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GENES, LEGITIMACY AND HYPERGAMY: ANOTHER LOOK AT THE ECONOMICS OF MARRIAGE

GILLES SAINT-PAUL
Paris School of Economics, New York University Abu Dhabi

Abstract: In order to credibly "sell" legitimate children to their spouse, women must forego more attractive mating opportunities. This paper derives the implications of this observation for the pattern of matching in marriage markets, the dynamics of human capital accumulation, and the evolution of the gene pool. A key consequence of the trade-off faced by women is that marriage markets will naturally tend to be *hypergamous* – that is, a marriage is more likely to be beneficial to both parties relative to remaining single, the greater the man's human capital, and the lower the woman's human capital. As a consequence, it is shown that the equilibrium can only be of two types. In the "Victorian" type, all agents marry somebody of the same rank in the distribution of income. In the "Sex and the City" (SATC) type, women marry men who are better ranked than themselves. There is a mass of unmarried men at the bottom of the distribution of human capital, and a mass of single women at the top of that distribution. It is shown that the economy switches from a Victorian to an SATC equilibrium as inequality goes up.

The model sheds light on how marriage affects the returns to human capital for men and women. Absent marriage, these returns are larger for women than for men but the opposite may occur if marriage prevails. Finally, it is shown that the institution of marriage may or may not favour human capital accumulation depending on how genes affect one's productivity at accumulating human capital.

Keywords: marriage markets, human capital accumulation, hypergamy, overlapping generations, legitimacy

JEL classification: D1, D13, D3, E24, I2, J12, J13, J16, K36, O15, O43

1. INTRODUCTION

This paper studies an economic model of marriage which is entirely based on the biological differences between men and women. The two most important differences are that, in nature, women know for sure whom their children are, while men don't; and that men can potentially have children with a large number of women, while the converse is not true for women.[1]

Because of the first of these biological differences, there are gains from trade between men and women. Women can sell to men a guarantee that her children are his – a property I call *legitimacy*. Men are willing to pay for legitimacy because they

can raise their utility by investing in their own children. This will hold provided men derive utility from the quantity and quality of children.

However, to provide such a guarantee, the woman must credibly commit to mate only with her husband – which is the key feature of the traditional marriage contract.[2] Furthermore, the second biological difference between men and women implies that women have an opportunity cost of marrying. Instead, they could mate with men with the most desirable characteristics, and improve the genotype of their offsprings. Because these men's gametes are not scarce, they have no cost of mating with as many women as possible, and they benefit from it as long as they derive utility from having illegitimate children.[3] By marrying, a woman foregoes the superior genetic material of the most attractive men[4]; on the other hand she benefits from the father's investment in the children's human capital and from any transfer from her husband. This trade-off will hold as long as men have different observable traits that are genetically heritable and valued by the parents in their children. In the model, it is assumed that children of more desirable men (the alpha men) are more productive in acquiring human capital.

This paper derives the implications of these observations for the pattern of matching in marriage markets, the dynamics of human capital accumulation, and the evolution of the gene pool. A key consequence of the trade-off faced by women is that marriage markets will naturally tend to be *hypergamous* – that is, a marriage is more likely to be viable, the greater the man's human capital, and the lower the woman's human capital. The reason is that the utility loss from marrying a beta man instead of an alpha man is not transferable; therefore, the greater a woman's human capital, the lower her marginal utility of consumption, and the larger the transfer that she must get from a man in order to be compensated for her foregone mating opportunities. The opposite logic is at work for men: the larger their human capital, the lower their marginal utility of consumption, and the greater their willingness to pay for legitimate children.

The paper is organized as follows. In Section 2 the model is set up, and we derive the equilibrium conditions for a "state of nature" where marriage does not exist, and for a society where marriage exists. We use a model of the intergenerational transmission of human capital with sexual reproduction, endogenous mating and household formation, and heritable genetic differences between people (alphas vs. betas). People derive utility from consumption, and the quantity and quality of their children. Their income is proportional to their human capital, which depends on their genes and on their parents' investment. They allocate their income between their consumption and their children's human capital accumulation. A key result is that in the State of Nature, only the alpha men mate; the beta men are driven out of the market as they cannot credibly buy legitimacy from women. We then derive a condition for marriage to yield a positive surplus relative to each party remaining single. This condition exhibits hypergamy: it is more likely to hold, the greater the man's human capital, and the smaller the woman's human capital.

Section 3 derives and discusses the model's predictions for the mating pattern. We characterize the equilibrium assignment of husbands to wives, and perform

comparative statics with respect to this assignment. A perfectly competitive marriage market is assumed. It is shown that perfect assortative matching arises and that this is due to the public good aspect of the children in the woman's and the man's utility function[5]. Because of the hypergamy effect, we can also show that the equilibrium can only be of two types. In the "Victorian" type, all agents marry somebody of the same rank in the distribution of income. In the "SATC" type, women marry men who have a greater rank than themselves. There is a mass of unmarried men at the bottom of the distribution of human capital, and a mass of single women at the top of that distribution.[6] It is shown that the economy switches from a Victorian to an SATC equilibrium as inequality goes up; one interpretation is that less skilled women underbid more skilled ones for their husbands, which in equilibrium drives the skilled woman's share in bargaining down. As a result, the most skilled women end up better-off unmarried, and mating with alpha men. The same mechanism explains why the equilibrium may be SATC even though all homogamous marriages would be viable: starting from a homogamous assignment, less skilled women would successfully underbid more skilled ones by accepting a lower share of the surplus, thus driving them out of the marriage market. This suggests that perfect competition in marriage markets may reduce the number of marriages relative to other institutional arrangements for matching husbands and wives together.[7]

The model sheds light on how marriage affects the returns to human capital for men and women. In the State of Nature, these returns are larger for women than for men because they use their human capital both to invest in their children and to increase their own consumption. When marriage exists, this effect is equalized between men and women, but additional interesting effects arise. The returns to human capital depend on how the surplus is split between men and women at different levels of human capital: when inequality is large, competition for mates from low-skill women generates a downward profile of the woman's share in output as her human capital goes up. This tends to reduce the returns to human capital for women relative to men. Another effect arises if the equilibrium is SATC: a man has a lower quality spouse than a woman with the same level of human capital; therefore his marginal utility of consumption and his return to human capital are higher. Finally, in an SATC equilibrium, acquiring human capital may make a man eligible for marriage, while it may eliminate the benefits of marriage for a woman. This, too, tends to reduce the return to human capital for women relative to men. On the other hand, in an SATC equilibrium beta men at the bottom of the distribution of skills are single and therefore have the same low return to human capital as in the State of Nature. Following this analysis, we may speculate that the decline of marriage may have something to do with men losing ground relative to women in higher education, relative to an initial situation where they did acquire more education than women[8].

These predictions are consistent with the microeconomic evidence provided by Bertrand et al. (2013). They show that the distribution of earnings shares in a couple is highly skewed toward the zone where the wife's share is below 1/2 – with a sharp drop as one moves from below 1/2 to above 1/2. They also

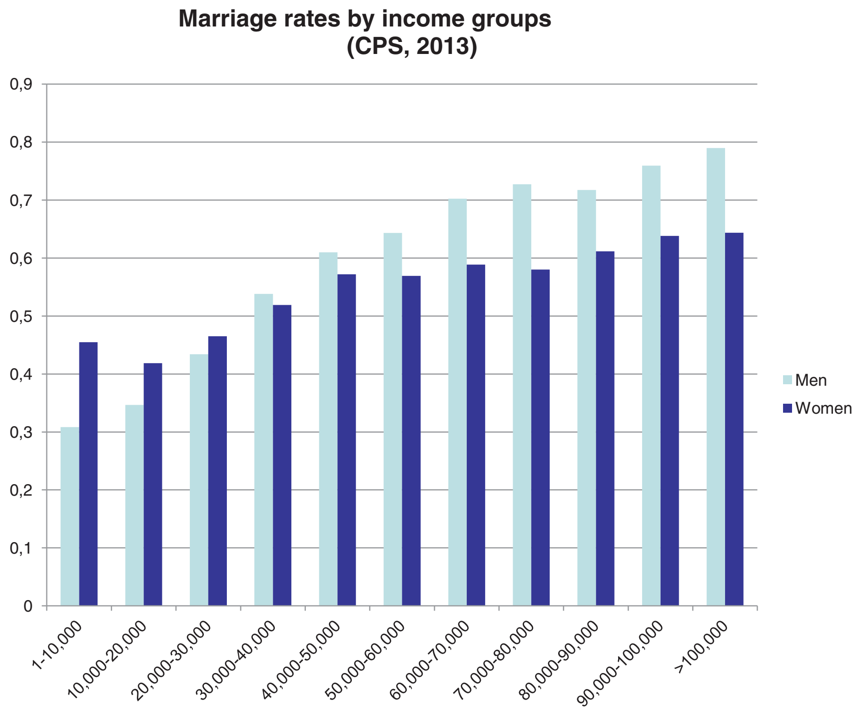

FIGURE 1. (Colour online) Marriage rates by income groups (CPS, 2013).

provide evidence of greater marital instability and lower marital satisfaction in couples where the wife earns more than her husband. This is consistent with the hypergamy property studied in this paper[9]. This phenomenon alone should, in the aggregate, be conducive to an SATC type equilibrium, more so as the earnings gap between women and men is being closed. However, the aggregate evidence is not so clear-cut. The Bureau of Labor Statistics (2013, Table 4) reports that in the United States the marriage rate increases systematically with men's educational level, while it peaks at high school for women and then falls somewhat, but by a small amount. Furthermore, the marriage rate of women with less than a bachelor's degree is significantly higher than for men of equal education (and the gap falls with the educational level), while the difference almost vanishes for women and men with a college degree. Similarly, the marriage rate by income groups rises much more steeply with income for men than for women, being virtually flat for the latter above 45,000$, and higher for women earning less than 10,000$ than for the next bracket (Figure 1). Nevertheless we do not observe a falling marriage rate for top female earners. The divergence predicted by the SATC equilibrium is more salient if one looks at divorce rates, which tend to fall with income for men but to go up with women, with the exception of the very top income bracket

TABLE 1. Proportion never married, 35–39 years old.

Income	< 5,000$	> 100,000$
Men	44.4	9.6
Women	15.7	18.1

Source: U.S. Census 2010.

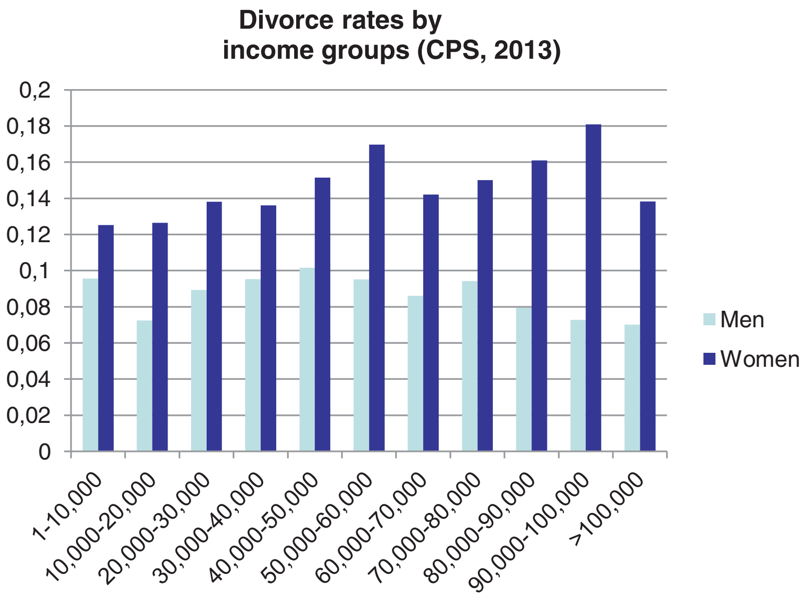

FIGURE 2. (Colour online) Divorce rates by income groups (CPS, 2013).

(Figure 2). Finally, for younger generations we do observe a greater proportion of never married women in high income groups than in low income groups[10] , while the corresponding figure is lower for men (Table 1)[11]. Overall, this evidence is suggestive that the forces analyzed in this paper are at work in the US marriage market, although the current model is not equipped to explain the lower marriage rate of female high-school dropouts.

While the evidence just discussed suggests that there may be some signs that an SATC-type equilibrium may be evolving in advanced societies – one example being the rise of single motherhood among high earning celebrities – throughout most of history many societies have imposed harsh penalties on out-of-wedlock births. At the end of Section 3, we use the model to study how such "sexual repression" affects the equilibrium. I show that a Victorian equilibrium then always exists and that the set of equilibrium bargaining shares is symmetrical and therefore not biased in favor of women[12]. Comparing such sexually repressed Victorian

equilibria with the SATC equilibrium that would prevail absent sexual repression allows to compute the distribution of gains and losses from sexual repression. I show that it unambiguously benefits beta men and harms beta women; alpha men lose while alpha women are indifferent (and would gain if they valued marital fidelity per se). Thus, societies are more likely to implement sexual repression, the more they are politically dominated by beta men[13].

A consequence of the trade-off between father's investment and good genes is that marriage does not necessarily enhance the quality of children. It increases parental investment but more children are of the less productive "beta" type. Whether marriage is beneficial for human capital accumulation depends on the productivity difference between alpha and beta types, as well as on the elasticity of a child's human capital to parental investment. These aspects are discussed in Section 4. I first study whether a particular marriage improves the children's human capital relative to the mother remaining single and mating with an alpha male. I show that the conditions are more stringent than for the marriage to just be viable; thus, for example, a marriage should be even more hypergamous than what is needed for its viability. These results clearly depend on the alphas being more productive rather than just more sexually attractive.

I then move to a general equilibrium dynamic analysis and study (in the Victorian case) the long-run distribution of human capital and genes in the marriage economy and compare it to the state of nature, and derive conditions for average steady state human capital to be larger under marriage than under the state of nature. Again, this need not always hold and will not if the productivity difference between the two genotypes is large enough. Section 5 summarizes and concludes.

This paper is related to the existing literature on marriage markets and on how this institution affects human capital accumulation. Overall, this literature has recognized that women are sellers in marriage markets either because of the sexual division of labor (Becker (1973, 1974)) or because of the role played by women in reproduction (Aiyagari et al. (2000), Edlund (2006)).[14] This paper's contribution is twofold. First, it brings back the abundance of male gametes and the existence of genetic differences in ability into the analysis, and accordingly identifies a trade-off for women as providers of legitimacy. Second, it fully analyses the consequences of that trade-off for the mating pattern and the evolution of the distribution of skills and genes[15]. Also, the assortative mating property follows a different logic than in most papers in the literature[16], as it is due to the public good aspect of children instead of complementarities between the husband's and wife's contributions[17].

2. THE MODEL

2.1. Basic Setup

At each generation, people are either male or female. They consume, produce offspring and invest in the human capital of their offspring. Generations are

non-overlapping and people only live one period, as far as their economically relevant activities are concerned.

Utility. People care about their consumption and their children's human capital. I assume the same utility for men and women:

$$U = \ln c + \gamma \sum_{i=1}^{n_c} E \ln h'_i,$$

where c is consumption, n_c the number of children, E the expectations operator, and h'_i the human capital of a child. People only care about the human capital of their true genetic offspring, and cannot transmit any other asset.

Genotypes. People differ in their genetic endowment. There are two genotypes: alpha (α) and beta (β). I will assume that the alphas have better genes in that it is easier for them to accumulate human capital (model A). People then prefer alpha offsprings, all else equal. Alternatively, though, one may assume that alpha people are more sexually attractive (model B): mating with an alpha then only yields a utility gain.[18] As long as the analysis is confined to marriage markets, the two models are equivalent. But they differ in their implications for human capital accumulation.

Production and human capital accumulation. The production structure is as follows: an individual with h units of human capital can produce Ah units of output. This can be used either to consume or to invest in the children's human capital. For an isolated individual, therefore, the budget constraint is $c + nz = Ah$, where z is the per-child investment in human capital.[19] The technology determining the offspring's human capital is then given by

$$h' = \alpha z^{\psi},$$

if the child is an alpha, and

$$h' = \beta z^{\psi},$$

if the child is a beta. We assume that $\alpha \geq \beta$ and that $0 \leq \psi \leq 1$.[20]

For simplicity, I will also assume that people invest in their children's human capital before their children's types are observed.

Mating and children. There is a perfectly competitive marriage market. Mating produces offsprings. Each intercourse produces one child. A woman can have up to n children. A man will have as many children as intercourses. I restrict the analysis to a zone where the contribution of children to utility is always positive, so that each woman will indeed have n children. I also assume that exactly $n/2$ of them are girls and $n/2$ are boys.

For simplicity, I assume that the type of a child (alpha vs. beta) only depends on the type of his or her father[21]. I assume that alpha (resp. beta) fathers sire alpha children in proportion p_α, (resp. $p_\beta \leq p_\alpha$).

In what follows, the variables pertaining to men will be denoted by a star. Hence, for example, in a couple, h will denote the wife's human capital and h^* the husband's human capital.

