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COHERENCE, LOCAL-INDICABILITY AND NONPOSITIVE IMMERSIONS

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Abstract Abstracts should be 250 words. It must be able to stand alone and so cannot contain citations to the paper's references, equations, etc. An abstract must consist of a single paragraph and be concise. Because of online formatting, abstracts must appear as plain as possible.

1. Nonpositive immersions

The goal of this paper is to examine implications of the following notion:

Definition 1.1 (Nonpositive immersions)**.** A map $Y \to X$ is an *immersion* if it is a local-injection. A 2-complex X has *nonpositive immersions* if for any immersion $Y \to X$ where Y is a compact connected 2-complex, either $\chi(Y) \leq 0$ or $\pi_1 Y$ is trivial.

Material handling is one of the widest spread application processes utilized in manufacturing today. While many manufacturers use material handling robot systems, several are moving toward material handling workcells, which consist of one or multiple robots, doing several tasks at one station. A material handling workcell is one of the most versatile workcells available on the market today. While welding workcells are usually confined to either spot or arc welding applications, material handling workcells can perform a dozen different tasks depending on the end-of-arm-tooling available. In an effort to make manufacturing more efficient, manufacturers are turning more to workcells for material handling. These workcells can perform several tasks within one workcell. Within these workcells, material handlers can work with other robots, CNC machines, or even human workers. They can be used to transfer parts into different machines that are positioned around it at one station, or they can take parts from an assembly robot within the material handling workcell and place them on a turntable or other positioner for a human

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worker to grab and take to the next area. A robotic manufacturing cell is shown in Figure 1.

According to statistics in 1989 (Gould, 1990), robot systeminstallations with respect to their application types were profiled as following: just-in-time delivery systems (56%), flexible manufacturing systems/flexible assembly system (FMS/FAS) transfer system (13%), storage load transfer, non-AS/RS (12%), AS/RS interface (8%), progressive assembly (7%), mini-load AS/RS interface (1%), and others (3%). Some other applications of AGV systems in non-manufacturing environments include, but are not limited to, delivering mail, messages, and packages in offices, and delivering meals and laundry in hospitals.

2. Background on Towers and Disc Diagrams

2.1. Towers

We recall here some background on towers which is due to Howie [How81]. A map $X \to Y$ between CW-complexes is *combinatorial* provided that its restriction to each open cell of X is a homeomorphism onto an open cell of Y. A CW-complex is *combinatorial* provided that the attaching map of each of its cells is a combinatorial map (after a suitable subdivision). Unless otherwise indicated, the spaces in this paper will be 2-dimensional combinatorial complexes, and the maps between these spaces will be cellular.

2.2. Disc diagrams

We now briefly summarize the basic definitions concerning disc diagrams. We refer the reader to [LS77, Ger87, MW02] for more detailed accounts.

All robots' transportation times in the simulation are set to zero. Thus, only driving robots are considered. For the "without turning point" strategy, only the average driven distance is used to compute a possible job performance without considering the effects of congestion. Hence, a linear relation between the number of robots and transport capacity is assumed. For the "with turning point" strategy, the turning point is used to reduce the number of deadlocks to zero. Thus, congestion results in nonlinear behavior when a large number of robots are employed.

3. Local indicability and Asphericity

Lemma 3.1. *Let H be a finitely generated group. There exists a finitely presented group K and a surjection $K \twoheadrightarrow H$ such that the map $\mathsf{H}_1(K) \to \mathsf{H}_1(H)$ is an isomorphism.*

