The Contagion Math

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Abstract. A comprehensive approach to mathematical modeling of an infectious epidemics development in time is outlined, that can be instrumental in developing specific models and provide insights into effective strategies and practices to containment and control of the contagion.

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1 Introduction

A rapid spread of Covid-19 has taken many of the disease control offices and jurisdictions by surprise, resulting in some places, in overwhelmed care facilities and significant impact on the society [1].

In addition to planning and management factors that will not be addressed here, an essential component of such outcomes is poorly understood factors of the dynamics of the infection, namely transmission channels, acquisition factors, courses of illness and such. Now, as more information is becoming available on these factors, more substantiated investigation and forecasting of the evolution of the contagion is possible and should be pursued.

However, constructing detailed mathematical models is not the intent of this work. Rather we would like to outline some general approaches to creating such models and derive some general conclusions that can be useful in developing immediate action plans and policies to control the spread of the pandemics.

2 Epidemiological Dynamics Model

2.1 Parametric dynamic model

The model is defined by a set of contagion agents $S_c = \{ a_1, .. a_N \}$ that represent the instant state of the contagion at the moment t_i in an area A. Each contagion agent can pass the infection to the entities in A such as population P, objects O, or even volumes of air H, sewer treatment system U and other, via a number of transmission channels C_T with transmission factors defined by a matrix of intra- and cross channel transmission factors $\{ V_{ij} \}$. A contaminated entity then itself becomes a contagion agent.

Contagion agents decay naturally with time with corresponding law and parameters. For example, according to available data, contaminated air volume can be contagion-free (clean) within 2-3 hours after contamination, while an infected patient can be contagion free in 2-3 weeks (and moreover, the law and parameters of the decay can vary within each category also, as the window of contamination can be different for a patient in a controlled hospital environment vs. an asymptomatic but still contagious person in the street).

Combined, the transmission V and decay D parameters can describe the dynamics of the evolution of the infectious epidemics in time as:

$$S(t') = C\{\, V, D \,\}\,(S(t)) \tag{1}$$

Equation (1) then describes a dynamic system with different scenarios or trajectories of development, depending on the choice of parameters of the identified transmission and decay channels that can be affected by the instruments and policies to arrest and contain the contagion. For example, total disinfection can severely suppress the intermediate surface transmission channel by regularly resetting the count of object contagion agents if not to zero, then to a realistically possible minimum. Placing everyone with symptoms under effective self-isolation can suppress both human-to-human and human-to-object transmission channels and so on.

Straightforwardly, (1) can be realized as a system of discrete or differential dynamics equations that describes the evolution of the contagion in multiple channels simultaneously.

The desired outcome is therefore $S(t) = \{\}$ (nil, or the empty set) at some point of time, at which the contagion cannot develop any further due to the absence of the contagion agents in the area.

$$S(t) = S_0 = nil \tag{2}$$

It is worth noting that just stating (2) clearly as an objective of the policy can be seen as an important step in controlling the spread of the epidemics, as it switches the long-term focus from secondary factors, such as the new cases, incremental and total mortality counts to a policy objective that ensures a solution that is sustainable in the long term and possibly, indefinitely. Note that it includes *all* types of contagion agents at least from the known and understood set such as, but not limited to, human population, animal population, including pets, objects and other intermediate channels, air, waters, and so on, in the area under analysis.

2.2 General Conclusions

A number of straightforward conclusions can be obtained from the assumptions and conditions (1) and (2):

1. Aggregation: the total contagion set is a sum of all channels: $S = S_1 + \ldots + S_N$
2. $S_0 = \{\}$ is a stable point of any contagion dynamics C(V, D): $C(S_0) = S_0$
3. Different phases can be defined in specific channels such as spread or expansion; containment; decline; and elimination: $S_k = nil$.
4. Suppression of the contagion in a single channel or a subset of channels *does not guarantee total elimination of the contagion in the area*: $S_k = nil: \neq S_0$ for this reason, the crossover transmission parameters V_{ij} between the channels can play important and sometimes, a defining role in the dynamics of the contagion.

5. If the cross-channel transmission is suppressed: $V_{ij} \rightarrow 0$, the dynamics factorizes into independent evolution in each channel: $C(V, D) = \{ C_k(V_k, D_k) \}$.
6. Isolation: if both cross-channel and intra-channel transmissions are suppressed, the evolution of the contagion in the given channel S_i is controlled by the decay factor in the channel: $V_{ii}, V_{ij} \rightarrow 0 : C_i(V, D) \sim D_i$.
7. Migration: the contagion can be augmented by the arrival of new agents into the contagion area *in all channels*: $\Delta S^{(M)} = \{\Delta S_1^{(M)}, .. \Delta S_N^{(M)}\}$.

