bell (1978) in terms of the spectrum of waves and particles at the shock and with the prediction of Vainio (2003) about the fluence of the particle population escaping upstream from the shock before the steady-state wave amplitudes are achieved.

Another simulation model by Ng & Reames (2008) utilizes a different numerical method (finite difference method) and employs many complications of particle and turbulence transport neglected in the model of Vainio & Laitinen (2007). It uses the full resonance condition, includes self-consistent turbulence transmission at the shock (Vainio & Schlickeiser 1999), and also follows particle propagation and wave growth in the downstream region. The main result of previous studies, i.e., efficient acceleration of particles up to hundreds of MeVs in some minutes in parallel CME-driven shocks is recovered also in this model. This implies that the simplified simulation model of Vainio & Laitinen (2007) probably captures the main physical ingredients of the theory.

How would the results of the simulation models, obtained for strictly parallel shocks, change with shock obliquity? Eq. (3.2) indicates that a shock propagating a fixed distance \( ds_{\parallel} = u_{n,1} dt / \cos \theta_{n,1} \) along the field accelerates particles at a rate \( dp/ds_{\parallel} \propto (\lambda \cos \theta_{n,1})^{-1} \propto I(\Omega/v) / \cos \theta_{n,1} \). Theory predicts that the growth rate and, hence, the steady-state intensity of the waves is \( I(\Omega/v) \propto f_{sh}^{(p)}(v; \theta_{n,1}) \cos \theta_{n,1} \). Thus, assuming a slow rate of change for \( \theta_{n,1} \), we can write \( dp/ds_{\parallel} \propto f_{sh}^{(p)}(v; \theta_{n,1}) \) for the dependence of the acceleration rate on the injection efficiency. There is no explicit dependence of the proton distribution function (3.1) on the shock obliquity, so at least in the quasi-parallel regime (say, \( \theta_{n,1} \lesssim 30^\circ \)) it should be rather insensitive to \( \theta_{n,1} \). In the intermediate obliquity...
regime (say, $30^\circ \lesssim \theta_{n,1} \lesssim 80^\circ$), simple estimates of the behavior are difficult to make, because the result depends on many details of the shock and the incident seed particle distribution. At nearly perpendicular shocks (say, $\theta_{n,1} \gtrsim 80^\circ$), ion injection is quenched and the number of accelerated particles at a given energy is probably strongly decreasing as a function of the shock normal angle. However, in these shocks we have to include perpendicular transport in the estimate of the acceleration rate, as discussed above.

5. Gradual events with impulsive composition signatures

As discussed above, a class of gradual SEP events shows impulsive-event-like composition at high energies. In these events, the iron-to-oxygen abundance ratio first starts to decrease around 1 MeV/nucl, but later increases to values resembling impulsive-flare abundances at tens of MeV/nucl. Tylka et al. (2005) suggested that these compositional signatures were due to diffusive acceleration at quasi-perpendicular coronal shocks of seed populations containing pre-accelerated flare material. Proposing that quasi-perpendicular shocks accelerate particles to higher energies than quasi-parallel ones, and assuming that it would be easier for a coronal shock to inject suprathermal flare ions than quasi-thermal coronal material, Tylka & Lee (2006) developed an analytical model of this scenario. Their model showed extremely good coincidence with the observational results. However, as the model contained several ad-hoc assumptions about the form of the accelerated particle spectra and injection efficiencies for different species and shock obliquities, Sandroos & Vainio (2007) performed test-particle simulations in such a scenario to verify the assumptions. The simulations employed an expanding spherical shock front centered in the low corona sweeping a radial (from the center of the Sun) coronal magnetic field line containing a seed population that was a mixture of low-energy ions with coronal composition and higher-energy ions with impulsive composition. The turbulence spectrum was assumed to be of the form $1/f$ with amplitudes consistent with those extrapolated backwards from the solar wind. The simulation results quantitatively confirmed the results of the model of Tylka & Lee (2006).

The models of Tylka & Lee (2006) and Sandroos & Vainio (2007) are consistent with the assumption that that the mean free path of the accelerated particles is proportional to particle rigidity up to the highest energies in the system. If the upstream turbulence was fully generated by protons accelerated at the shock, this would not be the case. Instead, there would be a low-wavenumber cutoff in the turbulence spectrum at $k_0 \sim m_p \Omega_{p,o}/p_{p,c}$, where $p_{p,c}$ is the cutoff momentum in the proton spectrum. Heavy ions (of species $i$) resonating with this wavenumber have momenta $p_{i,c} \sim m_i \Omega_{i,c}/k_0 = Q_i p_{p,c}$. For non-relativistic particles, this implies $E_{i,c}/A_i = p_{i,c}^2/(2A_i^2 m_p) = (Q_i/A_i)^2 E_{p,c}$, which becomes an upper limit of the cutoff energy. Recall that shock acceleration with $\kappa \propto v_p$ over a finite time yields $E_{i,c}/A_i = (Q_i/A_i) E_{p,c}$, which is the relation adopted by Tylka & Lee (2006) and obtained by Sandroos & Vainio (2007) in their simulation model. Furthermore, as deduced above, self-generated waves have lower intensities in oblique shocks, and the cutoff energies as a function of shock obliquity are most probably decreasing, not increasing as in the case of an external turbulence. Since the model of selective shock acceleration is relying on particles at the highest energies being accelerated by quasi-perpendicular shocks (requiring higher injection energies), it is inconsistent with turbulence being self-generated, or at least with this playing any role in the acceleration process.

