CORRESPONDENCE.

SOLUTION OF A PROBLEM.

To the Editor of the Journal of the Institute of Actuaries.

Sir,—In my paper entitled "How does an Increase of Mortality affect Policy-Values?" (J.I.A. xxi, 98) I proposed for solution a problem which may be stated thus:

If

\[
\frac{1 + a_x}{1 + a_{x+1}} > \frac{1 + a'_x}{1 + a'_{x+1}} > 1,
\]

then will

\[
\frac{a_x}{v(1 + a_x)} > \left( \frac{1}{1 + a'_x} - \frac{1}{1 + a_x} \right) \frac{1}{v - v'};
\]

\(a'_x\) being the value of an annuity calculated by the same mortality table as \(a_x\), but at a higher rate of interest. I have recently received from Mr. J. C. Hopkinson a solution of the problem, which you may perhaps think worthy of being brought under the notice of your readers.

We have

\[
\frac{1 + a_x}{1 + a'_x} - 1 = \frac{a_x - a'_x}{1 + a'_x}
\]

\[
= \frac{a_x}{a'_x} - \frac{1 + a_x}{1 + a'_x} a'_x
\]

\[
< \frac{a_x}{a'_x} - \frac{1 + a_{x+1}}{1 + a'_{x+1}} a'_x
\]

\[
< \left\{ \frac{a_x}{a'_x} - \frac{v'}{v} \cdot \frac{a_x}{a'_x} \right\} a'_x
\]

\[
< \frac{a_x}{a'_x} - \frac{c - c'}{v'}
\]

Then dividing both sides of this inequality by \((v - v')(1 + a_x)\), the desired result at once follows. It will be observed that we have made no use of the condition that \(\frac{1 + a_x}{1 + a_{x+1}}\) and \(\frac{1 + a'_x}{1 + a'_{x+1}}\) are both > 1; and it seems that this condition is unnecessary, and that it would be
sufficient to have \( \frac{1 + a_x}{1 + a_{x+1}} \) \( \frac{1 + a'_x}{1 + a'_{x+1}} \). It is worth noticing that this inequality leads to

\[
\frac{v p_{x-1}(1 + a_x)}{v p_x(1 + a_{x+1})} > \frac{v' p_{x-1}(1 + a'_x)}{v' p_x(1 + a'_{x+1})},
\]

whence

\[
\frac{a_{x-1}}{a_x} > \frac{a'_{x-1}}{a'_x}.
\]

I am, Sir,

Your obedient servant,

26 St. Andrew Square,

Edinburgh,

9 May 1885.

T. B. SPRAGUE.