2.2. The State of Nature: No Marriage

I now study the equilibrium in the "State of Nature", where individuals cannot contract on their mating behavior.

In the state of nature, men do not know who their offspring are. Consequently, they are not going to invest in the human capital of children[22]. I will denote by $a = p_\alpha \ln \alpha + (1 - p_\alpha) \ln \beta$ expected TFP in human capital accumulation if the father is alpha, and similarly for betas $b = p_\beta \ln \alpha + (1 - p_\beta) \ln \beta$. The following proposition characterizes the equilibrium in the State of Nature[23].

Proposition 1

Let

$$\underline{h}(\alpha) = \exp\left[1 - \ln A - \frac{a}{\psi} + \ln \frac{1 + n\gamma\psi}{\gamma\psi}\right]. \tag{1}$$

and

$$\pi_\alpha = \gamma n a + \gamma n \psi \ln(\gamma\psi) - (1 + \gamma n \psi) \ln(1 + \gamma n \psi).$$

Assume that the initial distribution of h is such that $h \geq \underline{h}(\alpha)$.
Then, in equilibrium

(i) *No β man mates.*
(ii) *Each woman mates n times with an α man, and invests*

$$z = \frac{\gamma\psi}{1 + n\gamma\psi} Ah, \tag{2}$$

in her offspring.
(iii) *Each α -man mates n/p_α times.*
(iv) *Utilities are given by*

$$\bar{U}^*_\beta(h^*) = \ln Ah^*, \tag{3}$$

for β men,

$$\bar{U}^*_\alpha(h^*) = \ln Ah^* + \frac{\gamma n \psi}{p_\alpha} \left(E \ln h - \ln \underline{h}(\alpha) + 1\right), \tag{4}$$

for α men, and

$$\bar{U}_\alpha(h) = (1 + \gamma n \psi) \ln Ah + \pi_\alpha, \tag{5}$$

for women.
Furthermore, if

$$\ln \underline{h}(\alpha) \leq \psi - p_\alpha \ln \frac{\alpha}{\beta}. \tag{6}$$

then

(*v*) *The distribution of h remains bounded from below by* $\underline{h}(\alpha)$ *in all periods.*

Because men do not observe who their children are, they do not invest in them and consume all their endowment. Furthermore, in this regime where children deliver positive net utility, men accept all sexual intercourse. Consequently, women can select which men they mate with. Since men do not provide resources to their children, the man's human capital is irrelevant to the woman's choice. Therefore, women will choose men on the basis of their genetic characteristics only, and only mate with alphas.

The State of Nature closely matches the world described by Trivers's (1972) seminal paper: males do not invest in their offspring and there is wide dispersion in their reproductive success: alpha men potentially have many mates while beta ones are excluded from reproduction.

2.3. Marriage

I now introduce marriage into the model. Marriage is a contract by which a woman commits to have intercourse with only one man-her husband[24]. A married man knows his children are his, a property I will call *legitimacy*. Legitimacy makes it desirable for the man to invest in the children's human capital. Women benefit from marriage because there is a surplus from the match, due to children's human capital being a public good to the household. That surplus makes it possible for the man to transfer income to both his wife and his children, while remaining better-off than if he were single.

I assume commitment in marriage is perfectly enforcible. Obviously that is a simplification, given that cuckoldry and illegitimacy are not rare. Imperfect commitment could be embodied into the model by assuming that women married with beta men could cheat with alpha men, with some probability of getting caught and some penalty. To the extent that marriage still raises the probability for a man of being able to invest his own children, the qualitative properties of the model should remain, although the return to marriage should be lower than in the full enforcement case.

It is costless for a woman to marry an alpha man, since she gets the same genetic material than if she were promiscuous. By marrying, an alpha man would lose mating opportunities. But, since he would not invest in any illegitimate offspring, and since the woman knows her children are hers, imposing faithfulness to the alpha man is inefficient for the couple. Instead, it is optimal to have a *double standard* by which the man can be promiscuous outside of the couple, while the woman cannot[25]. If one assumes a double standard, marrying is also costless for alpha men. Therefore, all marriage by alpha men are efficient, and for the sake of concision I will ignore them in the analysis: In what follows the man is beta unless otherwise specified.

If the husband is beta, the woman gets a lower genetic material than if she were single and promiscuous. She needs to be compensated for that loss by the man transferring enough resources to her. As we shall see, that is possible only if the man has enough human capital both in the absolute and relative to the woman. Thus a marriage between a woman and a beta man yields a positive net surplus only if the man's human capital is high enough and if the woman's human capital is low enough.[26]

I will assume, as in e.g. Browning and Chiappori (1998), that the joint allocation of resources within a household is efficient. Then

Proposition 2

Let

$$\underline{h}(\beta) = \exp\left[1 - \ln A - \frac{b}{\psi} + \ln \frac{1 + n\gamma\psi}{\gamma\psi}\right], \tag{7}$$

$$\pi_\beta = \gamma n b + \gamma n \psi \ln(\gamma\psi) - (1 + \gamma n \psi)\ln(1 + \gamma n \psi).$$

In any household such that $h + h^* \geq \underline{h}(\beta)$,

(i) *The couple has n intercourses, producing n children*

(ii) *The investment level in children is*

$$z = \frac{\gamma\psi}{1 + \gamma n \psi} A(h + h^*), \tag{8}$$

(iii) *There exists* $\theta \in [0, 1]$ *such that the wife's and husband's consumption level respectively are*

$$c = \frac{\theta}{1 + \gamma n \psi} A(h + h^*); \tag{9}$$

$$c^* = \frac{1 - \theta}{1 + \gamma n \psi} A(h + h^*). \tag{10}$$

(iv) *The wife's and husband's utility are*

$$U_\beta(h, h^*, \theta) = \ln\theta + (1 + \gamma n \psi)(\ln A + \ln(h + h^*)) + \pi_\beta, \tag{11}$$

$$U_\beta^*(h, h^*, \theta) = \ln(1 - \theta) + (1 + \gamma n \psi)(\ln A + \ln(h + h^*)) + \pi_\beta. \tag{12}$$

Let us now analyze when marriage is beneficial relative to being single. A marriage is efficient provided there exists some sharing parameter θ such that each party can get a utility greater than their outside option. These outside options are given by the beta man's and the woman's utilities in the State of Nature (since one can always mate with an alpha men, as they accept all intercourse). The following result then holds:

FIGURE 3. The marriage frontier.

Proposition 3
Let

$$k = \pi_\alpha - \pi_\beta.$$

Let

$$\tilde{h} = A^{-1} e^{\frac{-\pi_\beta}{\gamma n \psi}}.$$

A marriage is efficient relative to both parties remaining single, if and only if the marriage viability condition holds:

$$(h + h^*)^{1+\gamma n\psi} \geq \tilde{h}^{\gamma n \psi} h^* + e^k h^{1+\gamma n \psi}. \tag{13}$$

Figure 3 depicts the marriage viability set in the (h^*, h) plane. Given h, there exists a minimum value of h^* such that the match is viable. Furthermore, that value is increasing with h. Therefore, there is hypergamy in that the match is more viable, the more skilled the man relative to the woman.

A woman with zero human capital is marriageable because an arbitrarily small consumption level is enough to compensate her for the opportunity cost of not mating with an alpha male, while her husband gets a finite benefit from legitimacy. As her human capital goes up, the consumption equivalent of foregone mating opportunities with alpha men goes up, and only men with a high enough

level of human capital are willing to transfer that amount to her in exchange for legitimacy.[27]

3. WHO MARRIES WHOM? MARRIAGE MARKETS

I now derive and discuss the equilibrium assignment for a given generation, and therefore a given distribution of human capital, denoted by $f()$. I assume that $f()$ has as a bounded support $\in [h_{\min}, h_{\max}]$. I assume there exists a perfectly competitive marriage market. For simplicity, I also assume that alpha individuals and beta individuals cannot marry each other: there is a separate marriage market for each type[28]. However, single beta women can still mate with alpha men outside marriage. Their outside option of remaining single is therefore still given by (5). Finally, I assume that

$$h_{\min} > \underline{h}(\beta),$$

implying, since $\underline{h}(\beta) \geq \underline{h}(\alpha)$, that all women will mate n times, including single ones.

3.1. Defining an Equilibrium

The following definition clarifies the candidate equilibria. Note that it rules out equilibria where some individuals get married and other identical ones do not, except over a set of measure zero.

Definition 1 *Let $f()$ be the distribution of human capital among the beta individuals, which is assumed to be the same between men and women. Let $[h_{\min}, h_{\max}]$ be the support of $f()$. An assignment is*

(*i*) *A pair of sets $S, S^* \subseteq [h_{\min}, h_{\max}]$.*
(*ii*) *A mapping[29] $h^*()$ from S to S^* such that for any measurable set $\Sigma \subseteq S$*

$$\int_{\Sigma} f(h)dh = \int_{h^*(\Sigma)} f(h^*)dh^*. \tag{14}$$

The sets S, S^* tell us the set of women and men, respectively, who are married. The mapping $h^*()$ tells us who marries whom. Condition (14) ensures that each woman marries exactly one man, so that for any set of women Σ the measure of the set of their husbands is equal to the measure of Σ.

Definition 2 *An equilibrium is a quadruple $(S, S^*, h^*(), \theta())$ such that $(S, S^*, h^*())$ is an assignment and θ is a function $: S \to [0, 1]$ such that:*

(*i*) $\forall h \in S$,

$$U_\beta(h, h^*(h), \theta(h)) \geq \bar{U}_\alpha(h). \tag{15}$$

(*ii*) $\forall h \in S$,

$$U_\beta^*(h, h^*(h), \theta(h)) \geq \bar{U}_\beta^*(h^*(h)). \quad \textbf{(16)}$$

(*iii*) *Let* $V^*(h^*) = \bar{U}_\beta^*(h^*)$ *if* $h^* \notin S^*$ *and* $V^*(h^*) = U_\beta^*(h^{*-1}(h^*), h^*, \theta(h^{*-1}(h^*)))$ *if* $h^* \in S^*$. *Let* $V(h) = \bar{U}_\alpha(h)$ *if* $h \notin S$ *and* $V(h) = U_\beta(h, h^*(h), \theta(h))$ *if* $h \in S$.
For any $h, h^* \in [h_{\min}, h_{\max}]$, *let* $\hat{\theta}(h, h^*)$ *be such that* $U_\beta(h, h^*, \hat{\theta}^*(h, h^*)) = V(h)$. *Then the following must be true:*

$$\forall h, h^* \in [h_{\min}, h_{\max}], U_\beta^*(h, h^*, \hat{\theta}(h, h^*)) \leq V^*(h^*). \quad \textbf{(17)}$$

This definition spells out the three conditions for the equilibrium assignment to be better than any deviation. Condition (i) states that married women get a higher utility than if they were single. Condition (ii) states that married men get a higher utility than if they were single. Condition (iii) that no new couple can be formed so that one party gets at least his/her reservation utility and the other gets strictly more than his/her reservation utility. The function $\theta()$ defines the equilibrium consumption share of women. This set of prices clears the market in the sense that no married individual can be better-off by being single instead (conditions (i) and (ii)) or making an acceptable offer to another mate (condition (iii)).[30,31]

3.2. Properties of an Equilibrium

We now turn to analyzing the properties of the equilibrium assignment. A natural question to be asked is: will there be sorting? Intuitively, individuals with more human capital may be willing to pay more to get a higher quality mate. This is actually true here:

Proposition 4 *Any equilibrium assignment function* $h^*(h)$ *must be nondecreasing*.

This result, which in some way is a corollary of Lam (1988), comes from the public good aspect of children's human capital in the household. Since marriage provides benefits in the form of the children's human capital, it is an increasing returns technology: when the average human capital of a couple doubles, its output, in consumption-equivalent terms, more than doubles; not only can the consumption of each member double, but the quality of the children also goes up. For this reason, people with high human capital are willing to pay more to increase their spouse's human capital than people with low human capital; the usual sorting conditions hold.

In what follows, we will be able to elicit two types of equilibria, which we now define precisely.

Definition 3 *An assignment is "Victorian" if* $S = S^* = [h_{\min}, h_{\max}]$.

Definition 4 *An assignment has the "Sex and the City" (SATC) property if there exists* $\bar{h} \leq h_{\max}$ *and* $\underline{h}^* \geq h_{\min}$ *such that* $S = [h_{\min}, \bar{h}]$ *and* $S^* = [\underline{h}^*, h_{\max}]$.

A Victorian assignment is an assignment where everybody marries. Because $h()$ is nondecreasing, women must then marry men with the same rank in the distribution of income. Given that men and women have the same initial distribution of human capital, a Victorian assignment is *homogamous* , i.e. $h^*(h) = h$.[32]

An "SATC" assignment is such that any single woman has more human capital than any married woman, while the reverse holds for men. Because $h()$ is monotonous, an SATC assignment is *hypergamous*. Women must marry men who have a higher rank than them in the distribution of income: $h^*(h) \geq h$.[33]

The following proposition shows that Victorian and SATC are the only two possible equilibrium types, provided $F()$ has full support.

Proposition 5 *Assume $F()$ has full support. Then the equilibrium assignment must be either Victorian or SATC.*

Proposition 5 implies that in any equilibrium, the singles must be found at the top of the skilled distribution for women and at the bottom for the men. If a man is better-off married than single, then all men with greater skills could also be better-off than single by marrying his wife and give her her reservation utility, which is associated with a smaller share of the now larger surplus than what she got with her original husband. This implies that $S^* = [\underline{h}^*, h_{\max}]$ for some $\underline{h}^*$. Furthermore, one can show that S must be an interval: single women who are richer than some married women and poorer than some other married ones can successfully underbid one of these two. Finally, the poorest married woman ($\underline{h}$) must marry the poorest married man ($\underline{h}^*$). But if both of them are richer than $h_{\min}$, competition from poorer single men and women must drive the surplus of their match to zero–leaving them just as well off as if they were single. But this is not sufficient since, by the hypergamy property, a marriage between $\underline{h}^*$ and $h \leq \underline{h}$ would then generate a strictly positive surplus and therefore successfully break the original marriage. Therefore, there cannot be any single woman poorer than the poorest married one.

3.3. Existence of Victorian Equilibria

Having established results regarding how any equilibrium looks like, we are now able to construct equilibria. In this section, I characterize the existence conditions for Victorian equilibria.

Proposition 6

A. A Victorian equilibrium exists if and only if

$$(e^k - 2^{\gamma n\psi})h_{\max}^{1+\gamma n\psi} \leq 2^{\gamma n\psi} h_{\min}^{1+\gamma n\psi} - \tilde{h}^{\gamma n\psi} h_{\min}, \qquad \textbf{(18)}$$

B. The corresponding assignment such that

(*i*) $S = S^* = [h_{\min}, h_{\max}]$.
(*ii*) $h^*(h) = h$.

(iii)

$$\theta(h) = \frac{1}{2}\left(1 + \lambda h^{-(1+\gamma n\psi)}\right), \tag{19}$$

where λ *is any number such that*

$$\max\left((2^{-\gamma n\psi}e^k - 1)h_{\max}^{1+\gamma n\psi}, (2^{-\gamma n\psi}e^k - 1)h_{\min}^{1+\gamma n\psi}\right)$$
$$\leq \lambda \leq \left(1 - 2^{-\gamma n\psi}\tilde{h}^{\gamma n\psi}h_{\min}^{-\gamma n\psi}\right)h_{\min}^{1+\gamma n\psi}. \tag{20}$$

Because there are no singles, the sharing rule within the household is undetermined: the location of the $\theta(h)$ profile depends on an undetermined parameter λ. Sharing rules with a higher value of λ are more favorable for women. Also, it can be shown that the interval of acceptable values of λ favors women since it is centered around a strictly positive value[34].

3.4. Existence of a "Sex and the City" Equilibrium

If (18) is violated, can we construct an SATC equilibrium? While I cannot prove existence of a marriage market equilibrium for any set of parameters (and I conjecture that for some parameters existence will fail), one can construct an SATC equilibrium if (18) is not violated by too much. This is what the next proposition says:

Proposition 7 *Assume there exists* $B \geq 0$ *such that if*

$$\frac{2^{\gamma n\psi}h_{\min}^{1+\gamma n\psi} - \tilde{h}^{\gamma n\psi}h_{\min}}{e^k - 2^{\gamma n\psi}} \leq h_{\max}^{1+\gamma n\psi} \leq \frac{2^{\gamma n\psi}h_{\min}^{1+\gamma n\psi} - \tilde{h}^{\gamma n\psi}h_{\min}}{e^k - 2^{\gamma n\psi}} + B, \tag{21}$$

then if B *is small enough,*

(i) *An SATC equilibrium exists such that* $S = [h_{\min}, \bar{h}]$ *and* $S^* = [\underline{h}^*, h_{\max}]$.

(ii) *In this equilibrium, the assignment function is*

$$h^*(h) = F^{-1}(F(h) + F(\underline{h}^*)), \tag{22}$$

implying $h^*(h_{\min}) = \underline{h}^*$. *Furthermore,* $\bar{h} = F^{-1}(1 - F(\underline{h}^*))$, *implying* $h^*(\bar{h}) = h_{\max}$.