Proof. Let H be generated by $\langle h_1, \dots, h_r \rangle$. Since every finitely generated abelian group is finitely presented, we can choose a finite presentation $\langle h_1, \dots, h_r \mid R_1, \dots, R_s, [h_i, h_j] : i < j \rangle$ for the abelianization $\mathsf{H}_1(H)$ of H in terms of the original generators of H. For each i we have $R_i =_H W_i$ where W_i is a product of commutators. Regard $R_i(h_1, \dots, h_r)$ and $W_i(h_1, \dots, h_r)$ as words in $h_i^{\pm 1}$, and rewrite these in terms of new generators $\{k_1, \dots, k_r\}$

to obtain a group K presented by:

$$\langle\, k_1, \ldots, k_r \mid R_1(k_1, \ldots, k_r) = W_1(k_1, \ldots, k_r), \ldots, R_s(k_1, \ldots, k_r) = W_s(k_1, \ldots, k_r) \,\rangle.$$

There is an obvious surjection $K \twoheadrightarrow H$ induced by $k_i \mapsto h_i$, and it induces an isomorphism $\mathsf{H}_1(K) \to \mathsf{H}_1(H)$ by construction. □

Discrete event simulation (DES) refers to simulation that employs mathematical and logical models of a physical system to represent state changes at precise points in simulated time. Taking advantages of the computing advancement, DES has been intensively developed for modeling, simulating, and analyzing dynamic and complex systems. This is meant to enable research on advanced industrial system to be conducted. Among simulation-based researches for manufacturing applications include those conducted by Gupta, Singh, and Verma (2010) and Manup and Raja (2016). Consequently, ARENA and Simul8 simulation software are used to model material handling operation within a manufacturing cell. There are several advantages of the Simul8 software particularly in its ability to accommodate mathematical and logical procedure with relative ease through Visual Logic. Furthermore, it is also possible to integrate codes developed using Visual Basic into Simul8 simulation framework.

4. Negative immersions and coherence

The goal of this section is to prove coherence in the case of negative immersions. While the result is broader, the proof closely follows the argument proving coherence for negative generalized sectional curvature. We borrow an additional idea from the proof of the coherence of 3-manifold groups to easily ensure our immersions have no free faces or isolated edges.

The simulation experiment is carried out in accordance with the procedure follows here. The first test used to verify the simulation result is called chi-square goodness-of-fit test. Its purpose is to test for distributional adequacy. The chi-square test is used to test if a sample of data came from a population with a specific distribution. An attractive feature of the chi-square goodness-of-fit test is that it can be applied to any univariate distribution for which one can calculate the cumulative distribution function. The chisquare goodness-of-fit test is applied to binned data (i.e., data put into classes). This is actually not a restriction since for non-binned data one can simply calculate a histogram or frequency table before generating the chi-square test. However, the values of the chi-square test statistic are dependent on how the data is binned. Another disadvantage of the chi-square test is that it requires a sufficient sample size in order for the chi-square approximation to be valid.

The chi-square goodness-of-fit test can also be applied to discrete distributions such as the binomial and the Poisson rather than continuous ones. The Kolmogorov-Smirnov (K-S) and Anderson–Darling tests are restricted to continuous distributions.

5. Equations

Equations in LaTeX can either be inline or on-a-line by itself. For inline equations use the `$...$` commands. Eg: The equation $H\psi = E\psi$ is written via the command $H\psi = E\psi$.

For on-a-line by itself equations (with auto generated equation numbers) one can use the equation or eqnarray environments D.

$$\mathcal{L} = i\psi\gamma^{\mu}D_{\mu}\psi - \frac{1}{4}F^{a}_{\mu\nu}F^{a\mu\nu} - m\psi\psi \tag{5.1}$$

where,

$$\begin{aligned} D_{\mu} &= \partial_{\mu} - ig\frac{\lambda^{a}}{2}A^{a}_{\mu} \\ F^{a}_{\mu\nu} &= \partial_{\mu}A^{a}_{\nu} - \partial_{\nu}A^{a}_{\mu} + gf^{abc}A^{b}_{\mu}A^{a}_{\nu} \end{aligned} \tag{5.2}$$

Notice the use of `\nonumber` in the align environment at the end of each line, except the last, so as not to produce equation numbers on lines where no equation numbers are required. The `\label{}` command should only be used at the last line of an align environment where `\nonumber` is not used.

