The Correlation Between the Gravitational Constant and Distance from Matter

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Abstract

Here I show that we are able to consider a large value for the gravitational constant, G , at the vicinity of matter ($r < 1\text{\AA}$). Today physics and chemistry extremely need to this choice. The large value of G causes significant results in physics and chemistry. By considering a large value for G , at the vicinity of matter, the stability of atom and atomic and molecular bonds can be explained by a different method. Moreover, here I prove that the value of vacuum permittivity, ϵ_0 , at the vicinity of a charged particle ($r < 1\text{\AA}$), is larger than its present value and the Spacetime lines interact with electric field lines. Also, I obtain a new equation (not a new value) for a very important constant, namely fine-structure constant, with the presence of the gravitational constant, G .

Keywords: Gravitational Constant, Vacuum Permittivity, Fine-Structure Constant, Spacetime, Electric Field

Introduction: There is gravitational force at the subatomic scale ($r < 1\text{\AA}$) but its value is very negligible. By considering a large value for G , at the subatomic scale, the force of gravity can be increased at this scale. But the presence of a strong gravitational force along with the Coulomb force cause the collapse of the Bohr atom. To avoid this problem, we must choose a different value for ϵ_0 , at the subatomic scale. In the following, we will discuss about these issues and show that selecting a large value for G at the atomic scale has very valuable results.

Laboratory Limitations

First we should say that there is no laboratory limitations for selecting a large value for G at atomic distances because the value of G at the vicinity ($r < 1\text{\AA}$ or $r < 1nm$) of Protons, Neutrons, Electrons and nuclei of atoms has not been measured yet. The basis of most of the experiments to measure the value of G is the Newton's law of universal gravitation ($F = Gm_1m_2/r^2$) and torsion Balance [1]. In these experiments [2-5] the value of G is obtained according to the Newton's law and the values of m_1 , m_2 , F and r . In addition, the other various experiments with different methods have been carried out to measure the value of G such as the ones using very cold atoms and atom interferometry [6-9]. Yet, none of these experiments measured the value of G at distances of less than one nanometer or one angstrom ($r < 1\text{\AA}$ or $r < 1nm$). Therefore, we do not know the value of G through experiments, at subatomic distances; and thus we can theoretically consider a large value for it. We have:

$$\begin{cases} G(r) = G_0 = 6.67 \times 10^{-11} Nm^2 Kg^{-2} & r > 1 \text{ \AA} \\ G(r) = G_1 \gg G_0 & r < 1 \text{ \AA} \end{cases} \quad (1)$$

In the above equations, the boundary between G_0 and G_1 (namely $r = 1 \text{ \AA}$) is an approximate and selective one, with no emphasis on it.

Physical Concept of Change in the Gravitational Constant

The presence of an object in space causes the curvature of Spacetime (Fig. 1). The passing of gravitational waves creates the wrinkles in Spacetime [10,11], and variation in the value of $G(r)$ from G_0 to G_1 can be illustrated as the convergence and compression of Spacetime lines. It means that in Fig. 1, the Spacetime lines, at the vicinity of matter ($r < 1 \text{ \AA}$), converge and the squares become smaller.

Fig. 1. The presence of an object in space causes the curvature of Spacetime. The Spacetime lines are shown in this figure.

Stability of Atom and Gravitational Effects

By considering a large value for G at the subatomic scale, the gravitational effect of the nucleus along with the coulomb force must be considered as the reason for the stability of the electrons around the nucleus of atom. Based on this, in this section, we will investigate the simplest atom, namely the hydrogen atom. By using the Bohr atomic model we will obtain new equations for the radius of the hydrogen atom (1H), its mechanical energy, Rydberg constant and fine-structure constant. First, we need to review the Bohr atomic model.