(iii) *The married woman's share in bargaining satisfies*

$$\theta(h) = \left(h + h^*(h)\right)^{-(1+\gamma n\psi)}\left[(1+\gamma n\psi)\int_{h_{\min}}^{h}(z + h^*(z))^{\gamma n\psi}dz + \mu\right], \tag{23}$$

where μ *is a constant.*

(iv) *The equilibrium is locally unique.*

Proof – See Appendix

Thus, singleness arises at the top of the skill distribution for women, and at the bottom for men, as an outcome of competition in marriage markets. This, despite that the sex ratio is 1:1 and that all homogamous marriages might be preferred to being single.[35]

TABLE 2. Characteristics of the SATC equilibrium, for various values of $h_{\max}$

$h_{\max}$	$\underline{h}^*$	$\bar{h}$	μ	$\theta(h_{\min})$	$\theta(\bar{h})$	Share unmarried
1.326	1	1.326	3.26	0.816	0.68	0
1.5	1.06	1.44	3.44	0.816	0.65	0.11
1.6	1.09	1.51	3.55	0.816	0.64	0.14
2	1.21	1.79	3.98	0.817	0.61	0.21
2.5	1.35	2.15	4.52	0.82	0.58	0.23
3	1.48	2.5	5.08	0.823	0.57	0.24
3.8	1.69	3.1	6.01	0.828	0.55	0.25

To illustrate these results, consider the following simple numerical example: I assume $a = 2\psi, b = \psi, n = 1, \gamma = 1/\psi,\ A = 2,$ implying $k = 1, \tilde{h} = 2/e, \pi_\beta = 1 - 2\ln 2$, and $\underline{h}(\beta) = 1$. I assume that h is uniformly distributed over $[h_{\min} = 1, h_{\max}]$.

It can be checked that the condition for the Victorian equilibrium to prevail is $h_{\max} \leq \sqrt{\frac{2(1-1/e)}{e-2}} \approx 1.326$. The minimum value of λ is then $(e/2-1)h_{\max}^2$, which varies between 0.359 and $1 - 1/e = 0.632$ as $h_{\max}$ varies from 1 to 1.326. The maximum value of λ is equal to 0.632 regardless of $h_{\max}$. If λ is equal to its minimum, then the consumption share of the least skilled women is equal to $\frac{1}{2}(1 + (e/2-1)h_{\max}^2)$, which varies between 0.68 and 0.816 as $h_{\max}$ varies from 1 to 1.326. The consumption share of the most skilled women is equal to $e/4 = 0.68$, independently of $h_{\max}$. If λ is equal to its maximum, the consumption share of the least skilled women is equal to 0.816, independently of $h_{\max}$. The consumption share of the most skilled women is equal to $\frac{1}{2}(1 + (1 - 1/e)h_{\max}^{-2})$, which varies from 0.816 to 0.68 as $h_{\max}$ varies from 1 to 1.326.

For $h_{\max} \geq 1.326$, we can construct SATC equilibria – checking that all the sufficient conditions spelled out in the proof of Proposition 7 hold – as long as $h_{\max}$ remains smaller than 3.89. Thus, the range where the SATC equilibrium exists is wide; $h_{\max}$ can be well above the maximum value for which a Victorian equilibrium prevails. Table 2 reports the bounds of the set of married men and women, the constant μ, the consumption share of the most and least skilled married women, and the fraction of unmarried people, for increasing values of $h_{\max}$, starting from the critical level of 1.326.

3.5. Discussion

3.5.1. How is output being shared? In both the SATC and the Victorian equilibrium, $\theta()$ satisfies the following differential equation:

$$\frac{\theta'(h)}{\theta(h)} = \frac{1+\gamma n\psi}{h+h^*(h)}\left(\frac{1-\theta(h)}{\theta(h)} - h^{*\prime}(h)\right). \qquad \textbf{(24)}$$

This equation tells us that locally, women with an arbitrarily close level of human capital cannot profitably underbid a married woman for her husband (a similar condition for men leads to the same mathematical expression). The second term in parentheses, $h^{*\prime}(h)$, tells us that the larger the husband's human capital relative to the husbands of marginally less skilled women, the lower the share of output that this woman can get, due to competition from these women. The first term tells us that the larger a woman's output share, the smaller (more negative) $\theta'(h)$. This is because the greater her output share, the greater the incentives for marginally less skilled women to underbid her; for them to be deterred from doing that, their own output share must be higher, hence $\theta'(h)$ must be lower.

3.5.2. The role of inequality. A key property of (18) is that it is more likely to be satisfied, the greater $h_{\min}$ and the lower $h_{\max}$. Therefore, greater inequality, as defined by a larger $h_{\max}$ and/or a lower $h_{\min}$ makes it more likely that the equilibrium, if any, be of the SATC type. In other words, inequality destroys the Victorian equilibrium and therefore has an adverse effect on the number of marriages.

Let us try to provide some intuition for this result. The mechanism at work is an unraveling of marriage market competition throughout the distribution of income. If $e^k \leq 2^{\gamma n \psi}$, (18) always holds and the Victorian equilibrium always exists, regardless of $h_{\min}$ and $h_{\max}$. If $e^k \geq 2^{\gamma n \psi}$, then Proposition 6 implies that $\lambda \geq 0$, so that (i) women get more than 50% of the marriage's total consumption, and (ii) this share is lower, the greater the woman's human capital. Women get a large share of the surplus because k is large, implying that the value of the lost genetic material from mating with a beta man instead of an alpha man is large. But, as seen in Section 3.5.1, this large share of the surplus has an effect on competition between married beta people: as implied by (24), the woman's output share must be more steeply decreasing with h. Hence, if $e^k \geq 2^{\gamma n \psi}$, competition tends to reduce the share of high-skill women and to increase that of low-skill women. But, if there is too much inequality, this process will be defeated by the exit options of low-skill men who will be better-off single than transferring a large share of the surplus to their wives. And similarly, high-skill women will get too low a share of the surplus for them to get appropriate compensation for mating with a beta man. This destroys the Victorian equilibrium and triggers a transition to an SATC equilibrium.

3.5.3. The returns to human capital. While the model has no role for the returns to human capital, since people cannot change the level of h inherited from their parents, it is instructive to compute them among alternative arrangements. To do so, I compare the marginal utility of human capital for men and women at a given level of human capital for different institutions. In addition to any difference in monetary returns, there are differences between men and women in the returns to human capital expressed in terms of utility, to the extent that if men are single they do not expect to have to invest in their offsprings' human capital, while this

is not true for single women. This may help explain the recent educational gap in favor of women despite that the wage gap continues to favor men, in light of the increased prevalence of divorce rates and marital instability.[36]

State of Nature. In the state of nature, women have a greater return to human capital than men:

$$\frac{d\bar{U}_\beta^*(h^*)}{dh^*} = \frac{d\bar{U}_\alpha^*(h^*)}{dh^*} = \frac{1}{h^*};$$
$$\frac{d\bar{U}(h)}{dh} = \frac{1+\gamma n\psi}{h}.$$

This is because women invest their resources in both consumption and children, while men spend all on consumption. This suggest that if there were scope for accumulating human capital beyond what is inherited from parents, then in the state of nature women would acquire more human capital than men.

Victorian equilibrium. Let us now compute the rate of return to h in the Victorian equilibrium. For beta women, it is equal to

$$\begin{aligned}\frac{d\bar{U}_\beta}{dh} &= \frac{\theta'(h)}{\theta(h)} + \frac{1+\gamma n\psi}{h}\\ &= \frac{(1+\gamma n\psi)}{h+\lambda h^{-\gamma n\psi}}.\end{aligned}$$

Similarly, for men we get

$$\begin{aligned}\frac{d\bar{U}_\beta^*}{dh^*} &= \frac{-\theta'(h)}{1-\theta(h)} + \frac{1+\gamma n\psi}{h}\\ &= \frac{(1+\gamma n\psi)}{h-\lambda h^{-\gamma n\psi}}.\end{aligned}$$

If $\lambda \geq 0$, i.e. $\theta(h) \geq 1/2$, then men have a greater return to human capital than women. This is because by acquiring more human capital they end up marrying a woman with a smaller equilibrium share of output. The converse occurs for women. If $\lambda \leq 0$, i.e. if k is not too large, then the reverse holds: women have a greater return to human capital because in equilibrium their husband's output share falls as they climb the social ladder.

Thus in a Victorian equilibrium the only factor that may introduce a gap between men and women in terms of their incentives to acquire human capital is the indeterminate parameter λ which determines the sharing rule. If the economy coordinates on an equilibrium such that $\lambda \geq 0$, then women have a greater share of the surplus than men but this fuels competition for richer men, thus reducing the woman's share when her human capital goes up. This in turn reduces the returns to human capital for women. The converse occurs for $\lambda \leq 0$.

SATC equilibrium. The analysis is richer in the case of an SATC equilibrium. For married women, the returns to human capital come from three components:

- The effect of their own human capital on the quality of their mate, which is equal to

$$\frac{\partial U_\beta}{\partial h^*} h^{*\prime}(h) = \frac{(1+\gamma n \psi) h^{*\prime}(h)}{h + h^*(h)}.$$

  The larger $h^{*\prime}(h)$, the greater the increase in the husband's human capital when the wife's human capital goes up by one unit. Since $h^{*\prime}(h) = \frac{f(h)}{f(h^*)}$ by virtue of (22), this effect is stronger, the scarcer men are relative to women locally.
- The effect of their human capital on their output share, given by

$$\frac{\partial U_\beta}{\partial \theta} \theta'(h) = \frac{\theta'(h)}{\theta(h)}.$$

  The return to human capital is greater, the greater the increment in the woman's output share when she climbs the social ladder. As implied by (24), this effect can be further decomposed into the effect of husband's incremental human capital and the effect of marriage competition. Straightforward computations show that the former exactly cancels the $\frac{\partial U_\beta}{\partial h^*} h^{*\prime}(h)$ term, so that $h^{*\prime}(h)$ disappears from the final formula.
- Finally, there is a direct effect due to the decreasing marginal utility of human capital; this effect is equal to

$$\frac{\partial U_\beta}{\partial h} = \frac{1 + n\gamma\psi}{h + h^*(h)}.$$

Similar effects hold for men. Putting all these effects together, we get the net return to human capital for women:

$$V'(h) = \frac{1 + n\gamma\psi}{\theta(h)(h + h^*(h))}.$$

For men, the corresponding formula is

$$V^{*\prime}(h^*) = \frac{1 + n\gamma\psi}{(1 - \theta(h^{*-1}(h^*)))(h^* + h^{*-1}(h^*))}.$$

One effect tends to generate greater return to human capital for men than for women with the same skills: the former's mate has less skills than the latter's; because of hypergamy, total household human capital is smaller for men than for women with the same skills, hence the greater returns to skills for the former. This effect was not present in the homogamous Victorian equilibrium. The other effect is that of θ, which is the same as in the Victorian equilibrium. If $\theta() \geq 1/2$ then women will have a lower return to human capital because of the net effect of marriage competition. While we do not know in general whether this inequality holds, it does for the equilibria constructed in Proposition 6, because they are close to the limit Victorian equilibrium such that (18) holds exactly, and we know from Proposition 6 that $\lambda \geq 0$, i.e. $\theta() \geq 1/2$, for these equilibria.

3.6. Comparative Statics with Respect to the Assignment

In this section, I provide further results on the comparative statics of the SATC assignment constructed in Proposition 7. The following result can be proved:

Proposition 8 *Under the conditions of Proposition 7, the following comparative statics result hold:*

(i) $\dfrac{d\underline{h}^*}{dk} \geq 0; \dfrac{d\mu}{dk} \geq 0$

(ii) $\dfrac{d\underline{h}^*}{d\tilde{h}} \geq 0; \dfrac{d\mu}{d\tilde{h}} \leq 0$

(iii) *For a uniform distribution* $F()$, $\dfrac{d\underline{h}^*}{dh_{\max}} \geq 0$, $\dfrac{d\mu}{dh_{\max}} \geq 0$ *and the proportion of married people falls with* $h_{\max}$.

This result tells us that the proportion of married people will fall, and the equilibrium gap in human capital between husbands and wives rise,

- if k goes up, that is the opportunity cost of mating with a beta instead of an alpha goes up. This is natural, as an increase in k raises the outside option of celibacy for the most skilled women. For the same reason, women get a higher share of output, i.e. μ goes up.
- if $\tilde{h}$ goes up, holding other parameters constant. The parameter $\tilde{h}$ is inversely related to the hedonic value of having children with a beta father, relative to not having children. The greater $\tilde{h}$, the lower the value of children. When $\tilde{h}$ goes up, the least skilled married men find themselves better-off being single, as their willingness to pay for children is lower. As a result, too, the women's share falls, but they get assigned to richer husbands.
- if there is an increase in inequality due to a higher maximum level of human capital, in the case of a uniform distribution. This reduces the proportion of women below $\bar{h}$, but increases the proportion of men above $\underline{h}^*$. This creates an imbalance in the marriage market, which leads to an increase in the women's share of output as well as an increase in $\underline{h}^*$. The total effect on the number of marriages is negative.

3.7. Sexual Repression

We have studied the properties of the equilibrium assignment when women can exercise their choice between committing to a monogamous marriage and having children on their own. Traditionally, though, many societies put severe penalities on out-of-wedlock birth, which I will call "sexual repression". While such penalties are nonbinding if the equilibrium is Victorian, they would prevent an SATC equilibrium from arising.

In this section, I compute the equilibrium when there is sexual repression. I then compare it to an equilibrium without sexual repression: When the latter is of the SATC type, sexual repression reduces the welfare of beta women while increasing that of beta men. Alpha men lose to the extent that their mating opportunities outside marriage disappear, while alpha women are indifferent.

The analysis thus sheds light on the *political economy* of sexual repression. In a patriarchal society where men have more political power than women and where beta men are more numerous than alpha men, sexual repression is likely to arise. It allows the low-skilled men to marry and procreate, and men with greater skills get higher quality mates. Furthermore, they also get an improvement in their bargaining share as the outside option of single motherhood has disappeared for women. Hence, beta married women lose, and so do single ones.

Proposition 9 *Assume that no single woman can legally bear children. Then, there exists a Victorian equilibrium assignment such that*

(*i*) $S = S^* = [h_{\min}, h_{\max}]$.
(*ii*) $h^*(h) = h$.
(*iii*) $\theta(h) = \frac{1}{2}\left(1 + \lambda h^{-(1+\gamma n\psi)}\right)$, *where* λ *is any number such that*

$$-\left(1 - 2^{-\gamma n\psi}\tilde{h}^{\gamma n\psi}h_{\min}^{-\gamma n\psi}\right)h_{\min}^{1+\gamma n\psi} \leq \lambda \leq \left(1 - 2^{-\gamma n\psi}\tilde{h}^{\gamma n\psi}h_{\min}^{-\gamma n\psi}\right)h_{\min}^{1+\gamma n\psi}. \quad \textbf{(25)}$$

Proof – See Appendix.

This proposition tells us that under sexual repression a Victorian equilibrium always exists. Furthermore, a comparison of (25) with (20) shows that the upper bound of the admissible range for λ (which is determined by men's outside option) is the same as for a Victorian equilibrium absent sexual repression. On the other hand, the lower bound here is always negative and lower than absent sexual repression. And it is the opposite of the upper bound. Thus the bias in favor of women is eliminated: the interval for the equilibrium values of λ is now centered around zero. Sexual repression widens the range of equilibrium sharing rules and the additional values of λ are all more favorable to beta men than the ones that could prevail absent sexual repression.

Proposition 10 *Assume that in the absence of sexual repression there exists a Sex and the City equilibrium. Let* $V_S(h)$ *(resp.* $V_S^*(h^*)$*) be the utility of a beta woman (resp. beta man) with human capital* h *(resp.* h^**) in that SATC equilibrium. Let* $V_V(h;\lambda)$ *(resp.* $V_V^*(h^*;\lambda)$*) be the utility of a beta woman (resp. beta man) in a Victorian equilibrium with sexual repression and sharing parameter* λ. *Then*

$$\forall\lambda \in [-\left(1 - 2^{-\gamma n\psi}\tilde{h}^{\gamma n\psi}h_{\min}^{-\gamma n\psi}\right)h_{\min}^{1+\gamma n\psi}, \left(1 - 2^{-\gamma n\psi}\tilde{h}^{\gamma n\psi}h_{\min}^{-\gamma n\psi}\right)h_{\min}^{1+\gamma n\psi}],$$

$$\forall h \in [h_{\min}, h_{\max}],\ V_V(h;\lambda) \leq V_S(h),\ \textit{and}$$

$$\forall h^* \in [h_{\min}, h_{\max}],\ V_V^*(h^*;\lambda) \geq V_S^*(h^*).$$

Proof – See Appendix.

This proposition tells us that even if we compare the sexually repressed Victorian equilibrium which is most favorable to women to the SATC equilibrium, the latter leaves them better-off, while men are better-off in the former.

