“The Unreasonable Effectiveness of Mathematics”: A Universal Language of Contact?

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Abstract. In this essay we argue that the expectation that mathematics can serve as a universal foundation and conceptual framework for communication can be based on some implicit anthropocentric assumptions. In our view, the formation of foundations of mathematical thinking, including arithmetic and geometry can be determined to a large extent if not entirely, by the environment and the means of perception and processing of the sensory information about it, and for these reasons can be species-dependent and to a large degree, specific. The conclusion of this analysis is a form of semantic relativism in communication: that no absolute and universal frame of reference may exist for species attempting communication; and therefore, the success of such attempts is determined by finding or establishing a common conceptual frame of reference that is relative to them and specific to them.

Keywords: Mathematics, Semantics, Intelligent Systems.

1 Introduction

Following well-known exploration of the subject in “Contact” by Carl Sagan [2] many works, fictional and scientific, explored and promoted the concept that mathematics can be used as a foundation, or a common reference framework for communication between different species. In this way of thinking, π, “1 + 1 = 2”, intersecting and parallel lines, hydrogen wavelength and so on, are seen as universal semantic symbols that carry meaning for the interpreters regardless of their location in the Universe, origin, environment, evolution path and so on.

In this work arguments are made to the extent that such expectations can be based on implicit anthropocentric assumptions. Further, it is argued that forming frameworks and concepts of quantitative analysis of the phenomena in the environment of an intelligent species are likely defined by the type of the environment and sensory mechanisms of receiving and processing information about it. This line of argumentation, if substantiated by further analysis can lead one to the possibly unavoidable conclusion that an absolute, universal semantic framework, or frameworks may not exist, or, at least, its existence may not be ascertained; and that finding a common region of

“The Unreasonable Effectiveness of Mathematics in the Natural Sciences”, E. P. Wigner, [1]

perception that can be shared between the species and specific to their qualities and abilities can be the only foundation for successful communication.

2 A Natural Math in a Fuzzy World

Inspired in part by "Flatterland" [3], let us consider a thought experiment with intelligent species evolving in a significantly different physical environment. The world may look, for example, like:

- a dense, viscous media with high refraction;
- the species use sensory organs other than light-based to obtain information about the environment, for example, auditory, olfactory etc.
- the species feed on small and spread sources, like plankton. And are predated by populous schools of predators, like piranhas.

We would like to ask now, how likely it is that such a species would develop discrete whole number arithmetic to interpret a statement as "1 + 1 = 2"? There would be no possibility, nor evolutionary advantage in identification of individual constituents of either feeding or predating entities. A feeding area could be larger or smaller in size, denser or sparser but the count would always be one, or none. Such an environment may prompt the emergence of concepts of quantitative analysis that are continuous and fuzzy, with variable content whereas discrete and exactly defined finite sets would represent in it highly abstract and unusual limit case(s).

Basic operations could be entirely different in this world. "Addition" of two feeding areas may result in a feeding area with different characteristics rather than two identical feeding areas. In the event of an attack by a group of predators, it could be far more essential for the survival to know geometric and dynamical characteristics of the predator pack than the exact count of the group.

The simplest equation in the foundation of our natural arithmetic would have very little semantic meaning in such a world. And the need for multiplication, as a way of counting collections of identical entities may not emerge at all, or emerge much later. There goes "$2 \times 2 = 4$" as a universal symbol of inter-species communication. On the other hand, there can be more than type of the set union operation, always resulting in a binary count, but perhaps with very rich flavors and characteristics of the resulting sets.

Similar considerations can be applied to geometry. Complex and non-linear propagation of acoustic waves in a dense environment can trigger formation of geometric concepts that are "natural" in a complex possibly, multi-dimensional manifold defined by the acoustic characteristics of the physical environment, possibly very different from the lines and planes of Euclidean geometry.

Two parallel lines may not have essential meaning and carry little semantic significance for such an observer because homogeneousness and isotropy would not be natural or common notions in such an environment, and the trajectory of a "line", that is, the path of propagation of a sensory wave in the media may depend on the position it passes through. Far from universal, in this fuzzy world π may not have any essential meaning, as the ratio could depend on the position, size and orientation, again the

purported universality based on implicit assumptions of homogeneousness and isotropy of the perceived geometry of the surrounding environment.

All in all, the basic foundations of the mathematics as a conceptual framework of quantitative analysis of the environment in such a world could be very different from ours with a limited range of common semantic concepts that can be shared readily and interpreted unambiguously by the communicating parties.

3 Semantic Relativism

The thought experiment in the previous section suggests that the expectation of forming absolute and universal semantic frame of reference based on quantitative analysis of the environment can be overly optimistic. Not that such frameworks may not, or could not emerge; but based on the characteristics of the environment, its perception and interaction with it, the resulting quantitative frameworks can be defined to a large extent if not in entirety, by the characteristic features of the environment and the channels of perception and interpretation, with questionable universal content. Symbols and notions that are clear and familiar to us, like "π" and "$2 + 2 = 4$", as the very concepts of addition, multiplication and so on, may represent remote and odd cases for an observer from a different environment and of different origin; and vice versa, with little common semantic context and conceptual content that can be exchanged in communication.

Taken to the ultimate limit, this argument can lead to the conclusion that the expectation of existence of a universally shared semantic framework of reference may not be grounded or reasonable. In this eventuality, the only reasonable and possible alternative would be *semantic relativism*, whereby parties looking to communicate with an exchange of interpretable information would have to find a common frame of reference defined by their specific characteristics as only such common conceptual frame could provide a meaningful basis for exchange of information.

4 Conclusion

In this essay arguments were provided to a viewpoint that the expectations of mathematics or in fact, any other conceptual framework based on and derived from sensory information about the observed environment can be based on implicit anthropocentric assumptions. Further, it is argued that forming frameworks and concepts of quantitative analysis of the phenomena in the environment of an intelligent species are likely defined by the type of the environment and sensory mechanisms of receiving and processing information about it. It is possible that conceptual frameworks that reflect and describe essential patterns in the observed sensory environment form very early as a natural outcome of the principles of information processing, as a number of results in learning of both artificial and biologic systems seem to indicate [4-6]. As illustrated by the thought experiment, and based on these results, it is possible what conceptual frameworks, including those of quantitative nature, that form in the process of observation and processing of the sensory inputs from very different environments can be very different and carry little if any, common context and conceptual content.

This line of argumentation, if substantiated by further analysis can lead one to the conclusion that an absolute, universal semantic framework, or frameworks may not exist, or, at least, its existence cannot be ascertained; with a natural interpretation that finding a common region or context in the observed and perceived environment that can be shared between the species; that is relevant and specific to their qualities and capacities can be the only possible common frame of reference and a foundation for successful communication.

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