In the Bohr model, Bohr assumed that the electron revolves in a circular orbit about the nucleus. and he supposed that the mass of the electron is completely negligible compared to the mass of the nucleus [12] The potential energy for the hydrogen atom is coulomb potential [12][13][14]:

$$E_p(r) = - \int F(r) dr = - \frac{k_e e^2}{r} \quad (2)$$

And the centripetal force is:

$$\frac{k_e e^2}{r^2} = \frac{m_e v^2}{r} \Rightarrow v = \sqrt{\frac{k_e e^2}{m_e r}}$$

Where k_e is coulomb constant. In the Bohr model, the weak gravitational potential is ignored, so the mechanical energy of the system is [13][14]:

$$E = E_K + E_P = \frac{1}{2} m_e v^2 + \left(-\frac{k_e e^2}{r}\right) \xrightarrow{\frac{k_e e^2}{r^2} = \frac{mv^2}{r}} E = -\frac{k_e e^2}{2r} \quad (3)$$

In Bohr model the orbital angular momentum of the electron, $L = m_e v r$, must be a constant, because the force acting on the electron is entirely in the radial direction. Applying the Bohr quantization condition to L [13], we have

$$L = n\hbar \Rightarrow m_e v r = n\hbar \quad (4)$$

Therefore, we will obtain the following equations for atomic radius [13][14]:

$$m_e v r = n\hbar \xrightarrow{v = \sqrt{\frac{k_e e^2}{m_e r}}} r_n = \frac{n^2 \hbar^2}{k_e m_e e^2} \quad n = 1, 2, 3, \dots \quad (5)$$

$$r_1 = \frac{\hbar^2}{k_e m_e e^2} = 0.528 \text{ \AA} \quad (6)$$

And

$$r_n = n^2 r_1 \quad (7)$$

Where n is the Principal Quantum number. For orbital velocity of electron, we have:

$$v = \frac{n\hbar}{m_e r} \xrightarrow{(5)} v_n = \frac{k_e e^2}{n\hbar} \quad (8)$$

$$v_1 = \frac{k_e e^2}{\hbar} \quad (9)$$

As you know, the definition of fine-structure constant or Somerfield's constant is the ratio of the speed of electron in the first orbit of Bohr atom to the velocity of light in vacuum [14]:

$$\alpha = \frac{v_1}{c} \xrightarrow{(9)} \alpha = \frac{k_e e^2}{\hbar c} = \frac{1}{137} \quad (10)$$

In addition, by using the equation 3 we have:

$$E_n = -\frac{k_e e^2}{2r_n} = -\frac{k_e e^2}{2n^2 r_1} \xrightarrow{(6)} E_n = -\frac{k_e^2 e^4 m_e}{2n^2 \hbar^2} \quad (11)$$

$$E_1 = -\frac{k_e^2 e^4 m_e}{2\hbar^2} = -13.5 \text{ eV} \quad (12)$$

$$E_n = -\frac{E_1}{n^2} \quad (13)$$

Based on Bohr model, the difference between the two energy states in the hydrogen atom is equal to $h\nu$:

$$\nu = \frac{E_{up}-E_{low}}{h} \xrightarrow{(13)} \nu = \frac{E_1}{h} \left(\frac{1}{n_l^2} - \frac{1}{n_u^2} \right) \xrightarrow{c=\nu\lambda} \frac{1}{\lambda} = \frac{E_1}{hc} \left(\frac{1}{n_l^2} - \frac{1}{n_u^2} \right) \quad (14)$$

Where n_u and n_l are the quantum numbers for the upper and lower energy states, respectively; and λ is the wavelength of radiation emitted by the atom[13][14]. The equation 14 is of precisely the same mathematical form as the empirically derived Rydberg formula:

$$\frac{1}{\lambda} = R \left(\frac{1}{n_l^2} - \frac{1}{n_u^2} \right) \quad (15)$$

Therefore:

$$R = \frac{E_1}{hc} \xrightarrow{(12)} R = \frac{k_e^2 e^4 m_e}{4\pi\hbar^3 c} = 1.0974 \times 10^{-3} \text{ \AA}^{-1} \quad (16)$$

Compare this equation with the experimentally determined value $1.0968 \times 10^{-3} \text{ \AA}^{-1}$. This experimental value is determined by the precise Spectroscopic Methods [14]. The strong agreement of the experimental value and the theoretical value of Rydberg constant was a great success for the Bohr model.