Intuitively, if one were to start from an SATC equilibrium and introduce sexual repression, highly skilled single women would not be able to have children and it would be profitable for them to underbid the most skilled married women. This

process would trickle down throughout the distribution of skills, thus reducing the bargaining share of all women, while at the same time reassigning a husband with lower skills to each married woman. For these reasons all beta women lose, while all beta men gain for symmetrical reasons.

This result sheds light on the observation that patriarchal societies where men have disproportional political power compared to women tend to be associated with sexual repression. Such provisions both ensure that each beta man can get a wife while at the same time eliminating the source of the asymmetry between men and women in bargaining, namely the women's natural possibility to get their own illegitimate children from a mate they pick. Conversely, the introduction of women's suffrage in the West during the twentieth century has been followed by a relaxation of regulations and social norms that penalized single parenthood.[37]

4. THE DYNAMICS OF HUMAN CAPITAL ACCUMULATION

We now provide some results about the effect of marriage on the dynamics of human capital accumulation. The results clearly depend on whether mating with an alpha man enhances the children's human capital as assumed above (model A) or whether the gratification is purely hedonic (model B).

In model A, I show that marriage improves human capital accumulation provided the productivity difference between the two types is not too large. Otherwise, marriage will reduce the level of human capital in the long run despite that all marriages are voluntary and viable. I also show that imposing marriage as a social norm, for example in the form of sexual repression, further reduces the level of human capital accumulation.

In model B, however, marriage has unambiguous positive effects on human capital accumulation, since the mate's type has no impact on the innate ability of the children, who always get higher parental investment under marriage.

I first analyze the effects of marriage on the children's human capital for a given couple in partial equilibrium, and then provide some results on aggregate dynamics in general equilibrium.

4.1. Partial Equilibrium Analysis

One key aspect of marriage is that two parents, rather than one, now invest in the children's human capital. In the state of nature, parental investment is equal to $z = \frac{\gamma\psi}{1+\gamma n\psi} Ah$; in the matrimonial society, it is equal to $z = \frac{\gamma\psi}{1+\gamma n\psi} A(h + h^*)$. Because the child's human capital is a public good to his/her parents, establishing a link between fathers and their children works like a free lunch and may make all parties better-off. The woman, however, bears the opportunity cost of not mating with an alpha male. If she did instead of marrying, the resulting expected human capital of children would be equal to

$$E \ln h'_S = \psi(\ln A + \ln h) + a + \psi \ln(\gamma\psi) - \psi \ln(1 + \gamma n\psi).$$

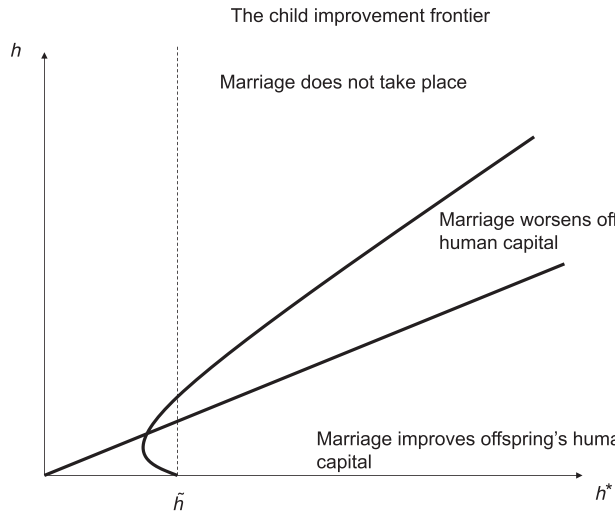

FIGURE 4. The child improvement frontier.

This is to be compared with its counterpart if the woman marries a beta man:

$$E \ln h'_{M\beta} = \psi(\ln A + \ln(h + h^*)) + b + \psi \ln(\gamma\psi) - \psi \ln(1 + \gamma n\psi).$$

We get that marriage improves the children's human capital, i.e. $E \ln h'_{M\beta} \geq E \ln h'_S$, if and only if

$$h^* \geq h(e^{k/(\gamma n\psi)} - 1). \qquad \textbf{(26)}$$

It is easy to see that if $h \geq \tilde{h}$, this condition is stronger than (13) – See Figure 4. There are marriages that are preferable to being single but yield a lower human capital to the children than if the woman mated with an alpha man instead. The additional investment in children is not enough to compensate for the poorer genetic material. This is possible because the mother gets a higher consumption level due to the "free lunch" aspect of legitimacy discussed above (while the father still gets the direct benefit of legitimacy). Marriage not only boosts investment in children but has consumption benefits as well.

To summarize: While marriage always boosts investment in children, it does not necessarily boost their human capital in model A, because of the implied reduction in the father's genetic quality. Human capital goes up if the marriage is sufficiently hypergamous or if the father is alpha.

4.2. The Effect of Marriage on Aggregate Human Capital Accumulation in the Long-Run

I now study the effect of marriage on human capital accumulation in the economy as a whole, comparing human capital accumulation in the State of Nature with a marital economy in a Victorian equilibrium.[38]

4.2.1. Aggregate human capital accumulation in the State of Nature. A first step is to characterize aggregate human capital accumulation in the State of Nature. This is easy, provided average human capital is defined in logarithms. Assume the conditions of Proposition 1 hold. We get from (2):

$$\begin{aligned} E \ln h' &= a + \psi E \ln z \\ &= \psi (E \ln h - \ln \underline{h}(\alpha) + 1). \end{aligned} \quad \textbf{(27)}$$

Thus, average log human capital converges to $\psi(1 - \ln \underline{h}(\alpha))/(1 - \psi)$.

4.2.2. Aggregate human capital accumulation in an economy with marriage. We now turn to the marital economy. An important technical step is to ensure that a marriage market equilibrium exists at all dates. To do so, we construct a Victorian equilibrium by checking that the inherited distribution of skills at each date satisfies the conditions of Proposition 6. As long as that is granted, it is straightforward to characterize the evolution of the economy's average human capital, as well as its genetic composition, and compare it to the state of nature.

Proposition 11 *Let*

$$h^{LR}_{\max,\alpha} = \exp\left[\frac{\psi(1 + \ln 2 - \ln \underline{h}(\alpha)) + (1 - p_\alpha)\ln \frac{\alpha}{\beta}}{1 - \psi}\right];$$

$$h^{LR}_{\max,\beta} = \exp\left[\frac{\psi(1 + \ln 2 - \ln \underline{h}(\beta)) + (\psi - p_\beta)\ln \frac{\alpha}{\beta}}{1 - \psi}\right]$$

$$h^{LR}_{\min,\beta} = \exp\left[\frac{\psi(1 + \ln 2 - \ln \underline{h}(\beta)) - p_\beta \ln \frac{\alpha}{\beta}}{1 - \psi}\right].$$

Assume that

$$\ln \underline{h}(\beta) \leq \psi(1 + \ln 2) - p_\beta \ln \frac{\alpha}{\beta}. \quad \textbf{(28)}$$

Assume that the support of the initial distribution of human capital for alpha (resp. beta) people is contained in $[h^{LR}_{\min,\beta}, h^{LR}_{\max,\alpha}]$ (resp. $[h^{LR}_{\min,\beta}, h^{LR}_{\max,\beta}]$). Assume (18) holds at $h_{\max} = h^{LR}_{\max,\beta}$ and $h_{\min} = h^{LR}_{\min,\beta}$. Let ρ_t be the proportion of alpha individuals. Then

(*i*) *There exists a path for the economy where the marriage market equilibrium is Victorian for both the alphas and the betas at each date.*
(*ii*) *Along this path ρ_t evolves according to*

$$\rho_{t+1} = p_\beta(1 - \rho_t) + p_\alpha \rho_t. \tag{29}$$

(*iii*) *The average log human capital of this economy, defined as $E \ln h_t$, evolves according to*

$$E \ln h_{t+1} = \psi[E \ln h_t + 1 + \ln 2 - \rho_t \ln \underline{h}(\alpha) - (1 - \rho_t) \ln \underline{h}(\beta)]. \tag{30}$$

The following proposition establishes conditions for such a Victorian equilibrium to deliver a higher average level of human capital.

Proposition 12 *Assume the conditions of Proposition 11 hold. Then*

(*i*) *the Victorian equilibrium has a greater offspring expected log human capital than the State of Nature at date t if and only if*

$$\psi \ln 2 - (1 - \rho_t)\left[p_\alpha - p_\beta\right] \ln \frac{\alpha}{\beta} \geq 0.$$

(*ii*) *The Victorian equilibrium steady state has a greater expected log human capital than the State of Nature steady state if and only if*

$$\psi \ln 2 \geq (p_\alpha - p_\beta) \frac{1 - p_\alpha}{1 - p_\alpha + p_\beta} \ln \frac{\alpha}{\beta}.$$

(*iii*) *At any given date, the Victorian equilibrium has a greater expected log human capital for the betas than the State of Nature if and only if*

$$\psi \ln 2 \geq \left[p_\alpha - p_\beta\right] \ln \frac{\alpha}{\beta}. \tag{31}$$

This is equivalent to (26) for $h^* = h$.

Proposition 12 suggests that marriage boosts society's aggregate human capital if

(i) The alphas are not too different from the betas in terms of the likelihood of getting an alpha offspring, or
(ii) The alpha's productivity in accumulating human capital is not too different from the betas', or
(iii) The proportion of alphas is sufficiently large.[39]

The condition for marriage to increase the human capital of the betas has the same qualitative properties but is more stringent, since one now ignore the alpha marriages that always enhance the offspring's human capital, since they involve no loss of genetic material.

4.3. Can Sexual Repression Enhance Human Capital?

The penalization of out-of-wedlock birth has been prevalent in most civilizations throughout history. The political economy analysis offered in Section 3.7 may

help to explain why that is the case. Another issue is whether sexual repression benefits future generations by improving their average human capital. If so, we might expect societies where it prevails to grow faster and potentially eliminate societies where it does not prevail.

Remember that sexual repression works by forcing a Victorian equilibrium upon a society that would otherwise be in an SATC equilibrium. This has two conflicting effects on the offsprings' human capital:

- Married women have children with greater parental investment, and therefore more human capital, in the SATC equilibrium. This is because they marry a man with better skills than in the Victorian assignment.
- Unmarried women have lower parental investment but their children are better endowed genetically.

Despite that, we can establish a non-ambiguous result:

Proposition 13 *Assume an SATC equilibrium exists absent sexual repression. For any woman with human capital* h, *let* $E_{SATC} \ln h'(h)$ *(resp.* $E_V \ln h'(h)$*) be her offspring's average log human capital in the SATC equilibrium (resp. in the sexually repressed Victorian assignment). Then*

$$\forall h \in [h_{\min}, h_{\max}],\ E_{SATC} \ln h'(h) \geq E_V \ln h'(h).$$

Again, this result would be overturned if the value of alpha men were purely hedonic (model B), in which case sexual repression would unambiguously enhance human capital accumulation.

5. SUMMARY AND CONCLUSION

By bringing fathers into the family, marriage allows to increase parental investment in children. But, for this to be credibly operational, monogamy must be enforced. As a result, women lose the opportunity of choosing more attractive mates.

Most of the results derive from this trade-off. Hypergamy arises from the fact that women must be compensated for the utility loss associated with the foregone mating opportunities. Assortative mating arises even though there are no complementarities between the skills of the two members of the couple, due to the public good aspect of children's human capital, which generates increasing returns to skills in the household.

The institution of marriage reduces the genetic quality of offspring, with that reduction being compensated by greater parental investment. As a result, a marital society does not necessarily imply greater human capital than the State of nature. As in Saint-Paul (2007), this is an example of institutions increasing the frequency of less fit genes as they provide alternative means of achieving fitness. But this result would be overturned if one holds the view that the mates with the best genes are not more productive but only more sexually attractive, in which case the marital society unambiguously achieves greater human capital.

Another key result is that inequality in skills in some sense intensifies competition in marriage markets and leads to "SATC" equilibria where a pool of single women arises at the top, while a corresponding pool of single men emerges at the bottom of the distribution.

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APPENDIX A

A.1. Proof of Proposition 1

Conditional on mating, a woman will always prefer an α, since she gets a higher utility for any parental investment. This proves (i). As a result, the maximization problem of women is

$$\begin{aligned}&\max_{c,\upsilon,z} \ln c + \gamma \nu (p_\alpha \ln(\alpha z^{\psi}) + (1 - p_\alpha) \ln(\beta z^{\psi})),\\ &s.t.\ c + \upsilon z \leq Ah,\\ &0 \leq \upsilon \leq n.\end{aligned}$$

Writing the Lagrangian as

$$\ln c + \gamma \nu (p_\alpha \ln(\alpha z^{\psi}) + (1 - p_\alpha) \ln(\beta z^{\psi})) + \lambda (Ah - c - \upsilon z) + \mu (n - \upsilon) + \varepsilon \upsilon,$$

it is easy to see that the full fertility regime ($\mu \geq 0$) prevails iff

$$a + \psi \ln z \geq \psi. \tag{32}$$

In this case, the solution is given by

$$\upsilon = n \tag{33}$$

$$z = \frac{\gamma \psi}{1 + \gamma n \psi} Ah; \tag{34}$$

$$c = \frac{1}{1 + \gamma n \psi} Ah. \tag{35}$$

Consequently, (32) holds iff

$$a + \psi\,(\ln A + \ln h) + \psi\,\ln(\gamma\psi) - \psi\,\ln(1 + \gamma n\psi) \geq \psi. \tag{36}$$

Clearly, from this, the minimum level of human capital for a woman to be in this regime is $\underline{h}$, as defined in (1). Therefore, all women are willing to mate n times with an α man as long as they all have more human capital than $\underline{h}$.

Next, a beta offspring with a mother of human capital h has a human capital given by βz^{ψ}. This is larger than $\underline{h}$ iff

$$\ln\beta + \psi\,(\ln A + \ln h) + \psi\,\ln(\gamma\psi) - \psi\,\ln(1 + \gamma n\psi) \geq \ln\underline{h}. \tag{37}$$

If the distribution of human capital is above $\underline{h}$, a sufficient condition for the distribution of human capital among offsprings to have that same property is that (37) holds at $h = \underline{h}$. Rearranging using (1), we see that this is equivalent to (6). This proves that (v) holds if the two assumptions in the Proposition hold. To check that (i) and (ii) hold, we need to check that it is rational for alpha men to mate. If they do so, their utility is clearly given by

$$\bar{U}^*_{\alpha}(h^*) = \ln A + \ln h^* + \frac{n}{p_{\alpha}} E\ln h'. \tag{38}$$

This is because the proportion of alpha men in the total population of men is p_{α}, therefore each alpha man mates n/p_{α} times. For this to be larger than utility without mating, it must be that $E\ln h \geq 0$. Clearly, the expected human capital of an offspring condition on the mother's human capital h is given by

$$\begin{aligned} E(\ln h' \mid h) &= a + \psi\ln z \\ &= a + \psi\,(\ln A + \ln h) + \psi\,\ln(\gamma\psi) - \psi\,\ln(1 + \gamma n\psi) \\ &= \psi(\ln h - \ln\underline{h} + 1) \geq 0. \end{aligned}$$

This proves that $E\ln h \geq 0$ and therefore that it is also in the interest of alpha men to mate. This proves claims (ii) and (iii). From the preceding formula we get that $E\ln h' = \psi(E\ln h - \ln\underline{h} + 1)$, which, once substituted into (38), proves (4). Equation (3) holds straightforwardly from (i). Equation (5) holds from substituting the optimal policies (33)–(35) into the woman's utility function. This proves (iv).

A.2. Proof of Proposition 2

The household maximizes

$$\max_{c,c^*,\upsilon,z} \theta(\ln c + \gamma\upsilon E\ln h') + (1-\theta)(\ln c^* + \gamma\upsilon E\ln h'),$$

for some $\theta \in [0, 1]$. The budget constraint for the couple is now

$$c + c^* + \nu z \leq A(h + h^*).$$

Furthermore

$$E\ln h' = b + \psi\ln z.$$

The Lagrangian is

$$L = \theta(\ln c + \gamma \upsilon E \ln h') + (1-\theta)(\ln c^* + \gamma \upsilon E \ln h') \\ + \lambda \left(A(h+h^*) - \left(c + c^* + \nu z \right) \right) + \mu(n-\upsilon) + \varepsilon \upsilon.$$

Looking at the FOCs, a regime such that $\upsilon = n$ arises iff (8)–(35) holds and if $\mu \geq 0$, i.e. $\partial L/\partial \upsilon \geq 0$, or equivalently

$$\gamma(b + \psi \ln z) \geq \lambda z.$$

From $\lambda = \theta/c$ and (8)–(35), one can check that this inequality is equivalent to $h + h^* \geq \underline{h}(\beta)$. One can then prove (11)–(12) by simple substitution of the optimal policies into the utility function.

