ON CERTAIN FORMULÆ IN MR. DAVID JONES'S WORK ON ANNUITIES.

To the Editor.

SIR,—I am not aware whether what seems to me to be a serious error in one of the expressions given in Jones's work On Annuities and Reversionary Payments, for a deferred assurance, has ever been noticed or not.

On page 170, vol. i., of the work referred to, the general value of a deferred assurance on any number of lives is given as

\[ A_{m, m_1, m_2, \&c.} = \frac{1}{1 - r} p_{(m, m_1, m_2, \&c.)} (1 - r) \alpha_{(m, m_1, m_2, \&c.)} \cdot \frac{1}{1 - r} p_{(m, m_1, m_2, \&c.)} \alpha_{(m, m_1, m_2, \&c.)} \ldots \]  

An expression correct enough: but wishing, I presume, to give this result in a more convenient form, the author goes on to manipulate the latter term of the second member of this equation, thus—

\[ (1 - r) \alpha_{(m, m_1, m_2, \&c.)} = (1 - r) \frac{1}{1 - r} p_{(m, m_1, m_2, \&c.)} \frac{1}{1 - r} p_{(m, m_1, m_2, \&c.)} \alpha_{(m + m_1 + m_2 + \&c.)} \alpha_{(m, m_1, m_2, \&c.)} ; \]

and in this way he deduces

\[ A_{m, m_1, m_2, \&c.} = \frac{1}{1 - r} p_{(m, m_1, m_2, \&c.)} \alpha_{(m + m_1 + m_2 + \&c.)} \alpha_{(m, m_1, m_2, \&c.)} . \]  

This latter expression it seems to me is quite erroneous, in consequence of a fatal error in the process that leads to it. In fact, \( a_{(m, m_1, m_2, \&c.)} \) is not, except in the single case when \( v = m + m_1 + m_2 + \&c. \)

the equivalent of \( (1 - r) \alpha_{(m, m_1, m_2, \&c.)} \alpha_{(m, m_1, m_2, \&c.)} \), as assumed in the work referred to.

This will at once be seen if we particularise the expression. Suppose two lives \( (m \text{ and } m_1) \) concerned, and \( v = 1 \); then

\[ (1 - r) \frac{1}{1 - r} p_{(m, m_1)} \frac{1}{1 - r} p_{(m, m_1)} \alpha_{m} \alpha_{m_1} \]  

\[ = r' (1 - r) \left( \frac{l_{m + m_1}}{l_m} a_{m + m_1} + \frac{l_{m_1}}{l_{m_1}} a_{m} - \frac{l_{m + m_1}}{l_m} a_{m} \right) , \]

while

\[ (1 - r) \frac{1}{1 - r} p_{(m, m_1)} \frac{1}{1 - r} p_{(m, m_1)} a_{m} a_{m_1} = r' (1 - r) \left( \frac{l_{m + m_1}}{l_m} a_{m + m_1} + \frac{l_{m_1}}{l_{m_1}} a_{m} - \frac{l_{m + m_1}}{l_m} a_{m} \right) ; \]

a very different expression indeed, and giving rise to a very serious error.

As an illustration, let us take an actual case:—Required the single premium for £1 payable on death of last of two parties now aged 30 and 40 respectively, provided that event happens after 10 years (Carlisle 3 per cent.)

Thus the true value, i.e., the result of equation (1) is 0.3118

while the value given by equation (2) is 0.2959

a result too little by nearly 2 per cent.

Hoping you will be good enough to inform me whether this point has ever been remarked on previously,

I am, Sir, your most obedient servant,

JAMES R. MACFADYEN.

City of Glasgow Life Assurance Company,
Glasgow, 18th July, 1866.