Now, we want to do recent calculations in the presence of the powerful gravitational effects of the nucleus of the hydrogen atom. Since the orbital velocity of the electron is much lower than the velocity of light ($v_1/c = 1/137$) [12], the gravitational potential (gP) of the hydrogen atom (1_1H) is non-relativistic, namely

$$E_{gP}(r) = -\frac{G_1 m_e m_p}{r} \quad r < 1 \text{ \AA} \quad (17)$$

Where m_e and m_p are the mass of electron and proton, respectively. In such a case, the new potential energy (E_{Pnew}) is equal to the sum of gravitational potential (E_{gP}) and electrical potential (E_{eP}). The mechanical energy is:

$$E = E_K + E_{Pnew} = E_K + [E_{gP} + E_{eP}]$$

It is clear, in such a case, the values of m_e , m_p and G_1 will enter in the calculations and the recent equations such as equation 6, 10 and 16 will change; therefore, the theoretical and experimental results will not be equal. To avoid this problem, new potential energy (E_{Pnew}) must be equal to old potential energy (equation 2). It means that, we will have a new value for Coulomb constant (namely k_{enew} instead of k_e), at the vicinity of electric charge ($r < 1 \text{ \AA}$). So we have:

$$E_{Pold} = E_{Pnew} \Rightarrow -\frac{k_e e^2}{r} = -\frac{k_{enew} e^2}{r} - \frac{G_1 m_e m_p}{r} \Rightarrow k_{enew} = k_e - \frac{G_1 m_e m_p}{e^2} \quad (18)$$

This is a very exciting idea. But, what does it mean? As you know, the definition of Coulomb constant is $1/4\pi\epsilon_0$. Therefore, the variation of Coulomb constant from k_e to k_{enew} means that when we approach to an electric charge, the amount of vacuum permittivity shifts from ϵ_0 to ϵ_{0new} . This is because of the variation of the gravity constant from G_0 to G_1 and the convergence and compression of Spacetime lines. Interestingly, because of the compression of Spacetime lines near the charged particle the vacuum permittivity changes. It means that the Spacetime lines interact with electric field lines.

So, based on equation 18, we have:

$$k_{enew} < k_e \xrightarrow{k_e = \frac{1}{4\pi\epsilon_0}} \epsilon_{0new} > \epsilon_0 \quad (19)$$

In the presence of gravity, the mechanical energy of the hydrogen atom is equal to

$$E = E_K + E_P = \frac{1}{2} m_e v^2 + \left(-\frac{k_{enew} e^2}{r} - \frac{G_1 m_e m_p}{r} \right) \quad (20)$$

And the centripetal force is:

$$\frac{m_e v^2}{r} = \frac{k_{enew} e^2}{r^2} + \frac{G_1 m_e m_p}{r^2} \quad (21)$$

$$v = \sqrt{\frac{k_{enew} e^2 + G_1 m_e m_p}{m_e r}} \quad (22)$$

By putting the equation 22 in equation 20, we have:

$$E = \frac{-(k_{enew} e^2 + G_1 m_e m_p)}{2r} \quad (23)$$

Compare this equation with equation 3. For atomic radius we have:

$$m_e v r_n = n\hbar \Rightarrow r_n = \frac{n\hbar}{m_e v} \xrightarrow{(22)} r_n = \frac{n^2 \hbar^2}{m_e (k_{enew} e^2 + G_1 m_e m_p)} \quad n = 1, 2, 3, \dots \quad (24)$$

$$r_1 = \frac{\hbar^2}{m_e (k_{enew} e^2 + G_1 m_e m_p)} \quad (25)$$

$$(23) \xrightarrow{(24)} E_n = \frac{-m_e (k_{enew} e^2 + G_1 m_e m_p)^2}{2n^2 \hbar^2} \quad (26)$$

$$E_{1new} = \frac{-m_e (k_{enew} e^2 + G_1 m_e m_p)^2}{2\hbar^2} \quad (27)$$

$$\frac{1}{\lambda} = \frac{E_{1new}}{hc} \left(\frac{1}{n_l^2} - \frac{1}{n_u^2} \right) \Rightarrow R_{new} = \frac{E_{1new}}{hc} \xrightarrow{(27)} R_{new} = \frac{k_{enew}^2 e^4 m_e}{4\pi \hbar^3 c} \quad (28)$$

$$R_{new} = \frac{k_{enew}^2 e^4 m_e}{4\pi \hbar^3 c} \xrightarrow{(18)} R_{new} = \frac{k_e^2 e^4 m_e - 2k_e G_1 m_e^2 m_p e^2 + G_1^2 m_e^3 m_p^2}{4\pi \hbar^3 c} \quad (29)$$