A.3. Proof of Proposition 3

Confronting (11) with (5), we see that for the woman to gain from marriage she must at least get a share $\theta_{\min}$ of consumption, where

$$\theta_{\min} = e^k \left(1 + \frac{h^*}{h} \right)^{-(1+\gamma n \psi)}.$$

The match is efficient if the man's utility $U^*_\beta(h, h^*, \theta)$, computed at $\theta = \theta_{\min}$, is greater than the man's outside option, which is given by (3). That defines the condition under which the marriage takes place:

$$1 - \theta_{\min} \geq \tilde{h}^{\gamma n \psi} h^* (h + h^*)^{-(1+\gamma n \psi)}.$$

Substituting the value of $\theta_{\min}$, we get Equation (13).

A.4. Proof of Proposition 4

Suppose it's not. Then we can find two married couples, (h_0, h_1^*) and (h_1, h_0^*), such that $h_0 \leq h_1$ and $h_0^* = h^*(h_1) \leq h_1^* = h^*(h_0)$. Let $\theta_0 = \theta(h_0)$ and $\theta_1 = \theta(h_1)$. For this assignment to be an equilibrium, condition (iii) in Definition 2 must hold. Let us apply it for $h^* = h_1^*$ and $h = h_1$. Using (11), we see that

$$\ln \hat{\theta}(h_1, h_1^*) = \ln \theta_1 - (1 + \gamma n \psi)(\ln(h_1 + h_1^*) - \ln(h_1 + h_0^*)). \tag{39}$$

Using (12) and (17), we see that we must have

$$\ln(1 - \hat{\theta}(h_1, h_1^*)) \leq \ln(1 - \theta_0) + (1 + \gamma n \psi)(\ln(h_1^* + h_0) - \ln(h_1^* + h_1)).$$

Substituting (39), we get that the following inequality must hold:

$$(h_1 + h_1^*)^{1+\gamma n \psi} \leq (1 - \theta_0)(h_1^* + h_0)^{1+\gamma n \psi} + \theta_1 (h_1 + h_0^*)^{1+\gamma n \psi}.$$

If we now apply condition (iii) to $h^* = h_0^*$ and $h = h_0$, we get a similar condition

$$(h_0 + h_0^*)^{1+\gamma n \psi} \leq (1 - \theta_1)(h_1 + h_0^*)^{1+\gamma n \psi} + \theta_0 (h_1^* + h_0)^{1+\gamma n \psi}.$$

Adding these two inequalities, we get that the following must hold

$$(h_1 + h_1^*)^{1+\gamma n\psi} + (h_0 + h_0^*)^{1+\gamma n\psi} \leq (h_1 + h_0^*)^{1+\gamma n\psi} + (h_1^* + h_0)^{1+\gamma n\psi}. \quad \textbf{(40)}$$

However, the strict convexity of the function $\psi(x) = x^{1+\gamma n\psi}$ precludes it. Let $\mu = \frac{h_1 - h_0}{h_1 - h_0 + h_1^* - h_0^*} \in [0, 1]$. One has $h_1 + h_0^* = \mu(h_1 + h_1^*) + (1 - \mu)(h_0 + h_0^*)$, and $h_1^* + h_0 = (1 - \mu)(h_1 + h_1^*) + \mu(h_0 + h_0^*)$. Therefore, $\psi(h_1^* + h_0) \leq (1 - \mu)\psi(h_1 + h_1^*) + \mu\psi(h_0 + h_0^*)$ and $\psi(h_1 + h_0^*) \leq \mu\psi(h_1 + h_1^*) + (1 - \mu)\psi(h_0 + h_0^*)$. Adding these two inequalities, we clearly contradict (40).

A.5. Proof of Proposition 5

The first step is to show that if a man is married, all men with greater human capital must also be married. To see this, consider a man with human capital h_0^*, married with a woman with human capital h_0. Let $\theta_0 = \theta(h_0)$ be her corresponding equilibrium output share. A man with human capital $h_1^* \geq h_0^*$ can marry her if he gives her a share $\hat{\theta}(h_0, h_1^*)$ such that $U_\beta(h_0, h_1^*, \hat{\theta}(h_0, h_1^*)) = V(h_0) = U_\beta(h_0, h_0^*, \theta_0)$, or equivalently, using (11),

$$\ln \hat{\theta}(h_0, h_1^*) = \ln \theta_0 + (1 + \gamma n\psi)\left[\ln(h_0 + h_0^*) - \ln(h_0 + h_1^*)\right]. \quad \textbf{(41)}$$

If h_1^* is single, then he must not be better-off by marrying h_0 and offering her an output share equal to $\hat{\theta}(h_0, h_1^*)$; otherwise, (17) would be violated. Therefore, we must have $U_\beta^*(h_0, h_1^*, \hat{\theta}(h_0, h_1^*)) \leq V(h_1^*) = \bar{U}_\beta^*(h_1^*)$, or equivalently, using (12) and (3),

$$\ln(1 - \hat{\theta}(h_0, h_1^*)) \leq \gamma n\psi \ln \tilde{h} + \ln h_1^* - (1 + \gamma n\psi)\ln(h_0 + h_1^*). \quad \textbf{(42)}$$

Substituting (41) into (42), we see that the following inequality must hold:

$$(h_0 + h_1^*)^{1+\gamma n\psi} - \theta_0(h_0 + h_0^*)^{1+\gamma n\psi} \leq \tilde{h}^{\gamma n\psi} h_1^*. \quad \textbf{(43)}$$

At the same time, h_0^* must be better-off married with h_0 than single, otherwise (16) would be violated. Using (12) and (3), this is equivalent to

$$(h_0 + h_0^*)^{1+\gamma n\psi} - \theta_0(h_0 + h_0^*)^{1+\gamma n\psi} \geq \tilde{h}^{\gamma n\psi} h_0^*. \quad \textbf{(44)}$$

Putting together (44) and (43), we see that the following inequality must hold:

$$(h_0 + h_0^*)^{1+\gamma n\psi} - \tilde{h}^{\gamma n\psi} h_0^* \geq (h_0 + h_1^*)^{1+\gamma n\psi} - \tilde{h}^{\gamma n\psi} h_1^*. \quad \textbf{(45)}$$

Observe that $\underline{h}(\beta) = \tilde{h}\exp(1 - \ln(1 + \gamma n\psi)/\gamma n\psi) \geq \tilde{h}$, and that the expression $(h_0 + x)^{1+\gamma n\psi} - \tilde{h}^{\gamma n\psi} x$ is strictly increasing in x over the relevant range provided $h_{\min} \geq \frac{1}{2}(1 + \gamma n\psi)^{-\gamma n\psi}\tilde{h}$, which is clearly implied by our assumption that $h_{\min} \geq \underline{h}(\beta)$. Therefore, (45) cannot hold for $h_1^* \geq h_0^*$. Therefore, h_1^* must be married too. Consequently, it must be that $S^* = [\underline{h}^*, h_{\max}]$.

Next, we show that the inverse assignment function $h^{*-1}()$ must be continuous over S^*. Suppose it is not the case. Since it is monotonic, the set of its discontinuity points is at most countable. Then there exists some $h_0^* \in \mathring{S}$ such that $h_1 = \lim_{h^* \to h_0^{*+}} h^{*-1}() \geq h^{*-1}(h_0^*) = h_0$.[40] Then, all women in (h_0, h_1) must be single. Furthermore, a man with human capital h_0^* must be indifferent between marrying a woman with human capital h_0 or a woman with human capital (arbitrarily close to) h_1, since he prefers the former but a

man with an arbitrarily close human capital to his prefers the latter. Denoting $\theta_0 = \theta(h_0)$ and $\theta_1 = \lim_{h \to h_1^+} \theta(h)$, this can be written as

$$\ln(1 - \theta_1) + (1 + \gamma n\psi)\ln(h_0^* + h_1) = \ln(1 - \theta_0) + (1 + \gamma n\psi)\ln(h_0^* + h_0). \tag{46}$$

Another equilibrium condition is that all women such that $h \in (h_0, h_1)$ could not be better-off if they married h_0^*. The woman's output share that would leave him indifferent between marrying h_0 or h_1 and marrying h is $\hat{\theta}^*(h, h_0^*)$ such that

$$\ln(1 - \hat{\theta}^*(h, h_0^*)) = (1 + \gamma n\psi)\ln(h_0^* + h_0) + \ln(1 - \theta_0) - (1 + \gamma n\psi)\ln(h_0^* + h). \tag{47}$$

That a woman with $h \in (h_0, h_1)$ prefers to be single than marrying h_0^* under these terms can be written as

$$k + (1 + \gamma n\psi)\ln h \geq \ln \hat{\theta}^*(h, h_0^*) + (1 + \gamma n\psi)\ln(h_0^* + h). \tag{48}$$

Note that $\hat{\theta}^*(h_0, h_0^*) = \theta_0$ and $\hat{\theta}^*(h_1, h_0^*) = \theta_1$. Taking limits in (48) for $h \to h_0$ and $h \to h_1$ and noting that a woman with $h = h_0$ or h arbitrarily close to h_1 is married in equilibrium and thus not worse-off than single, we see that (48) must hold with equality at the bounds of (h_0, h_1), i.e.

$$k + (1 + \gamma n\psi)\ln h_0 = \ln \theta_0 + (1 + \gamma n\psi)\ln(h_0^* + h_0); \tag{49}$$

$$k + (1 + \gamma n\psi)\ln h_1 = \ln \theta_1 + (1 + \gamma n\psi)\ln(h_0^* + h_1). \tag{50}$$

Using (49) and (50) to eliminate θ_0 and θ_1 in (46), we get that

$$(h_0^* + h_1)^{1+\gamma n\psi} - e^k h_1^{1+\gamma n\psi} = (h_0^* + h_0)^{1+\gamma n\psi} - e^k h_0^{1+\gamma n\psi}.$$

Let $\phi(h)$ be the function defined by $\phi(h) = (h_0^* + h)^{1+\gamma n\psi} - e^k h^{1+\gamma n\psi}$. It is easy to see that $\phi'(h)$ is positive and then negative as h goes from zero to infinity. Since $\phi(h_0) = \phi(h_1)$, $\phi()$ must be hump-shaped between h_0 and h_1, implying that

$$\phi(h) \geq \phi(h_0) = \phi(h_1) \text{ for } h \in (h_0, h_1).$$

But, substituting (47) into (48), we see that we must also have

$$(h + h_0^*)^{1+\gamma n\psi} - e^k h^{1+\gamma n\psi} \leq (1 - \theta_0)(h_0^* + h_0)^{1+\gamma n\psi}.$$

Substituting again the value of θ_0 from (49), we see that this is equivalent to

$$(h + h_0^*)^{1+\gamma n\psi} - e^k h^{1+\gamma n\psi} = \phi(h) \leq \phi(h_0) = (h_0 + h_0^*)^{1+\gamma n\psi} - e^k h_0^{1+\gamma n\psi},$$

which is clearly a contradiction. Therefore, the inverse assignment function must be continuous, implying that S is an interval.

Let $\underline{h}$ be the lower bound of S. It must be that $h^*(\underline{h}) = \underline{h}^*$. If $\underline{h}^* = h_{\min}$, then all men are married, so must all women, and one must have $S = [h_{\min}, h_{\max}]$. The equilibrium is then Victorian. Assume then that $\underline{h}^* > h_{\min}$. Assume $\underline{h} > h_{\min}$. Then, all women such that $h \leq \underline{h}$ are single. We can use similar steps as the ones used to derive (47)–(50) to show that $\underline{h}$ is just indifferent between being married and single, i.e.

$$k + (1 + \gamma n\psi)\ln \underline{h} = \ln \theta(\underline{h}) + (1 + \gamma n\psi)\ln(\underline{h}^* + \underline{h}).$$

By the same token, $\underline{h}^*$ is also indifferent between being married and single, that is

$$\ln(1-\theta(\underline{h})) + (1+\gamma n\psi)\ln(\underline{h}^* + \underline{h}) = \gamma n\psi \ln \tilde{h} + \ln \underline{h}^*.$$

Putting these two conditions together, we see that the marriage viability condition (13) must be satisfied with equality at $h = \underline{h}$ and $h^* = \underline{h}^*$. But this implies that it is satisfied strictly for any $h < \underline{h}$ and $h^* = \underline{h}^*$. Therefore, a woman with human capital $h < \underline{h}$ can underbid $\underline{h}$ to marry $\underline{h}^*$ and give him a positive surplus, meaning that condition (iii) in Definition 2 must be violated. Hence, it cannot be that $\underline{h} > h_{\min}$, implying that if $\underline{h}^* > h_{\min}$ the equilibrium must be SATC.

A.6. Proof of Proposition 6

Assume (18) holds. We show that a Victorian assignment along with a sharing rule defined by (19) matches all equilibrium conditions in Definition 2. Let us start with condition (iii). Since all women are married, their reservation utility is given by

$$V(h) = \ln\ \theta(h) + (1+\gamma n\psi)(\ln A + \ln(h + h^*(h))) + \pi_\beta. \tag{51}$$

By marrying another man with human capital h^* and get a fraction θ of consumption, their utility would be given by (11). Therefore, the consumption share that would make them indifferent between their marriage and this alternative marriage is given by

$$\ln \hat{\theta}(h, h^*) = \ln \theta(h) + (1+\gamma n\psi)(\ln(h + h^*(h)) - \ln(h + h^*)). \tag{52}$$

The new husband utility is now

$$U^*_\beta(h, h^*, \hat{\theta}(h, h^*)) = \ln\ (1 - \hat{\theta}(h, h^*)) + (1+\gamma n\psi)(\ln A + \ln(h + h^*)) + \pi_\beta.$$

It must not exceed the utility he had in his assigned marriage

$$V^*(h^*) = \ln(1 - \theta(h^{*-1}(h^*))) + (1+\gamma n\psi)(\ln A + \ln(h^{*-1}(h^*) + h^*)) + \pi_\beta. \tag{53}$$

Using (52) to eliminate $\hat{\theta}(h, h^*)$, and rearranging, we see that the condition $U^*_\beta(h, h^*, \hat{\theta}(h, h^*)) \le V^*(h^*)$ is equivalent to

$$(h + h^*)^{1+\gamma n\psi} \le \theta(h)(h + h^*(h))^{1+\gamma n\psi} + (1 - \theta(h^{*-1}(h^*)))(h^{*-1}(h^*) + h^*)^{1+\gamma n\psi}, \tag{54}$$

and, since the actual equilibrium assignment defines $V^*(h^*)$, this will hold with equality for $h^* = h^*(h)$.

In our candidate equilibrium, we have $h^*(h) = h$. The preceding formula can simply be rewritten as

$$(h + h^*)^{1+\gamma n\psi} \le \theta(h)(2h)^{1+\gamma n\psi} + (1 - \theta(h^*))(2h^*)^{1+\gamma n\psi}.$$

Suppose now that we have $\theta(h) = \frac{1}{2}\left(1 + \lambda h^{-(1+\gamma n\psi)}\right)$, this boils down to

$$\left(\frac{h + h^*}{2}\right)^{1+\gamma n\psi} \le \frac{h^{1+\gamma n\psi} + h^{*\,1+\gamma n\psi}}{2},$$

which is true by convexity. This proves that our candidate equilibrium satisfies (iii) in Definition 2.

Let us now check condition (i), i.e. that women are better-off married than single. Comparing (51) and (5), we see that the condition $V(h) \geq \bar{U}_\alpha(h)$ is equivalent to

$$\theta(h) \geq e^k 2^{-(1+\gamma n\psi)}.$$

Given (19), this is equivalent to

$$\lambda \geq \max(\left(e^k 2^{-\gamma n\psi} - 1\right) h_{\min}^{1+\gamma n\psi}, \left(e^k 2^{-\gamma n\psi} - 1\right) h_{\max}^{1+\gamma n\psi}).$$

This defines the set of values of λ for which (i) in Definition 2 holds.

Turning now to condition (ii), using (53) and (3), and the fact that $h^{*-1}(h^*) = h^*$, the condition $V^*(h^*) \geq \bar{U}_\beta^*(h^*)$ is equivalent to

$$1 - \theta(h^*) \geq 2^{-(1+\gamma n\psi)} \tilde{h}^{\gamma n\psi} h^{*-\gamma n\psi}. \tag{55}$$

If $\lambda \geq 0$ then the LHS goes up with h^*; since the RHS falls with h^*, then for this to hold for all h^* it must hold for $h^* = h_{\min}$. We get the condition that

$$\lambda \leq \left(1 - 2^{-\gamma n\psi} \tilde{h}^{\gamma n\psi} h_{\min}^{-\gamma n\psi}\right) h_{\min}^{1+\gamma n\psi}. \tag{56}$$

Note that the RHS to this inequality is always positive.

If $\lambda \leq 0$ then the LHS of (55) is always greater than 0.5, and therefore always exceeds the RHS since $h^* \geq \underline{h}(\beta) \geq \tilde{h}$.

Summarizing all these findings, we see that there exist values of λ which satisfy (i) and (ii) if and only if (18) holds – which is always the case if $e^k \leq 2^{\gamma n\psi}$. The relevant interval of values of λ is then clearly defined by (20).

The preceding steps imply that if (18) holds, then a Victorian equilibrium exists. Furthermore, the claims of part B hold by construction. We now prove that those conditions are necessary.