A straightforward issue from the above observations is that any comprehensive policy to control the contagion in a given area should focus on several key factors of the epidemiological dynamics and actions to control the infection, such as:

- Suppression of cross-channel transmission leading to isolation of contagion channels;
- Effective control, suppression and elimination of transmission within each channel;
- Effective control over the migration into the managed area, in all channels;
- The conclusion of containment or full elimination of the contagion can be made based on the comprehensive progress toward the condition (2), rather than selected channel-specific statistics;
- Clearly, such a determination cannot be made without certain level of confidence in the monitoring of the situation in different channels and continuously.

2.3 Channels and Parameters

Rather than proceeding here with development of specific models that may show varying levels of agreement with the factual numbers depending on the choice of parameters that may not be not known with the appropriate confidence at the time of writing, we shall briefly outline a possible taxonomy of contagion agents and approaches to identifying the factors and parameters that can be used for developing quantitative models in the further studies.

Human Contagion Agent

An infected human (a contagion agent of type *H*) can transmit the infection in a number of channels:

Direct-to-human, by touching, kissing or other close contact such as poorly protected personnel in a crowded hospital.

Human-to-object, by touching, spitting, coughing with aerosol setting on a surface where it can survive for some, often considerable, time.

Human-to-air: by coughing, sneezing, spitting.

Human-to-water: via sewage system, spitting, coughing and so on, outside.

Human-to-animal: e.g. with pets, feeding wildlife, etc.

Decay channels

Depending on the type of a human agent, different scenarios are possible.

Symptomatic patient, robust treatment: agent recovers within certain time window t_R with no residual contagiousness: $a_h(t_R) \rightarrow 0$.

Symptomatic patient, poor treatment: recovery time can be longer with a higher probability of transmitting the infection, and a higher likelihood of a more serious outcome.

Non-symptomatic patient, all stages: recovers within the time window of the natural recovery t_{nr} that needs to be confirmed for better understanding and modeling of the dynamics of the contagion and development of effective policies. Transmission factors can be expected to be higher in this case as the agent may believe to be in a perfect state of health and neglect some transmission control procedures. The importance of across the board spectrum testing as conducted in some jurisdictions cannot be overemphasized in this case: "early results from deCode Genetics indicate that a low proportion of the general population has contracted the virus and that about half of those who tested positive are non-symptomatic…" [3]. Hence, the non-symptomatic cohort needs to be treated as one of the major potential factors of transmission.

More detailed studies are needed for confident understanding of the transmission channels for this group of contagion agents. Some studies indicate that human-to-human [4], as well as the intermediary channel via a contaminated object [2] can both be effective in propagating the infection through the population. Good understanding of these scenarios and associated transmission factors is needed for developing effective approaches to controlling the spread.

Vulnerable group: while the transmission parameters for this group can be similar to those of the symptomatic group with lower probability of transmission, special policy attention should be given to the maximum risk protection of this group due to significantly higher risk of heavier course of the disease.

Object Contagion Agent

Similarly, a contaminated object (a contagion agent of type *O*) can transmit the infection by the following channels:

Object-to-human, by touching or otherwise contact with the contaminated object.

Object-to-object, by passing the contagion from object to object e.g. in a strong wind, hurricane and so on.

Object-to-air: e.g. by evaporation, need to be confirmed.

Object-to-water: e.g. in a rain, by street cleaning.

Object-to-animal: pets, street animals.

Decay channels

Some data on survivability of the infection on some surfaces is available [2].

Air Contagion Agent

An air contagion agent is a contaminated volume of air that is independent of the agent that created it. It can be defined as any contaminated volume of air that is detached from the personal space of its progenitor object (that in its own turn needs to be defined for example, as the volume to which normal, non-symptomatic breath extends). Thus, a close contact within the limits of the personal space is considered a human-to-human transmission while that via a detached and independent volume or air, a separate air transmission agent. As shown in [4] this distinction indeed makes sense, as under certain conditions, contaminated volumes of air can both travel considerable distances and survive over extended periods of time.

Similarly, a contaminated volume of air (a contagion agent of type a-) can transmit the infection through a number of channels. At this time, a detailed understanding of the characteristics and parameters of the channel is needed, such as characteristic size and volume of air agents, their evolution over time and their contagiousness to the entities in the area. These questions are beginning to be addressed in the studies [4].

Air-to-human, by direct inhalation of the contaminated air.

Air-to-object, by settling on an object(s).

Air-to-air: for example, if the initial agent is split into multiple parts evolving independently, the case needs to be analyzed further.

Air-to-water: by settling in puddles, ponds, rivers, etc.

Air-to-animal: similar to air-to-human.

Decay channels

Some data on survivability of the infection in the air is available [1,4].

Water Contagion Agent: Transmission, Decay

Some data is available [1]. More studies are needed for evaluation of the effect of this channel on the spread and control of the contagion.