Where R_{new} is the new equation of Rydberg constant with the presence of G_1 , m_e and m_p . Compare equations 28 and 29 with equation 16. This new value of Rydberg constant is equal to its recent value ($1.0974 \times 10^{-3} \text{ \AA}^{-1}$) because of using equation 18. Addition to equation 22 the orbital velocity of electron is:

$$m_e v r = n \hbar \xrightarrow{(24)} v_n = \frac{(k_{enew} e^2 + G_1 m_e m_p)}{n \hbar} \quad (30)$$

And

$$\alpha_{new} = \frac{v_1}{c} = \frac{(k_{enew} e^2 + G_1 m_e m_p)}{c \hbar} \quad (31)$$

This is the new equation of the fine-structure constant. Compare this equation with equation 10. According to equation 18, the value of α_{new} is the same value as α ($1/137$). In the new equation of the fine-structure constant, α_{new} , special Relativity (c), electromagnetism (k_{enew}), Quantum mechanics ($\hbar$) and General relativity (G_1 and m) are unified by the number $1/137$.

Eventually, based on all of these discussions, the Coulomb force between the two charged particles like two electrons is equal to

$$F = \frac{k_e e^2}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2} \quad r > 1 \text{ \AA} \quad (32)$$

$$F = \frac{k_{enew} e^2}{r^2} = \frac{1}{4\pi\epsilon_{0new}} \frac{e^2}{r^2} \quad r < 1 \text{ \AA} \quad (33)$$

Atomic and Molecular Bonds

In this article, we show that a large value for G can be considered at the atomic scale. Based on this, in this section, we will investigate how the hydrogen molecule is composed of two Hydrogen atoms. A Hydrogen molecule is formed of the collision of two Hydrogen atoms in the laboratory and inside a star. Consider two hydrogen atoms (1H) collide with each other. Because of the strong gravitational force between the two electrons ($F = G_1 m_e^2 / r^2$), the electrons of the two hydrogen atoms will be attracted to each other (Fig. 2). In such a case, the distance (r) between the two electrons gradually narrows; and the Coulomb force ($F = k_{enew} e^2 / r^2$) gradually increases. Thus, the two electrons will approach together until the attraction gravitational force equals the repulsion Coulomb force. In this way, the bond between the two atoms of hydrogen is formed and the hydrogen molecule is formed. Here the nuclei of the two hydrogen atoms also interact with each other and the electrons. But, the electron-electron interaction is the most effective interaction in the hydrogen molecule.

Fig. 2. Hydrogen molecule formation. The two hydrogen atoms are attracted and form a bond. In the hydrogen molecule, the repulsion Coulomb force between the two electrons is equal to the attraction Gravitational force, based on this article. In the hydrogen molecule and in Quantum shape, the electron clouds of two atoms do not overlap because of the repulsion Coulomb force between the two electrons.

By using this description about the hydrogen molecule formation, it is easy to explain other atomic and molecular bonds. In the bonds between atoms and molecules with multiple electrons, several attraction and repulsion forces (arising from the electrons of inner and outer shells of atoms and nuclei) are involved in the formation of bonds and the magnitude of Bond-dissociation energy. For example, in the C–H bond, the electron of the hydrogen atom interacts with six electrons of Carbon atom and therefore, the bond energy of C–H is different from H–H. For this reason, the energy required to break one bond is different from the other. Here and in atomic bonds and molecular bonds, the gravitational interactions and Coulomb interactions between the electron-electron, electron-nucleus and nucleus-nucleus, depending on the distance between the two particles, can be explained through G_1 and K_{enew} or G_0 and K_e . In other words, in atomic bonds and molecular bonds, the particles which are close to each other ($r < 1\text{\AA}$), interact with each other with constants G_1 and K_{enew} and the particles which are at a distance interacts with each other with constants G_0 and K_e .

Conclusion

In this article, we proved that considering a large value for G , at the vicinity of matter, is a very appropriate choice for physics and chemistry. The justification of atomic and molecular bonds is very acceptable and understandable than covalent bond [15], van der Waals bond [15] and some other bonds [15] in chemistry. This article revealed that Spacetime lines interact with electric field lines. This can be a way to know the nature of electric field, electric charge and also magnetic field. Moreover, in this article, we obtained new equations for Rydberg constant and fine-structure

constant. The results and discussions of this paper will cause a great revolution in physics and chemistry.

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