We have seen that for the equilibrium condition (87) to hold (see footnote 30), it must be that (54) holds. We also know that (54) holds with equality at $h^* = h^*(h)$. Thus, $h^*(h)$ must be a local extremum of the RHS of (54) minus its LHS, as a function of h^*. Locally, this means that:

$$\begin{aligned}(1 + \gamma n\psi)&(h + h^*(h))^{\gamma n\psi} \\ &= (1 - \theta(h))(1 + \gamma n\psi)(h + h^*(h))^{\gamma n\psi} \times \left(1 + h^{*\prime}(h)^{-1}\right) \\ &\quad - (h + h^*(h))^{1+\gamma n\psi} \frac{\theta'(h)}{h^{*\prime}(h)}.\end{aligned} \tag{57}$$

In a Victorian equilibrium $h^*(h) = h$, and this simplifies to

$$\theta'(h) = \frac{1 + \gamma n\psi}{2h}(1 - 2\theta(h)).$$

The solution to this differential equation is given by (19), which is therefore a necessary condition for a Victorian equilibrium. We then have already seen that (20) is necessary for (i) and (ii) in Definition 2 to be satisfied, and that this interval of values of λ is nonempty if and only if (18) holds. This proves that (18) is necessary and that the conditions in B necessarily hold.

A.7. Proof of Proposition 7

A. Constructing the assignment for the SATC equilibrium

We now show that an SATC equilibrium can be constructed. For this, we look for a pair $(\underline{h}^*, \bar{h})$ such that $\underline{h}^* \geq h_{\min}$, $\bar{h} \leq h_{\max}$, and the assignment given by

$$h^*(h) = F^{-1}(F(h) + F(\underline{h}^*)), \tag{58}$$

is an equilibrium one. Clearly, given $\bar{h}$, if we choose

$$\underline{h}^* = F^{-1}(1 - F(\bar{h})) = \underline{h}^*(\bar{h}), \tag{59}$$

the candidate assignment will map $S = [h_{\min}, \bar{h}]$ to $S^* = [\underline{h}^*, h_{\max}]$ and satisfy (14). Therefore, it is indeed an assignment:

- We have proved that given any $\bar{h}$, the value of $\underline{h}^*$ given by (59) and the $h^*()$ function defined by (58) are an assignment.

B. Checking that married people cannot underbid one another

Next, let us assume that the sharing function $\theta(h)$ satisfies (23). We show that (iii) in Definition 2 holds for $h \in S$ and $h^* \in S^*$. As shown in the proof of Proposition 6, this is equivalent to (54). Substituting (23), we get that this is equivalent to

$$(h + h^*)^{1+\gamma n\psi} \leq (h^{*-1}(h^*) + h^*)^{1+\gamma n\psi} + (1 + \gamma n\psi) \int_{h^{*-1}(h^*)}^{h} (z + h^*(z))^{\gamma n\psi} dz, \tag{60}$$

for all $h, h^* \in S \times S^*$. Clearly, equality holds for $h^* = h^*(h)$. Furthermore, the derivative of the RHS of (60) with respect to h^* is $(1 + \gamma n\psi)(h^{*-1}(h^*) + h^*)^{\gamma n\psi}$, while the derivative of the LHS is $(1 + \gamma n\psi)(h + h^*)^{\gamma n\psi}$. Given that $h^*(h)$ is increasing, the former is clearly larger than the latter for $h^* \geq h^*(h)$, and smaller for $h^* \leq h^*(h)$. Consequently, the difference between the RHS and the LHS reaches it minimum at $h^* = h^*(h)$; hence (60) holds.

- We have proved that if $\theta(h)$ satisfies (23), then condition (iii) holds for $(h, h^*) \in S \times S^*$.

*C. Deriving the value-matching conditions at the frontier of S and S**

Next, we show that there exist values for μ, $\bar{h}$ and $\underline{h}^*$ such that, in addition to (59), the two following conditions hold:

$$\bar{U}_\alpha(\bar{h}) = U_\beta(\bar{h}, h_{\max}, \theta(\bar{h})); \tag{61}$$

$$\bar{U}^*_\beta(\underline{h}^*) = U^*_\beta(h_{\min}, \underline{h}^*, \theta(h_{\min}). \tag{62}$$

These two conditions mean that the reservation utilities $V()$ and $V^*()$ do not jump as one crosses the boundaries of S and S^*. Otherwise, the equilibrium conditions would be violated. Suppose, for example, that a woman such that h is marginally higher than $\bar{h}$ has a utility higher than $U_\beta(\bar{h}, h_{\max}, \theta(\bar{h}))$ by a discrete amount. Then, since the $\theta()$ function is continuous over S, women with h below $\bar{h}$ but arbitrarily close to it would be better-off being single, and condition (i) in Definition 2 would be violated. Suppose now that a woman with h marginally higher than $\bar{h}$ has a utility lower than $U_\beta(\bar{h}, h_{\max}, \theta(\bar{h}))$ by a discrete amount. Then $\hat{\theta}(h, h_{\max}) \leq \theta(\bar{h})$: since these women are arbitrarily close to $\bar{h}$, but have a

discretely lower utility than the married women with $\bar{h}$, they can reach that same utility by marrying a man with $h_{\max}$ and get a lower fraction of the surplus. But the $h_{\max}$ man would then be better-off and this would violate condition (iii). Therefore, (61) must hold. A similar reasoning applies to (62). Using (5) and (11), we see that (61) is equivalent to

$$\ln\theta(\bar{h}) = k + (1+\gamma n\psi)\ln\bar{h} - (1+\gamma n\psi)\ln(\bar{h}+h_{\max}).$$

Substituting (23), we see that this is equivalent to

$$\mu = e^k\bar{h}^{1+\gamma n\psi} - (1+\gamma n\psi)\int_{h_{\min}}^{\bar{h}}(z+h^*(z))^{\gamma n\psi}dz = \mu_H(\bar{h}). \quad \textbf{(63)}$$

Similarly, we can substitute (3) and (12) into (62) and get

$$\ln(1-\theta(h_{\min})) = -\gamma n\psi\ln A - \pi_\beta + \ln\underline{h}^* - (1+\gamma n\psi)\ln(h_{\min}+\underline{h}^*),$$

or equivalently, given (23),

$$\mu = (h_{\min}+\underline{h}^*)^{1+\gamma n\psi} - \tilde{h}^{\gamma n\psi}\underline{h}^* = \mu_L(\bar{h}). \quad \textbf{(64)}$$

Equations (65) and (64) define a 2×2 system in $\bar{h}$ and μ, where $\underline{h}^*$ is implicitly treated as a function of $\bar{h}$ defined by (59).

- We have proved that μ and $\bar{h}$ must satisfy (63) and (64) in equilibrium.

D. Showing that there is a solution, for $\mu, \bar{h}, \underline{h}^*$which satisfies the value-matching conditions as well as condition (iii) in Definition 2 for pairs of singles

To prove that it has a solution, we use the intermediate value theorem. First, we show that $\mu_H(h_{\max}) \geq \mu_L(h_{\max})$. If $\bar{h} = h_{\max}$, then $h^*(h) = h$. Therefore, $\mu_H(h_{\max}) = e^k h_{\max}^{1+\gamma n\psi} - 2^{\gamma n\psi}\left(h_{\max}^{1+\gamma n\psi} - h_{\min}^{1+\gamma n\psi}\right)$, and $\mu_L(h_{\max}) = 2^{1+\gamma n\psi}h_{\min}^{1+\gamma n\psi} - \tilde{h}^{\gamma n\psi}h_{\min}$. Clearly, the condition $e^k h_{\max}^{1+\gamma n\psi} - 2^{\gamma n\psi}\left(h_{\max}^{1+\gamma n\psi} - h_{\min}^{1+\gamma n\psi}\right) \geq 2^{1+\gamma n\psi}h_{\min}^{1+\gamma n\psi} - \tilde{h}^{\gamma n\psi}h_{\min}$ is equivalent to (18) being violated, which is true by assumption.

Next, let $\hat{h}$ be the minimum possible value of $\bar{h}$ such that condition (iii) in Definition 2 holds for $h \geq \bar{h}$ and $h^* \leq \underline{h}^*$. The threshold $\hat{h}$ is such that a marriage between the least skilled single woman and the most skilled single man is barely viable, i.e.

$$(\hat{h}+\underline{h}^*(\hat{h}))^{1+\gamma n\psi} = \tilde{h}^{\gamma n\psi}\underline{h}^*(\hat{h}) + e^k\hat{h}^{1+\gamma n\psi}. \quad \textbf{(65)}$$

Observe that (59) implies that $\underline{h}^{*\prime}() \leq 0$, while (65) states that $(\underline{h}^*(\hat{h}), \hat{h})$ lies on the upward sloping marriage viability frontier. Thus, (65) holds for at most one value of $\hat{h}$. Furthermore, at $\hat{h} = h_{\max}$, we have that $\underline{h}^*(h_{\max}) = h_{\min}$ and that

$$(h_{\max}+h_{\min})^{1+\gamma n\psi} < \tilde{h}^{\gamma n\psi}h_{\min} + e^k h_{\max}^{1+\gamma n\psi}. \quad \textbf{(66)}$$

To see this, note that since (18) is violated by assumption, we have that

$$e^k h_{\max}^{1+\gamma n\psi} > 2^{\gamma n\psi}h_{\min}^{1+\gamma n\psi} + 2^{\gamma n\psi}h_{\max}^{1+\gamma n\psi} - \tilde{h}^{\gamma n\psi}h_{\min}.$$

Clearly, the inequality

$$2^{\gamma n\psi}h_{\min}^{1+\gamma n\psi} + 2^{\gamma n\psi}h_{\max}^{1+\gamma n\psi} > (h_{\max}+h_{\min})^{1+\gamma n\psi},$$

holds. This proves that (66) holds and therefore that the LHS of (65) is smaller than its RHS at $\hat{h} = h_{\max}$. Next, for B small enough, at $\hat{h} = h_{\min}$, we have that $\underline{h}^*(h_{\min}) = h_{\max}$ and that

$$(h_{\max} + h_{\min})^{1+\gamma n\psi} > \tilde{h}^{\gamma n\psi} h_{\max} + e^k h_{\min}^{1+\gamma n\psi}. \tag{67}$$

To see this, note that if (18) holds with equality, then

$$e^k h_{\max}^{1+\gamma n\psi} = 2^{\gamma n\psi} h_{\min}^{1+\gamma n\psi} + 2^{\gamma n\psi} h_{\max}^{1+\gamma n\psi} - \tilde{h}^{\gamma n\psi} h_{\min},$$

or equivalently

$$\begin{aligned} e^k - 2^{\gamma n\psi} &= 2^{\gamma n\psi} \frac{h_{\min}^{1+\gamma n\psi} - \tilde{h}^{\gamma n\psi} h_{\min}}{h_{\max}^{1+\gamma n\psi}} \\ &< 2^{\gamma n\psi} - \tilde{h}^{\gamma n\psi} h_{\min}^{-\gamma n\psi}, \end{aligned}$$

where the inequality comes from the fact that $h_{\min} > \underline{h}(\beta) \geq \tilde{h}$, hence the numerator in the fraction is ≥ 0. This inequality is then equivalent to

$$e^k h_{\min}^{1+\gamma n\psi} \leq 2^{\gamma n\psi+1} h_{\min}^{1+\gamma n\psi} - \tilde{h}^{\gamma n\psi} h_{\min}. \tag{68}$$

Observe that the function $(x + h_{\min})^{1+\gamma n\psi} - \tilde{h}^{\gamma n\psi} x$ is increasing with x for $x \geq 0$ since $h_{\min} > \tilde{h}$, which implies that (67) holds since (68) does. Since (67) holds if (18) holds with equality, by continuity it will also hold if it is marginally violated, i.e. if B is small enough.

Together, (66) and (67) imply that $\hat{h}$ exists and satisfies $h_{\min} \leq \hat{h} \leq h_{\max}$.

We now show that $\mu_H(\hat{h}) \leq \mu_L(\hat{h})$. Substituting (65) into (63), we see that this is equivalent to

$$(\hat{h} + \underline{h}^*(\hat{h}))^{1+\gamma n\psi} - (1+\gamma n\psi) \int_{h_{\min}}^{\hat{h}} (z + h^*(z))^{\gamma n\psi} dz \leq (h_{\min} + \underline{h}^*(\hat{h}))^{1+\gamma n\psi}.$$

This inequality always holds.[41] Thus, by the intermediate value theorem, there exists a solution to (63)–(64) such that $\hat{h} \leq \bar{h} \leq h_{\max}$. Since $\bar{h} \geq \hat{h}$, by construction, no single woman in this solution wants to marry a single men; hence condition (iii) in Definition 2 holds for pairs of singles.

- We have proved that there exists a pair $(\mu, \bar{h})$ such that (63)–(64) hold and that condition (iii) in Definition 2 holds for $(h, h^*) \in [h_{\min}, h_{\max}] - S \times [h_{\min}, h_{\max}] - S^*$.

E. Checking that the constructed solution satisfies (iii) for a single woman and a married man

Another requirement is that condition (iii) hold for $h \geq \bar{h}$ and $h^* \in S^*$. Using (5) and (11), we must have

$$\ln \hat{\theta}(h, h^*) = (1+\gamma n\psi)(\ln h - \ln(h + h^*)) + k. \tag{69}$$

Using (12), we see that (17) is equivalent to

$$\begin{aligned} &\ln(1 - \hat{\theta}(h, h^*)) + (1+\gamma n\psi)(\ln A + \ln(h + h^*)) + \pi_\beta \\ &\leq V^*(h^*) \\ &= \ln(1 - \theta(h^{*-1}(h^*))) + (1+\gamma n\psi)(\ln A + \ln(h^* + h^{*-1}(h^*))) + \pi_{\beta.} \end{aligned} \tag{70}$$

Substituting (69) and (23), we see that this is equivalent to

$$\mu \leq (h^* + h^{*-1}(h^*))^{1+\gamma n\psi} - (1+\gamma n\psi)\int_{h_{\min}}^{h^{*-1}(h^*)} (z + h^*(z))^{\gamma n\psi} dz \tag{71}$$
$$+e^k h^{1+\gamma n\psi} - (h+h^*)^{1+\gamma n\psi}$$
$$= \phi(h, h^*).$$

Since (63) holds by construction, we have that $\phi(\bar{h}, h_{\max}) = \mu$. Furthermore,

$$\frac{\partial \phi}{\partial h} = (1+\gamma n\psi)\left[e^k h^{\gamma n\psi} - (h+h^*)^{\gamma n\psi}\right].$$

Therefore, $\frac{\partial \phi}{\partial h} \geq 0$ if and only if $e^k \geq (1+\frac{h^*}{h})^{\gamma n\psi}$. Let us assume that

$$e^k \geq (1+\frac{h_{\max}}{\bar{h}})^{\gamma n\psi}. \tag{72}$$

It must then be that $e^k \geq (1+\frac{h^*}{h})^{\gamma n\psi}$ for all $h^* \in S^*$ and for all $h \geq \bar{h}$. Consequently, $\phi(h, h^*) \geq \phi(\bar{h}, h^*)$.

Furthermore, since $\bar{h} \in S$, (17) is equivalent to (54) for $(\bar{h}, h^*)$, and we already know that from part B of this proof that (54) holds for $(\bar{h}, h^*)$. Using (23) and rearranging, we see that this is equivalent to

$$(\bar{h} + h^*)^{1+\gamma n\psi} \leq (1+\gamma n\psi)\int_{h^{*-1}(h^*)}^{\bar{h}} (z+h^*(z))^{\gamma n\psi} dz + (h^* + h^{*-1}(h^*))^{1+\gamma n\psi};$$

but inspection of (71) and making use of (63) shows that this condition is equivalent to $\phi(\bar{h}, h^*) \geq \mu$. Therefore, $\phi(h, h^*) \geq \mu$.

Hence, condition (72) is sufficient for (iii) in Definition 2 to hold for $h^* \in S^*$ and $h \geq \bar{h}$. Furthermore, if condition (18) holds with equality, we have that $e^k \geq 2^{\gamma n\psi}$,and in this limit case the solution to (63)–(64) is $\bar{h} = h_{\max}$. Condition (72) then strictly holds. By continuity, if $h_{\max}$ is such that (18) is not violated by too much, i.e. B in (21) is not too large, then (72) will hold.

- We have proved that we can choose B such that the values of μ and $\bar{h}$ constructed in D are such that condition (iii) in Definition 2 holds for $(h, h^*) \in [h_{\min}, h_{\max}] - S \times S^*$.

