Exercise 14

Jeans mass

Study time: 90 minutes

Summary

In this spreadsheet activity you will create a formula to determine the Jeans mass of interstellar clouds. You will then use it to investigate the influence of the different cloud properties on the tendency of the cloud to contract.

You should have read Chapter 5 of *An Introduction to the Sun and Stars* before attempting this activity.

Learning outcomes

- Gain an awareness of the range of conditions under which a uniform spherical interstellar cloud is likely to collapse.
- Create a complex formula in a spreadsheet and use it to perform calculations.

Background to the activity

If the mass of a uniform spherical cloud exceeds the Jeans mass, the force of gravitational attraction will overcome the opposing pressure due to the motion of the particles, and contraction will occur. This critical mass depends on the density, temperature and composition of the cloud, and is given by the expression (*An Introduction to the Sun and Stars*, Equation 5.1):

$$M_{\mathrm{J}} = \frac{9}{4} \times \left(\frac{1}{2\pi n} \right)^{1/2} \times \frac{1}{m^2} \times \left(\frac{kT}{G} \right)^{3/2} \qquad (1)$$

Although this equation provides a very simplified criterion for the collapse of an interstellar cloud it does provide a useful guide to the conditions under which that collapse is likely to occur. You will not consider here the complex aspects of cloud collapse such as inhomogeneities in properties of the cloud, rotation or magnetic fields. You will use the Jeans mass as a tool in investigate the conditions under which an ideal cloud is likely to collapse and to develop further your spreadsheet skills in use of formulae.

Part 1 The Jeans mass spreadsheet

The instructions given here assume that you are using the spreadsheet in Sun Microsystems' StarOffice™ 6.0[1]. While it should be possible to carry out the activity using other spreadsheet packages (such as Microsoft Excel), you should be aware that the detailed instructions given here relate specifically to the StarOffice spreadsheet.

Set up spreadsheet

- If you have not already done so, start StarOffice. From the main StarOffice menu, select **File | New | Spreadsheet** to create a new, blank, sheet.

Don't forget to save your work regularly in your work folder. From time to time make a backup copy of your work (using a different filename) in case you need to go back to an earlier stage.

Consider carefully the layout of your spreadsheet. In this activity you will construct a complex formula stage by stage so it is vital that you include sufficient *labelling* and other information to allow any other user of the spreadsheet (or yourself if you return to the spreadsheet after some time) to understand what is going on.

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[1] StarOffice is a trademark or registered trademark of Sun Microsystems Inc. in the United States and other countries.

The Jeans mass formula

In order to use the formula in a spreadsheet it is sensible to break it down into sections containing all the constants (which will be fixed in all calculations) and the variable quantities.

Question 1

For each symbol in the Jeans mass equation, write down its meaning, whether it is a constant or a variable, its value (if a constant) and its SI units.

- Enter a section in your spreadsheet with headings 'Constant' 'Symbol' 'Value' 'Unit' and insert the appropriate constants from the Jeans mass equation. To enter the value of π in the spreadsheet, use the function `=PI()`.
- Format the cells appropriately (scientific notation is required for most of the numbers).

Your spreadsheet should look like Figure 2 in Note 1 (at the end of these instructions).

Question 2

Rearrange the Jeans mass equation so that all the constants are separated from the variables.

- Insert a new constant in your spreadsheet table (you can call it A), which is the combined constants you defined in the answer to Question 1. You can calculate the value of this constant using a calculator or you can enter it as a formula.

Question 3

What is the value of A? What are the units of A?

If you want to enter A as a formula you need to remember the basic rules for arithmetic. The order for arithmetic operations to be performed is:

^ (to the power of)

× and / (multiply and divide)

+ and – (add and subtract)

For example, when entering a number such as $5^{2/5}$ you will get the wrong answer if you enter `=5^2/5` since this represents $5^2/5$. The correct formula is `=5^(2/5)`. (Alternatively, you could write `=5^0.4`). If in doubt always use brackets to separate different parts of a formula and check your answers.

Question 4

What is the spreadsheet formula for the constant A?

Before you enter the formula for the Jeans mass equation, you need to set up cells containing the variables.

- Below your list of constants make a new small table, with similar headings for the variables in the Jeans mass equation. It is worth highlighting the 'Value' cells in a different colour since this is where you will want to enter some numbers. (See Figure 3 in Note 2.)

So far you have entered all numbers in SI units. It always makes sense to perform all calculations in SI units so that there is no confusion. However, you may want to enter numbers (or answers) in more convenient, non-SI units. Examples are molecular masses in units of the mass of a hydrogen atom (m_H), (or the Jeans mass in terms of solar mass ($M_\odot$)).

- In your table, enter additional columns showing the value and units of quantities in non-SI units.
- Enter formulae in the appropriate cells to convert them into SI units.

Question 5

What is the formula you would enter into the spreadsheet for the Jeans mass?

- Now enter the Jeans mass formula into your spreadsheet. (If you get an error message this may be because you have forgotten to enter values in the cells for the variables.)
- Format the cell to display the result in scientific notation. It is also a good idea to colour highlight the cell containing the result.

- Now enter an additional cell to convert the Jeans mass into units of solar mass.

(Your spreadsheet should now look like Figure 4 in Note 3.)