$$Y_{\infty} = \left(\frac{m}{\mathrm{GeV}}\right)^{-3}\left[1 + \frac{3\ln(m/\mathrm{GeV})}{15} + \frac{\ln(c_2/5)}{15}\right] \tag{5.3}$$

The class file also supports the use of `\mathbb{}`, `\mathscr{}` and `\mathcal{}` commands. As such `\mathbb{R}`, `\mathscr{R}` and `\mathcal{R}` produces $\mathbb{R}$, $\mathscr{R}$ and $\mathcal{R}$ respectively.

6. Figures

As per the LaTeX standards eps images in `latex` and pdf/jpg/png images in `pdflatex` should be used. This is one of the major differences between `latex` and `pdflatex`. The images should be single page documents. The command for inserting images for latex and pdflatex can be generalized. The package that should be used is the graphicx package. See Figure 1.

7. Tables

Tables can be inserted via the normal table and tabular environment. To put footnotes inside tables one has to use the additional "fntable" environment enclosing the tabular environment. The footnote appears just below the table itself.

8. Cross referencing

Environments such as figure, table, equation, align can have a label declared via the `\label{#label}` command. For figures and table environments one should use the

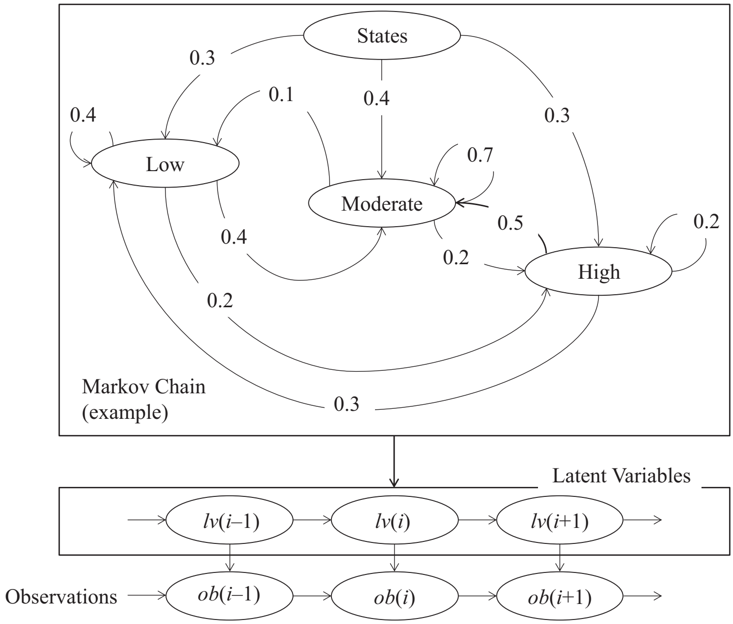

Figure 1. The concept of hidden Markov model.

`\label{}` command inside or just below the `\caption{}` command. One can then use the `\ref{#label}` command to cross-reference them. As an example, consider the label declared for Figure 1 which is `\label{fig1}`. To cross-reference it, use the command `Figure \ref{fig1}`, for which it comes up as "Figure 1". The reference citations should used as per the "natbib" packages. Some sample citations: [CT00, FH99, Ger87, How81].

9. Lists

List in LaTeX can be of three types: enumerate, itemize and description. In each environments, new entry is added via the `\item` command. Enumerate creates numbered lists, itemize creates bulleted lists and description creates description lists. List in LaTeX can be of three types: enumerate, itemize and description. In each environments, new entry is added via the `\item` command. Enumerate creates numbered lists, itemize creates bulleted lists and description creates description lists.

1. This is the 1st item
2. Enumerate creates numbered lists, itemize creates bulleted lists and description creates description lists.
3. Numbered lists continue.

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- This is the 1st item
- Itemize creates bulleted lists and description creates description lists.
- Bullet lists continue.

10. Conclusion

Some Conclusions here.

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