F. Checking that the constructed solution satisfies (i)

The condition that married women are better-off than if they were single can be written

$$\ln\theta(h) + (1+\gamma n\psi)\ln(h + h^*(h)) \geq (1+\gamma n\psi)\ln h + k, \forall h \leq h^*, \tag{73}$$

or equivalently using the formula for $\theta(h)$:

$$\mu \geq e^k h^{1+\gamma n\psi} - (1+\gamma n\psi)\int_{h_{\min}}^{h} (z+h^*(z))^{\gamma n\psi} dz = \phi(h). \tag{74}$$

Again, (63) implies that it holds with equality at $h = \bar{h}$. Furthermore, $\phi'(h) = (1+\gamma n\psi)(e^k h^{\gamma n\psi} - (h+h^*(h))^{\gamma n\psi})$. We have $\phi'(h) \geq 0$ if and only if $h^*(h)/h \leq e^{\frac{k}{\gamma n\psi}} - 1$. This is again true in the limit case where (18) holds with equality, since we then have $h^*(h) = h$ and $e^{\frac{k}{\gamma n\psi}} \geq 2$. Therefore, in this limit equilibrium we have $\phi(h) \leq \phi(\bar{h})$. By

continuity, this remains true if (18) is not violated by too much. Then (74) holds, and condition (i) in Definition 2 is satisfied.

- We have proved that we can choose B such that, in addition to the properties spelled out above, condition (i) in Definition 2 holds.

G. Checking that the constructed solution satisfies (iii) for a single man and a married woman

We now prove that (iii) holds when $h^* \leq \underline{h}^*$ and $h \leq \bar{h}$. Here, it is more convenient to use the alternative formulation defined in footnote 30. Using (3) and (12) allows us to compute $\hat{\theta}^*(h, h^*)$:

$$\hat{\theta}^*(h, h^*) = 1 - \frac{\tilde{h}^{\gamma n \psi} h^*}{(h + h^*)^{1+\gamma n \psi}}. \tag{75}$$

Comparing (11) for $\theta = \hat{\theta}^*(h, h^*)$ and for $\theta = \theta(h)$ and $h^*(h)$ instead of h^*, we see that the condition in footnote 30 holds if and only if

$$\hat{\theta}^*(h, h^*) \leq \theta(h) \frac{(h + h^*(h))^{1+\gamma n \psi}}{(h + h^*)^{1+\gamma n \psi}}.$$

Substituting (23) and (75), we get the following condition

$$(h + h^*)^{1+\gamma n \psi} - \tilde{h}^{\gamma n \psi} h^* \leq \mu + (1 + \gamma n \psi) \int_{h_{\min}}^{h} (z + h^*(z))^{\gamma n \psi} dz. \tag{76}$$

Note that this holds with equality for $h = h_{\min}$ and $h^* = \underline{h}^*$, by virtue of (64). Next, note that the LHS is an increasing function of h^*. Therefore, (76) holds for all $h^* \leq \underline{h}^*$ if and only if it holds for $h^* = \underline{h}^*$.˜Next, note that the derivative of the RHS with respect to h is $(1 + \gamma n \psi)(h + h^*(h))^{\gamma n \psi}$, while the derivative of the LHS with respect to h at $h^* = \underline{h}^*$ is $(1 + \gamma n \psi)(h + \underline{h}^*)^{\gamma n \psi}$. The former is clearly larger than the latter since $h^*(h) \geq \underline{h}^*$. Therefore, the difference between the RHS of (76) and its LHS at $h^* = \underline{h}^*$ is an increasing function of h. Since (76) holds with equality for $h = h_{\min}$ and $h^* = \underline{h}^*$, it also holds for any $h \geq h_{\min}$ and $h^* = \underline{h}^*$. As we have already seen, that in turn implies that it holds for any $h \geq h_{\min}$ and $h^* \leq \underline{h}^*$. This completes the proof that (iii) holds for single men underbidders.

- We have proved that the values of μ and $\bar{h}$ constructed in D are such that condition (iii) in Definition 2 holds for $(h, h^*) \in S \times [h_{\min}, h_{\max}] - S^*$.

H. Proof that (ii) holds for the constructed solution

The last thing we have to check is that (ii) holds, that is, married men are better-off than if they were single. Denoting by h the wife of a married man and by $h^*(h)$ this man, we see that this is equivalent to

$$\ln A + \ln h^*(h) \leq \ln(1 - \theta(h)) + (1 + \gamma n \psi) \left(\ln A + \ln(h + h^*(h)) \right) + \pi_\beta.$$

Substituting in (23), we see that this is equivalent to

$$\mu \leq (h + h^*(h))^{1+\gamma n \psi} - \tilde{h}^{\gamma n \psi} h^*(h) - (1 + \gamma n \psi) \int_{h_{\min}}^{h} (z + h^*(z))^{\gamma n \psi} dz. \tag{77}$$

Again, this holds with equality for $h = h_{\min}$, because of (64). Furthermore, the RHS's derivative with respect to h is equal to $h^{*\prime}(h) \left[(1 + \gamma n \psi)(h + h^*(h))^{\gamma n \psi} - \tilde{h}^{\gamma n \psi} \right]$, which

is clearly positive since $h^*()$ is increasing and $h + h^*(h) \geq \underline{h}(\beta) \geq \tilde{h}$. Therefore, the RHS of (77) is an increasing function of h and is always greater for $h \geq h_{\min}$ than for $h = h_{\min}$, where it holds with equality. Hence, (77) always holds:

- We have proved that the values of μ and $\bar{h}$ constructed in D are such that condition (ii) in Definition 2 holds.

This completes the proof of Proposition 7.

A.8. Proof of Proposition 8

The discussion in D in the proof of proposition 7 implies that we can always pick an equilibrium such that locally, the RHS of (63) as a function of $\underline{h}^*$ is flatter than that of (64). It is then clear that a rise in k shifts the RHS of (63) up, and has no effect on (64). Therefore, both $\underline{h}^*$ and μ go up, which proves claim (i). Similarly, a greater $\tilde{h}$ reduces the RHS of (64), with no effect on (63), so that $\underline{h}^*$ goes up again while μ falls. This proves claim (ii).

Finally, note that $h_{\max}$ does not enter in (64) and that for a uniform distribution, (63) is equivalent to

$$\mu = e^k (h_{\max} - \delta)^{1+\gamma n\psi} - 2^{\gamma n\psi} \left[(h_{\max} - \delta/2)^{1+\gamma n\psi} - (h_{\min} + \delta/2)^{1+\gamma n\psi} \right], \quad \textbf{(78)}$$

where $\delta = \underline{h}^* - h_{\min}$, and $h^*(h) = h + \delta$. The constructed equilibrium is such that (72) holds. This also implies that the RHS of (78) is increasing in $h_{\max}$, holding $\underline{h}^*$ or equivalently δ constant. Consequently, a greater $h_{\max}$ raises the RHS of (63), so that the equilibrium values of μ and $\underline{h}^*$ go up. The proportion of married people is $1 - \frac{\delta}{h_{\max} - h_{\min}}$, and it must go down. It falls iff $\frac{d\delta}{dh_{\max}} \geq \frac{\delta}{h_{\max} - h_{\min}}$, which is true if δ is small enough, which is true in the constructed equilibrium of Prop. 7.

A.9. Proof of Proposition 9

Since the formula for $\theta(h)$ is the same as in Proposition 6, it is clear that (54) is still satisfied by our candidate equilibrium and therefore that equilibrium condition (iii) in Definition 2 holds.

The utility of a beta woman outside marriage is now given by $\bar{U}^*_\beta(h)$; like beta men, they cannot have children and are therefore in a symmetrical situation. Using (51) and (3), we see that condition (i) in Definition 2 holds if and only if

$$\ln(1 + \lambda h^{-(1+\gamma n\psi)}) + \gamma n\psi \ln h \geq -\gamma n\psi \ln 2 + \gamma n\psi \ln \tilde{h}, \forall h \epsilon [h_{\min}, h_{\max}].$$

This condition always holds for $\lambda \geq 0$ since $h \geq \tilde{h}$. For $\lambda \leq 0$, the LHS is clearly an increasing function of h, so (i) holds provided the above holds for $h = h_{\min}$, that is $\lambda \geq -\left(1 - 2^{-\gamma n\psi} \tilde{h}^{\gamma n\psi} h_{\min}^{-\gamma n\psi}\right) h_{\min}^{1+\gamma n\psi}$, which defines the lower bound of (25).

For men, the proof that (ii) holds if $\lambda \leq \left(1 - 2^{-\gamma n\psi} \tilde{h}^{\gamma n\psi} h_{\min}^{-\gamma n\psi}\right) h_{\min}^{1+\gamma n\psi}$ is the same as in the proof of Proposition 6.

A.10. Proof of Proposition 10

We compare utility in the two types of equilibria for all agents. In all what follows, $h^*(h)$ refers to the assignment function in the SATC equilibrium, since in the Victorian equilibrium we can readily replace it by h.

We start with women who are married in the SATC equilibrium. Using (11) for both equilibria, we see that $V_S(h) \geq V_V(h;\lambda)$ if and only if

$$\ln\theta_S(h) + (1+\gamma n\psi)\ln(h+h^*(h)) \geq \ln\theta_V(h;\lambda) + (1+\gamma n\psi)\ln(2h), \tag{79}$$

where $\theta_S(h)$ and $\theta_V(h;\lambda)$ are the appropriate shares, i.e.

$$\theta_V(h,\lambda) = \frac{1}{2}(1+\lambda h^{-(1+\gamma n\psi)}),$$
$$\theta_S(h) = \left(h+h^*(h)\right)^{-(1+\gamma n\psi)}\left[(1+\gamma n\psi)\int_{h_{\min}}^{h}(z+h^*(z))^{\gamma n\psi}dz + \mu\right].$$

Condition (79) is equivalent to

$$(1+\gamma n\psi)\int_{h_{\min}}^{h}(z+h^*(z))^{\gamma n\psi}dz + \mu \geq 2^{\gamma n\psi}(\lambda + h^{1+\gamma n\psi}). \tag{80}$$

Clearly, if it holds for the maximum possible value of λ, it must hold for any equilibrium value of λ. Hence, substituting $\lambda = \left(1-2^{-\gamma n\psi}\tilde{h}^{\gamma n\psi}h_{\min}^{-\gamma n\psi}\right)h_{\min}^{1+\gamma n\psi}$ into (80), we get

$$\mu \geq 2^{\gamma n\psi}h_{\min}^{1+\gamma n\psi} - \tilde{h}^{\gamma n\psi}h_{\min} + 2^{\gamma n\psi}h^{1+\gamma n\psi} - (1+\gamma n\psi)\int_{h_{\min}}^{h}(z+h^*(z))^{\gamma n\psi}dz. \tag{81}$$

By differentiating the last two terms with respect to h, we see that together they are a decreasing function of h. Therefore the preceding inequality will hold for all h iff it holds for $h_{\min}$, that is

$$\mu \geq 2^{\gamma n\psi+1}h_{\min}^{1+\gamma n\psi} - \tilde{h}^{\gamma n\psi}h_{\min}.$$

Subsituting (64), which must hold in any SATC equilibrium, we get

$$(h_{\min}+\underline{h}^*)^{1+\gamma n\psi} - \tilde{h}^{\gamma n\psi}\underline{h}^* \geq 2^{\gamma n\psi+1}h_{\min}^{1+\gamma n\psi} - \tilde{h}^{\gamma n\psi}h_{\min}.$$

Noting that there is equality at $\underline{h}^* = h_{\min}$ and that the LHS is increasing with $\underline{h}^*$, we conclude that this always holds. Consequently, (81) holds for all $h \leq \bar{h}$. Therefore, (80) holds for all λ and $h \leq \bar{h}$. Hence married beta women have a greater utility under the SATC equilibrium than under the Victorian one.

We next consider single women. Comparing (5) and (11), we get that $V_S(h) \geq V_V(h;\lambda)$ iff

$$(1+\gamma n\psi)\ln h + \pi_\alpha \geq \ln\theta_V(h) + (1+\gamma n\psi)\ln(2h),$$

or equivalently

$$e^k 2^{-\gamma n\psi} \geq 1 + \lambda h^{-(1+\gamma n\psi)}. \tag{82}$$

Again, if this inequality holds for the maximum equilibrium value of λ, it will hold for any of them. Furthermore, at this maximum λ, which is positive, the RHS is decreasing

in h, so that the inequality holds for all $h \geq \bar{h}$ iff it holds for $h = \bar{h}$. Substituting both $\lambda = \left(1 - 2^{-\gamma n\psi}\tilde{h}^{\gamma n\psi}h_{\min}^{-\gamma n\psi}\right)h_{\min}^{1+\gamma n\psi}$ and $h = \bar{h}$ into (82) we get, after rearranging:

$$e^k\bar{h}^{1+\gamma n\psi} \geq 2^{\gamma n\psi}\bar{h}^{1+\gamma n\psi} + h_{\min}^{1+\gamma n\psi}2^{\gamma n\psi} - \tilde{h}^{\gamma n\psi}h_{\min}.$$

Substituting (63) and then (64), this is equivalent to

$$\begin{aligned}(h_{\min} + \underline{h}^*)^{1+\gamma n\psi} - \tilde{h}^{\gamma n\psi}\underline{h}^* + (1+\gamma n\psi)\int_{h_{\min}}^{\bar{h}}(z + h^*(z))^{\gamma n\psi}dz \\ \geq 2^{\gamma n\psi}\bar{h}^{1+\gamma n\psi} + h_{\min}^{1+\gamma n\psi}2^{\gamma n\psi} - \tilde{h}^{\gamma n\psi}h_{\min}.\end{aligned} \quad \mathbf{(83)}$$

Since $(h_{\min} + \underline{h}^*)^{1+\gamma n\psi} - \tilde{h}^{\gamma n\psi}\underline{h}^* \geq h_{\min}^{1+\gamma n\psi}2^{1+\gamma n\psi} - \tilde{h}^{\gamma n\psi}h_{\min}$, a sufficient condition for (83) to hold is

$$\begin{aligned}h_{\min}^{1+\gamma n\psi}2^{1+\gamma n\psi} - \tilde{h}^{\gamma n\psi}h_{\min} + (1+\gamma n\psi)\int_{h_{\min}}^{\bar{h}}(z + h^*(z))^{\gamma n\psi}dz \\ \geq 2^{\gamma n\psi}\bar{h}^{1+\gamma n\psi} + h_{\min}^{1+\gamma n\psi}2^{\gamma n\psi} - \tilde{h}^{\gamma n\psi}h_{\min},\end{aligned}$$

that is

$$\begin{aligned}(1+\gamma n\psi)\int_{h_{\min}}^{\bar{h}}(z + h^*(z))^{\gamma n\psi}dz \\ \geq 2^{\gamma n\psi}(\bar{h}^{1+\gamma n\psi} - h_{\min}^{1+\gamma n\psi}).\end{aligned}$$

This inequality holds since the RHS equals the LHS for $\bar{h} = h_{\min}$, while differentiation shows that the LHS increases faster with $\bar{h}$ than the RHS. Therefore, (82) holds for the highest λ and for $\bar{h}$, i.e. for any equilibrium λ and all $h \geq \bar{h}$. Thus single women prefer the SATC equilibrium as well.

We now turn to married men. Using the same steps as for married women, we get that they prefer the Victorian equilibrium iff

$$(1+\gamma n\psi)\int_{h_{\min}}^{h^{*-1}(h^*)}(z + h^*(z))^{\gamma n\psi}dz + \mu \geq 2^{\gamma n\psi}(\lambda - h^{*1+\gamma n\psi}) + (h^{*-1}(h^*) + h^*)^{1+\gamma n\psi}.$$

Again, this holds for all equilibrium values of λ if it holds for the largest one, $\lambda = \left(1 - 2^{-\gamma n\psi}\tilde{h}^{\gamma n\psi}h_{\min}^{-\gamma n\psi}\right)h_{\min}^{1+\gamma n\psi}$. Substituting, we get

$$\mu \geq 2^{\gamma n\psi}h_{\min}^{1+\gamma n\psi} - \tilde{h}^{\gamma n\psi}h_{\min} + \phi(h^{*-1}(h^*)), \quad \mathbf{(84)}$$

where $\phi(h) \equiv (h + h^*(h))^{1+\gamma n\psi} - 2^{\gamma n\psi}h^*(h)^{1+\gamma n\psi} - (1+\gamma n\psi)\int_{h_{\min}}^{h}(z + h^*(z))^{\gamma n\psi}dz$. Now, note that $\phi'(h) = h^{*\prime}(h)(1+\gamma n\psi)\left[(h + h^*(h))^{\gamma n\psi} - 2^{\gamma n\psi}h^*(h)^{\gamma n\psi}\right] \leq 0$. Thus, (84) holds for all $h^* \geq \underline{h}^*$ iff it holds at $h^* = \underline{h}^*$, that is

$$\mu \geq 2^{\gamma n\psi}h_{\min}^{1+\gamma n\psi} - \tilde{h}^{\gamma n\psi}h_{\min} + (h_{\min} + \underline{h}^*)^{1+\gamma n\psi} - 2^{\gamma n\psi}\underline{h}^{*1+\gamma n\psi}.$$

Substituting (64), we see that this is equivalent to

$$2^{\gamma n\psi}\left(\underline{h}^{*1+\gamma n\psi} - h_{\min}^{1+\gamma n\psi}\right) \geq \tilde{h}^{\gamma n\psi}(\underline{h}^* - h_{\min}). \quad \mathbf{(85)}$$

As the RHS equates the LHS for $\underline{h}^* = h_{\min}$, and as the LHS increases more with $\underline{h}^*$ than the RHS, this inequality clearly holds, which proves that married beta men are better-off in the sexually repressed Victorian equilibrium than in the SATC equilibrium regardless of λ.