Animal Contagion Agent: Transmission, Decay

Possibility of an animal transmission has been indicated in [5-7]: “Some infected animals also suffer from having diarrhea, and the virus present in the feces can cause further disease transmission.”.

The case needs further study and evaluation of the impact on the spread and control of the contagion.

2.4 Contagion Propagation Map

In time and with targeted research, a detailed map of intra- and inter-channel transmission factors and parameters can be constructed to provide a factual foundation for development of precise models of the dynamics of the epidemics in the area, as well as effective targeted measures and policies allowing to control the spread of the contagion in each channel, and overall.

3 Policy Implications

Based on comprehensive descriptive modeling outlined in this work, one can envisage a sustainable even in a longer-term approach to management of difficult and resistant infections. It encompasses a multi-pronged strategy aimed at effective control of the contagion in all essential propagation channels based on extensive, widespread and constant measurement and scientifically validated measures of effective control specifically targeted to each channel and based on sound, verified and current data and research.

Other than in the initial period, the strategy does not envisage prolonged severe quarantine limitations for everyone, however reasonable social distancing measures to limit the spread of the contagion should indeed be maintained. The strategy is based on the following action areas:

1) Measure: widespread, even ubiquitous, random voluntary testing

2) Infrastructure and resources: care and support services available in sufficient quantities for each policy area

3) Effective, channel specific control policies: policies are constantly validated by a measurable improvement in the intended area of application

4) Measure, measure and measure again! Confirm that the set of policies is effective and is achieving the intended objectives.

Without preempting the results of a detailed evaluation of such a strategy, several points can be made. The highest risk of a serious outcome is carried by the vulnerable group. Effective policies can be developed to minimize the risk of exposure for this group. These policies must be designed from the outset to last for a considerable period in a sustainable manner, based not so much on bureaucratic prescriptions and rules but on an array of working and easily accessible support services for those in the vulnerable group. The risk isolation measures targeted to this group can be lifted only in the latest phases of the program when reliable containment of the epidemics is ascertained.

An important and not yet well understood transmission factor is that of the non-symptomatic group, comprised of those contaminated who have not yet developed the symptoms and those who will not develop them, while still capable of spreading the infection. The channels of transmission in this category should be studied carefully in which task the importance of the widespread random and voluntary testing cannot be overemphasized. With time, such studies will allow to minimize the factors of transmission by this group without heavy strains on their normal life, leading to the natural decay of the contagion in this channel (Section 2.2, p.6). Possibilities can include, for example, effective masks, virus resistant hand / body cover available amply and freely, as in p.2 (infrastructure and resources) and such.

Test and treatment facilities that are ample, easily accessible and available on demand. Once in a while, a case in the general group would require an immediate medical attention and the appropriate treatment option and the resources should be available on a shortest notice to minimize the risk of a negative outcome. Additional measurement and tracking actions can be conducted in such cases to eliminate the risk of a resurgence. The importance of development of an effective self-testing options cannot be overlooked.

Effective containment policies developed for other essential channels such as disinfection of common areas, maintaining public transport infection-free and so on. Possible secondary transmission channels such as water; street and wild animals should be studied as well.

Finally, ongoing widespread measurement (p.4) will allow to gauge the success of the containment policies across multiple channels. When the objectives of the first phase (Containment) are achieved and the number of new cases is stabilized, policies can be tuned to advance to the next stage (Suppression) and so on, eventually to elimination of the epidemics in the area.

Unlike the hard quarantine scenario, such a strategy would not set prohibitively severe restrictions on the normal functioning of the society and for that reason can proceed in a planned and controlled flow based on the accurately measured actual state of the process and phased objectives, with policies and measures continuously tuned to achieve the best outcome.

4 Conclusion

In this work we presented an approach and a methodology for development of parametric models of the dynamics of an infectious epidemics in time. Investigating such models can provide grounded and informative inputs for development of effective strategies and policies that with consistent and intelligent implementation can lead to effective control, suppression and the longer perspective, realistic possibility of a complete elimination of the contagion.

For example, despite convincing results reported in a number of cases [3], general advice policies in several OECD jurisdictions [9-11] does not, in the authors view, sufficiently emphasize the need for wide testing that would result in a better understanding of the current situation, as well as intra- and inter-channel dynamics in the important, human-to-human channel.

An alternative to drastic across-the-board quarantine measures that may not be sustainable over an extended period for a number of mentioned reasons, could be a set of scientifically justified and meticulously executed effective policies to control the contagion in each channel. Obtaining a correct and current data would be paramount in this approach; for example, the development of effective policies to control the transmission from non-symptomatic human agents, that is proving to be a significant proportion in the channel cannot be achieved without comprehensive and reliable data and good understanding of the transmission channels and processes.

A selective and systematic approach would allow to focus the research and policy development attention on determining the factors and parameters that control the spread of the infection in all essential channels thus providing a solid scientific base for its control and eventual elimination.

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