If self-generated waves cannot produce the impulsive composition signatures at high energies, we can study the proton fluences of the events to find out if they suggest that wave growth would be important in these events. Tylka et al. (2005) analyzed 30–40 MeV/nucl iron-to-oxygen ratio as a function of $>30$ MeV proton fluence for 44 gradual
Particle acceleration and turbulence in heliospheric plasmas

SEP events of the solar cycle 23. A clear organization of the events is evident. All events with enhanced high-energy iron-to-oxygen ratio have small integral proton fluences, below \( F \sim 10^7 \text{ cm}^{-2} \text{ sr}^{-1} \). We can use the model of Vainio (2003) to estimate the threshold fluence for efficient wave generation in the corona. This requires \( E \frac{dN}{dT} \sim 10^{33} \text{ protons} \) to be injected into the flux tube per steradian at the solar surface at the resonant energies. This number translates into a time-integrated net flux per unit logarithmic energy range at 1 AU of \( E \frac{dG_{\text{thr}}}{dT} \sim 4 \cdot 10^6 \text{ cm}^{-2} \). This quantity is related to the fluence in a unit logarithmic energy range, \( E \frac{dF}{dT} \), by \( E \frac{dG}{dT} = 4\pi \langle \mu \rangle E \frac{dF}{dT} \), where \( \langle \mu \rangle \) is the average value of the pitch-angle cosine at 1 AU during the event in the considered energy channel. Thus, \( E \frac{dG}{dT} = 4\alpha \langle \mu \rangle F \), where \( \alpha \) is the spectral index of the integral proton fluence assumed to be of form \( F \propto E^{-\alpha} \). Noting that the first-order anisotropy \( 3\langle \mu \rangle \approx \lambda/L \), where \( L \) is the focusing length (\( L \sim 1 \text{ AU} \) at 1 AU), we find values of \( E \frac{dG}{dT} \lesssim \alpha (\lambda/L) 4 \cdot 10^7 \text{ cm}^{-2} \) for events with composition anomalies in Tylka’s (2005) sample. Noting that for \( \alpha (\lambda/L) \sim 0.1 \), this estimate agrees with the threshold for wave growth, but reasonable values of the mean free path and spectral index may also produce estimates of the time-integrated flux extending up to an order of magnitude above the threshold. However, as the waves produced by 30-MeV protons resonate with iron ions of similar rigidity, the effects of these waves on iron would be most prominent at about an order of magnitude lower energies than the channel considered here. Thus, we may safely state that wave growth is unlikely to have significantly influenced turbulence responsible for iron acceleration at the highest energies in those gradual events showing composition anomalies at the highest energies. This strongly suggests that the shocks accelerating these ions are quasi-perpendicular.

6. Conclusions

We have reviewed some recent modeling efforts to understand particle acceleration in gradual SEP events assuming that they are accelerated by CME-driven shock waves. The following internally consistent picture of the acceleration process emerges:

- Particle acceleration in gradual events can be understood in terms of diffusive shock acceleration in the solar corona and interplanetary medium.
- Large gradual events, with proton fluences exceeding the wave-generation threshold at resonant wavenumbers, can be understood in terms of particle acceleration at shocks propagating through self-generated waves. These events show particle abundances consistent with high-rigidity particles being less effectively accelerated at the shock, i.e., abundances of low-charge-to-mass ratio ions decreasing as a function of energy. This is consistent with the bulk of the acceleration at these energies occurring at the quasi-parallel phase of the shock propagation.
- Smaller gradual events, with proton fluences below the wave-generation threshold, are accelerated in coronal shocks without self-generated turbulence. Extrapolations of turbulence levels from the solar wind to the corona imply that proton acceleration beyond 10 MeV in these events occurs in quasi-perpendicular shocks (\( \theta_{n,1} > 70^\circ \) for CME speeds of the order of \( \sim 1000 \text{ km s}^{-1} \)). These events show both decreasing and increasing abundances of low-charge-to-mass ratio ions as a function of energy. As injection in a quasi-perpendicular shock propagating in weak turbulence requires high-velocity ions to be present in the upstream region, we attribute the variations in the high-energy abundance ratios to variations in the seed-particle composition: when iron-rich suprathermal material is present in the ambient plasma, iron-rich composition at the highest energies is obtained.
GLEs can be understood either in terms of nearly perpendicular shock acceleration, where the acceleration time is governed by Bohm-like diffusion in the downstream region, or in terms of quasi-parallel shock acceleration by self-generated waves in very strong CME-driven shocks with high injection efficiency driving the ambient waves close to non-linear amplitudes. By studying the fluences and time-intensity profiles of the associated >100-MeV proton events it is possible to infer whether wave generation may bootstrap the acceleration to relativistic energies.

More work is still needed to develop simulation models that treat the injection and acceleration process self-consistently in terms of the local shock structure, global and local shock geometry, and waves and instabilities within the shock complex.

References
Marsch, E. & Tu, C.-Y. 1990, J. Geophys. Res., 95, 8211
Discussion

Vlahos: In your model the presence of turbulence before the shock arrival is an essential parameter and in the solar wind this is true but inside the corona we have no idea if turbulence is always there.

Vainio: For the selective acceleration process we indeed assume that particles are accelerated in ambient turbulence and the values for turbulence power are extrapolated with a transport model from measurements in solar wind. Of course, this is uncertain to some extent and we really don’t know if the turbulence is there (in large events, our model does not need almost any ambient turbulence, because the waves are self-generated.)

Tsap: Why did you not consider the drift acceleration mechanism and the mechanism proposed by Sagdeev?

Vainio: Our models for parallel shocks only employ pitch-angle diffusion, because in such shocks possible drifts do not lead to particle acceleration. In oblique shocks models, we compute particle trajectories in full, so drifts are included (Sagdeev’s model, shock surfing, is, however, not included, because we do not employ any cross-shock potential in our shock model. We hope to address this in the future).