We finally consider the case of single men. Using (3) and (12) and substituting $\theta = \theta_V(h^*, \lambda) = \frac{1}{2}(1 + \lambda h^{*-(1+\gamma n\psi)})$, then rearranging, we see that they prefer the Victorian equilibrium iff

$$\lambda \leq h^{*1+\gamma n\psi} - 2^{-\gamma n\psi}\tilde{h}^{\gamma n\psi}h^*.$$

This holds for the maximum λ, $\left(1 - 2^{-\gamma n\psi}\tilde{h}^{\gamma n\psi}h_{\min}^{-\gamma n\psi}\right)h_{\min}^{1+\gamma n\psi}$, iff

$$h_{\min}^{1+\gamma n\psi} - 2^{-\gamma n\psi}\tilde{h}^{\gamma n\psi}h_{\min} \leq h^{*1+\gamma n\psi} - 2^{-\gamma n\psi}\tilde{h}^{\gamma n\psi}h^*.$$

This is clearly satisfied since the RHS grows with h^* and is equal to the LHS at $h^* = h_{\min}$. Therefore single men also prefer the Victorian outcome. This completes the proof of Proposition 10.

A.11. Proof of Proposition 11

First, note that a Victorian equilibrium among the alphas always exists. This is because the alpha's marriage problem is identical to the beta's except that $k = 0$ in this case: women marrying an alpha would access the same genetic material if they were single instead. Since Proposition 6 holds for $k = 0$, such an equilibrium exists.

Next, assume that at date t, all alpha agents have a human capital level lower than $h_{\max,\alpha}^{LR}$, and all beta agents have a human capital lower than $h_{\max,\beta}^{LR}$.

Clearly, for generation $t+1$, the human capital level of an alpha cannot exceed that of the alpha children of a couple such that the man is alpha and both spouses have human capital $h_{\max,\alpha}^{LR}$. This can be written as

$$\ln h \leq \psi \ln 2 + \ln \alpha + \psi \ln \frac{\gamma\psi A}{1 + n\gamma\psi} + \psi \ln h_{\max,\alpha}^{LR}.$$

By construction, the RHS is equal to $h_{\max,\alpha}^{LR}$. Therefore, all the alphas in generation $t+1$ have a human capital which cannot exceed $h_{\max,\alpha}^{LR}$. As for the betas of that generation, they cannot do better than the beta children of an alpha couple with human capital $h_{\max,\alpha}^{LR}$:

$$\ln h \leq \psi \ln 2 + \ln \beta + \psi \ln \frac{\gamma\psi A}{1 + n\gamma\psi} + \psi \ln h_{\max,\alpha}^{LR} = \ln h_{\max,\beta}^{LR} \leq \ln h_{\max,\alpha}^{LR}.$$

Therefore, the property that $h \leq h_{\max,\beta}^{LR}$ for all the betas and $h \leq h_{\max,\alpha}^{LR}$ for all the alphas will remain true across all generations, regardless of how the betas mate, as long as the conditions of Proposition 2 hold.

Assume that at date t, all agents have a human capital level larger than $h_{\min,\beta}^{LR}$. We know that (18) is more likely to hold, the smaller $h_{\max}$ and the larger $h_{\min}$. By assumption, (18) holds for $h_{\min} = h_{\min,\beta}^{LR}$ and $h_{\max} = h_{\max,\beta}^{LR}$. By assumption, the distribution of the beta's human capital at t is such that $h_{\min,\beta}^{LR} \leq h_{\min,t} \leq h_{\max,t} \leq h_{\max,\beta}^{LR}$. Therefore, (18) holds. Also, by virtue of (28), so does the condition $h_{\min} \geq \underline{h}(\beta)$ and therefore the conditions of Proposition 2. Hence, there exists a marriage market equilibrium at t which is Victorian for the betas. Furthermore, in generation $t+1$, the lowest human capital level of a beta cannot exceed that of a beta offspring of a beta couple with human capital $h_{\min,\beta}^{LR}$:

$$\ln h \geq \psi \ln 2 + \ln \beta + \psi \ln \frac{\gamma\psi A}{1 + n\gamma\psi} + \psi \ln h_{\min,\beta}^{LR} = \ln h_{\min,\beta}^{LR}.$$

Therefore, the property that $h \geq h^{LR}_{\min,\beta}$ still holds among the betas (and, a fortiori, among the alphas[42]) of generation $t+1$, implying that a Victorian equilibrium also exists for them. By induction, $h \geq h^{LR}_{\min,\beta}$ for all generations and a Victorian equilibrium exists at all dates. This proves claim (i) in Proposition 11.

Claim (ii) derives straightforwardly from the fact that all agents marry, so that a fraction ρ_t of children have alpha fathers.

Claim (iii) derives straightforwardly from aggregating log human capital among all offsprings of types alpha and beta.

A.12. Proof of Proposition 12

The proof is straightforward from (27), (29), (30) and from noting that the expected human capital of the betas evolves according to

$$E \ln h_{t+1,\beta} = \psi(\ln 2 + 1 + E \ln h_{t,\beta} - \underline{h}(\beta)).$$

Finally note that (31) is equivalent to

$$e^k \leq 2^{\gamma n \psi}, \tag{86}$$

and compare with (26).

A.13. Proof of Proposition 13

The required inequality holds for married women, since they get a higher quality beta husband in the SATC assignment than in the Victorian one. Let us thus focus on unmarried ones. In the Victorian assignment, their average offspring log human capital would be equal to

$$E_V \ln h'(h) = b + \psi \ln(2h) + \ln \frac{A\gamma\psi}{1+\gamma n\psi}.$$

In the SATC equilibrium, we get

$$E_{\text{SATC}} \ln h'(h) = a + \psi \ln h + \ln \frac{A\gamma\psi}{1+\gamma n\psi}.$$

The inequality holds iff

$$\psi \ln 2 \leq (p_\alpha - p_\beta) \ln \frac{\alpha}{\beta}.$$

This condition is equivalent to $e^k \geq 2^{\gamma n \psi}$, i.e to (86) being violated. We can show that this is a necessary condition for an SATC equilibrium to exist. Going back to the proof of Proposition 5, part E, we can see that (72) is a necessary condition for an SATC equilibrium. If it does not hold, we can show that women slightly above $\bar{h}$ can profitably underbid the women at $h = \bar{h}$, by just inverting the reasoning in part E and (72) clearly implies that $e^k \geq 2^{\gamma n \psi}$.

NOTES

1 There is a deep link between these two differences: female gametes are scarce because women provide the investment in natural resources to turn an embryo into a baby. This feature implies that they cannot produce a very large number of children *and* that they know for sure that they are theirs. The opposite is true for men.

2 The model developed below aims at understanding marriage for most of human history; but in the last few decades contraception, IVF, (selective) abortion, and DNA testing have appeared. Clearly, these features decouple sexual intercourse from legitimacy. A new marriage contract may evolve. See Edlund (2005) for a thorough discussion.

3 In the model, this is true; parents derive the same utility from their legitimate and illegitimate children. Because the latter are not known to the father, though, they cannot invest in their human capital, and, for this reason, do prefer to marry and have legitimate children.

4 Unless she happens to marry a man with the highest genetic quality (an alpha in this paper's model), in which case marriage entails no opportunity cost to the woman. Nor would the alpha man have an opportunity cost under the double standard discussed further below in the paper.

5 That assortative mating is perfect clearly is a stylized feature of the model. It is commonly believed that there is more assortative mating than in the past in advanced countries, a trend that the model cannot capture since mating is perfectly assortative both in the Victorian and SATC equilibrium. Note however that in a society where women specialize in household production their actual human capital is poorly measured by their formal educational achievements. Consequently, assortative mating may have been underestimated in the past. Furthermore, the data are not so clear-cut about whether assortative mating has gone up in recent decades. See Eika et al. (2014).

6 Thus, there is no ingredient in the model that would account for the prevalence of out of wedlock birth at the bottom of the distribution of income. Indeed, in a world where marriage is enforceable and there are available potential husbands, it is unclear why such an outcome would arise. Phenomena that could account for this include an inadequate sex ratio, imperfect enforceability of the marriage contract, mistakes and behavioral problems, and an implicit tax on marriage associated with welfare programs such as AFDC. See Rosenzweig (1999) and Willis (1999).
Such ingredients are absent from the current model but it could be enriched by introducing them.

7 For example, in the case just discussed, there will be more marriages if instead of a competitive market, a social norm allocates a single partner of the same genetic type and human capital to each individual, thus replicating the Victorian assignment.

8 The assessment depends on how the decline of marriage is interpreted. As show below, a transition from a Victorian equilibrium to an SATC *holding the distribution of human capital constant* increases the returns to human capital for married men because they are mated to women with less human capital than in the Victorian equilibrium. So the returns to human capital only fall for the men who end up single at the bottom of the skill distribution. On the other hand, a decline in marriage due to lower enforceability of monogamy and thus lower prospects for men to have legitimate children will uniformly move the economy closer to the State of Nature and unambiguously reduce the returns to human capital for men and increase them for women.

9 While Bertrand et al. interpret their results as evidence of socially constructed gender roles, the present paper suggests that it may instead be related to features of human biology.

10 I am grateful to an anonymous referee for pointing out these figures to me.

11 The data contain some anomalies; at low income levels the proportion never married steeply rises with income for women, reaching 31% for the 15,000–25,000$ income group, which is much higher than for any other income groups.

12 Therefore, an SATC equilibrium would not have emerged during the Victorian era, despite the high level of inequality, due to sexual repression.

13 While arguably beta men have lower social status and commanded low political power in traditional societies, literary sources like Molière's *Don Juan* suggest they in fact managed to impose a substantial social control, in particular through the Church. Similar views are proposed in MacDonald

(1995). Furthermore, the distinction between alphas and betas here is a matter of biological attractiveness, not social or economic status, although in the model alpha children of legitimate marriages will have more human capital than their beta counterparts.

14 See also Edlund and Korn (2002), Edlund and Lagerlöf (2004) and Gould et al. (2004).

15 Two related papers are Bethmann and Kvasnicka (2007) and Edlund (2005b). The former, unlike the present paper, assume paternal investment in the State of Nature and do not study the role of inequality. In Edlund, as here, marriage entails a non-transferable utility loss for women in the form of (potential) transfer of custody rights to the husband. Accordingly, the implications for the transmission of human capital and long-term growth, as well as the predictions for the mating pattern of the non-married women, are entirely different. An independent contribution by Chiappori et al. (2008) uses the same perfectly competitive marriage market, with a similar derivation of the sharing rule. But their model deals with divorce, not with hypergamy and human capital accumulation.

16 See for example Fernandez et al. (2001), Burdett and Coles (1997). In these models, as in Aiyagari et al., the marriage market is frictional. In Fernandez et al. and Aiyagari et al. people are entitled to two draws, while Burdett and Coles use a matching function framework. In contrast, here, we follow Becker (1973) and characterize a perfectly competitive assignment in marriage markets.

17 This is somewhat similar to Kremer (1993) and Saint-Paul (2001).

18 This may be the case, for example, if the genetic advantage of the alphas evolved in an obsolete environment, in which the productivity advantage of the alphas was hard wired in the form of greater attractiveness.

19 I assume for simplicity that all children get the same investment in human capital.

20 The specific implications of "model B" are discussed in Section 4.

21 The key feature here is that no woman, either alpha or beta, has to settle for a beta man, because they can access the unscarce gametes of an alpha man instead. Given relative gamete scarcities, the converse cannot be true, so that allowing offspring's types to also depend on the mother's type will not qualitatively change the analysis.

22 More generally, though, they could weigh the $\gamma n E \ln h'$ term in their utility function by the probability that they are the actual father.

23 All proofs are in the Appendix.

24 For space reasons, the analysis of polygyny in the context of the current model is left for further research. The interested reader can refer to De La Croix and Mariani (2012), and Gould et al. (2004).

25 The existence of a double standard in the treatment of adultery is widely documented throughout cultures. For example, Holmes (1995) shows that "The Divorce and Matrimonial Causes Act of 1857 included a double standard in its provisions. While a wife's adultery was sufficient cause to end a marriage, a woman could divorce her husband only if his adultery had been compounded by another matrimonial offence."

The model could clearly be extended to impose restrictions on men as well. These restrictions would only be binding for alpha men and one would then have to analyze their trade-off between promiscuity/quantity of offspring and marriage/quality of offspring. See Willis (1999) for an analysis.

26 Note that the double standard is not binding in that case since no promiscuous woman wants to have intercourse with a beta male.

27 Also, hypergamy is more stringent at low levels of human capital, in that the maximum h/h^* ratio goes up with h^*.

28 While such a restriction on who can marry whom may capture a realistic feature of at least some traditional societies, and may endogenously arise in equilibrium, here this assumption is made for simplicity only. See Cole et al. (1998).

Whether segregation would endogenously arise if intermarriage between the two types were allowed probably depends on the model's parameters. Since in the SATC equilibrium discussed below beta men marry less skilled women, by the same token it is likely that alpha men may marry beta women in some configurations.

29 If $h^*()$ is continuous, then to be a mapping it must be monotonic. But we do not actually require that it be continuous, so other configurations are possible. Note though that it is not the most general

formulation, as it implies that each woman type marries exactly one man type, and vice versa. A more general formulation would introduce a measure of marriages over $[h_{\min}, h_{\max}]^2$.

30 For any $h, h^* \in [h_{\min}, h_{\max}]$ let $\hat{\theta}^*(h, h^*)$ be such that $U^*_\beta(h, h^*, \hat{\theta}^*(h, h^*)) = V^*(h^*)$. Then, (17) is equivalent to

$$\forall h, h^* \in [h_{\min}, h_{\max}], U_\beta(h, h^*, \hat{\theta}^*(h, h^*)) \leq V(h). \quad \textbf{(87)}$$

To see this, just note that both conditions are equivalent to $\hat{\theta}^*(h, h^*) \leq \hat{\theta}(h, h^*)$.

31 Note that this condition (17) holds with equality if the couple we consider is indeed married in equilibrium. Indeed, we can check that for $h^* = h^*(h)$, $\hat{\theta}(h, h^*) = \hat{\theta}^*(h, h^*) = \theta(h)$.

32 Formally, denote by $\mu()$ the measure associated with $f()$ and by $F()$ the c.d.f. Then all people marry. For property (14) to hold, it must be that $\mu([h_{\min}, h]) = \mu([h_{\min}, h^*(h)])$ since, by monotonicity (Proposition 1), $h^*()$ maps $[h_{\min}, h]$ into $[h_{\min}, h^*(h)]$. That is equivalent to $F(h) = F(h^*(h))$ for all h, i.e. $h^*(h) = h$. Similarly, in an SATC equilibrium, we must have $F(h) = F(h^*(h)) - F(\underline{h}^*)$, so that $h^*(h) \geq h$.

33 If the distribution of human capital has full support and no mass point, this inequality holds strictly; if it is not degenerate in a single mass point, it holds strictly for some h.

If the assumption that men and women have the same distribution of human capital were relaxed, then these properties would still hold in terms of how the spouses are ranked in their own sex's distribution of human capital.

34 This is because $\underline{h}(\beta) \geq \tilde{h}$. See Appendix.

35 See the working paper version of this article (Saint-Paul, 2008). This stands in contrast to Edlund's (2005a) analysis of the "Sex and the City phenomenon".

36 This explanation is different from the ones proposed by Chiappori et al. (2009), and Iyigun and Walsh (2007).

37 Arguably, the depenalization of adultery and the introduction of no-fault and unilateral divorce has also made the marriage contract less enforcible, leading to a partial reversion to the State of Nature.

38 Computing aggregate capital accumulation in an SATC equilibrium proved analytically intractable.

39 The difference between the two long-run levels is then equal to

$$\frac{\ln 2 - (p_\alpha - p_\beta)\frac{1-p_\alpha}{1-p_\alpha+p_\beta}\ln\frac{\alpha}{\beta}}{1-\psi}.$$

It is greater, the higher the proportion of alphas, the weaker the decreasing returns in the transmission of human capital, the lower the genetic loss from mating with a beta rather than an alpha.

40 Here, we assume the discontinuity takes place on the right of h^*_0. Nothing would change in the argument if it were on the left.

41 Denoting the LHS by $\phi(\hat{h}, \underline{h}^*(\hat{h}))$, and its RHS by R, it can be checked that $\phi(h_{\min}, \underline{h}^*(\hat{h})) = R$ and that $\phi'_1 \leq 0$.

42 The alphas will, from date $t = 1$ on, have a human capital strictly above $h^{LR}_{\min,\alpha} = \psi \ln 2 + \ln\alpha + \psi \ln \frac{\gamma\psi A}{1+n\gamma\psi} + \psi \ln h^{LR}_{\min,\beta} \geq h^{LR}_{\min,\beta}$.

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