At this stage it is worth testing your results to ensure you have not made an error. With $T = 100\,\text{K}$, $m = 1m_{\text{H}}$ ($= 1.67 \times 10^{-27}\,\text{kg}$) and $n = 10^{10}\,\text{m}^{-3}$, the Jeans mass is $3.03 \times 10^{32}\,\text{kg}$ or $152.2M_{\odot}$.

Question 6

Use your spreadsheet to determine if the following clouds are likely to collapse:

(a) A cloud of mass $5M_{\odot}$ consisting entirely of neutral hydrogen with $T = 30\,\text{K}$ and $n = 10^{11}\,\text{m}^{-3}$.

(b) A cloud with the same properties as in part (a) but consisting entirely of molecular hydrogen.

Part 2 Alternative versions of the Jeans mass spreadsheet

The spreadsheet you have created allows you to determine the Jeans mass for any set of cloud parameters. However, if you were asked the question:

'To what density would a molecular hydrogen cloud of mass $10M_{\odot}$ and temperature 20 K need to be compressed before it is likely to collapse?'

It would require some trial and error to determine the answer using your spreadsheet. It is not difficult to modify the spreadsheet to answer this question directly.

Set up a new worksheet

The new version can be entered in a different *worksheet*.

- At the bottom of the spreadsheet you will see a series of tabs marked **Sheet1**, **Sheet2** and **Sheet3**. The current worksheet will be **Sheet1**.
- Select **Format | Sheet | Rename…** from the main menu and rename the sheet **Jeans mass**. Select the tab **Sheet2** and rename it **Density**.

Since most of the **Density** worksheet will be the same as the **Jeans mass** worksheet, it is worthwhile copying the entire contents and editing them.

- Select the entire contents of the worksheet by clicking the square in the corner of the spreadsheet (shown in Figure 1). The worksheet will go dark to show the area selected. Select **Edit | Copy** from the main menu.

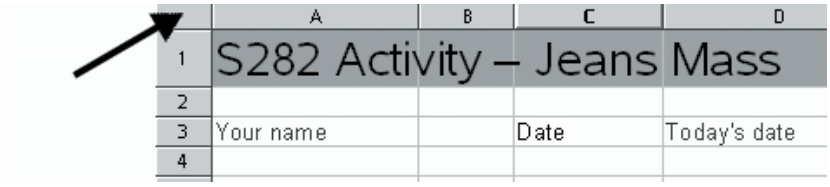

Figure 1 Select this cell to select the whole worksheet.

- Move to the **Density** worksheet and select the top left corner again. The worksheet will again go dark.
- Select **Edit | Paste** from the main menu and an exact copy of the **Jeans mass** worksheet will appear.

The density worksheet

The only difference required in the new worksheet is that the Jeans mass is to be specified and the density calculated.

- In the *variables* section of your worksheet, replace the row containing the density information with the Jeans mass (in SI units) as well as solar masses.
- Make sure you enter the appropriate conversion formula in the *value* cell to convert from solar masses to SI units. Enter an initial value of $1M_{\odot}$.

- Clear the row containing the Jeans mass formula by selecting the row (click on the row number to highlight the row) then select **Edit | Delete contents** from the main menu. Click **OK** in the window that appears.

You will now need to rearrange the Jeans mass equation to obtain the number density.

Question 7

What is the equation for the number density? How would you write this as a spreadsheet formula?

- Enter the formula for the number density in your spreadsheet. (See Note 4.) Test your working using the same data as in the Jeans mass worksheet: $T = 100\,\text{K}$, $m = 1m_{\text{H}}$ ($= 1.67 \times 10^{-27}\,\text{kg}$) and $M_{\text{J}} = 3.03 \times 10^{32}\,\text{kg}$ or $152.2M_{\odot}$ gives $n = 10^{10}\,\text{m}^{-3}$.

Question 8

Use your spreadsheet to determine the density above which the following clouds are likely to collapse:

(a) A cloud of mass $20M_{\odot}$ consisting entirely of neutral hydrogen with $T = 15\,\text{K}$.

(b) A cloud with the same properties as in part (a) but consisting entirely of molecular hydrogen.

Explain why you would expect the result for part (b) to be lower than for part (a).

The temperature worksheet

In this final part of the activity you will use your experience in setting up the **Density** worksheet to prepare a third worksheet to answer Question 9.

In order to answer Question 9 you will need to:

- Rearrange the Jeans mass equation in terms of the temperature.
- Create a new **Temperature** worksheet.
- Copy the contents of the **Jeans mass** worksheet into it.
- Modify the variables and formulae to derive the critical temperature for collapse.

(See Note 5 after you have attempted the question.)

Question 9

Use your spreadsheet to determine the temperature below which the following clouds are likely to collapse:

(a) A dense cloud of mass $3M_{\odot}$ and number density $10^{11}\,\text{m}^{-3}$ consisting entirely of molecular hydrogen.

(b) A diffuse cloud with the same mass and composition as the dense cloud in part (a) but with a number density of only $10^{7}\,\text{m}^{-3}$.

(c) A diffuse cloud with the same number density and composition as the diffuse cloud in part (b) but with a mass of $30M_{\odot}$.

Do the results match your expectations?

Exercise 14 was written by Simon Green.