“Two bits less” after quantum-information conservation and their interpretation as “distinguishability / indistinguishability” and “classical / quantum”

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Abstract: The paper investigates the understanding of quantum indistinguishability after quantum information in comparison with the “classical” quantum mechanics based on the separable complex Hilbert space. The two oppositions, correspondingly “distinguishability / indistinguishability” and “classical / quantum”, available implicitly in the concept of quantum indistinguishability can be interpreted as two “missing” bits of classical information, which are to be added after teleportation of quantum information to be restored the initial state unambiguously. That new understanding of quantum indistinguishability is linked to the distinction of classical (Maxwell-Boltzmann) versus quantum (either Fermi-Dirac or Bose-Einstein) statistics. The latter can be generalized to classes of wave functions (“empty” qubits) and represented exhaustively in Hilbert arithmetic therefore connectible to the foundations of mathematics, more precisely, to the interrelations of propositional logic and set theory sharing the structure of Boolean algebra and two anti-isometric copies of Peano arithmetic.

Key words: Bose-Einstein statistics, Fermi-Dirac statistics, Hilbert arithmetic, Maxwell-Boltzmann statistics, qubit Hilbert space, quantum indistinguishability, quantum-information conservation, teleportation

I BACKGROUND: THE ORIGIN OF QUANTUM INDISTINGUISHABILITY

Two or many particles in classical mechanics can be always distinguished from each other due to the four space-time coordinates available always and ambiguously for each of them independently of their masses at rests, energies and momenta. Their trajectories (or “world lines” in relativity) are smooth, which is equivalent to their distinguishability.

Quantum mechanics, in virtue of the complementarity of space-time coordinates, on the one hand, and energy-momentum coordinates, on the other, does not allow for the above distinguishability: a restriction denoted as *quantum indistinguishability*.

Furthermore, the statistics of an ensemble of quantum entities (defined as quantum for the commensurability of their physical actions with the Planck constant) is different from the Maxwell-Boltzman statistics and subdivided into two types: either Bose-Einstein boson statistics or Fermi-Dirac fermion statistics. Both latter pass approximately into the former after actions much bigger than the Planck constant, what those in classical physics are.

After the Born interpretation of $|\Psi|^2$as the probability of the state with the wave function $\Psi$to be measured in a long enough series of experiments, fermion statistics refers to the pair solutions: $(+\ \Psi,\ -\ \Psi)$ and $(-\ \Psi,\ +\ \Psi)$, and boson statistics, to the pair of $(-\ \Psi,\ -\ \Psi)$ and $(+\ \Psi,\ +\ \Psi)$.

The link between quantum indistinguishability versus classical distinguishability, on the one hand, and any of both quantum statistics and Maxwell-Boltzmann statistics, on the other hand, is the following. If wave function $\Psi$is granted to distinguish quantum entities rather than smooth trajectory (in each point of which both pairs of space-time and energy-momentum coordinates are defined unambiguously), not more than two fermions can share the same wave function (the Pauli principle), but this restriction is not valid as to bosons and thus, an arbitrary number of them can share it. One might state that the wave function as an identifier is able to "name" all fermions almost unambiguously (i.e. to the nearest pair), but not as to all bosons.

The conjecture of "supersymmetry" doubling all fermions with their boson counterparts identical with the former ones in all quantities different from spin (as well as vice versa) is not confirmed experimentally even minimally. In the present context, this means that the indistinguishability of all bosons and the distinguishability of all fermions are not relative to each other, but a characteristic property of both groups.

The absence of supersymmetry is consistent with the experimentally very well confirmed "dark matter" and "dark energy" under a few additional conditions which follows:

(1) Fermion is linked to time irreversibility however in both possible time arrows (i.e. correspondingly "backwards in time" and "forwards in time").

(2) Boson corresponds to time reversibility (that of all coherent states in quantum mechanics).

(3) All bosons are able to interact physically out of space-time (Einstein's "spooky action at a distance") and that interaction can be identified directly as entanglement.

Additionally (4), if the wave-function pair distinguishability can be conserved, there exist troubles about the definition of entanglement directly as to fermions (e.g. Bañuls, Cirac, Wolf 2009)[1]. Indeed, the wave-function pair distinguishability may imply for the wave functions $\{\Psi\}$ of all pairs $\{+\ \Psi,\ -\ \Psi\}$ of fermions to be orthogonal to each other and thus sharing zero entanglement. The modality of "may" in the previous sentence is used for the following:

The conceptual relation of "quantum indistinguishability" and "entanglement" is rather confused or at least unclear and ambiguous (e.g. Benatti et al 2020). The deep reason is: entanglement is defined as a relation of wave functions relevant *only to the complement* of all Hermitian operators to all operators, both in the separable complex Hilbert space thus involving implicitly, but necessarily a continuous intermediate area of quantum entities being neither bosons, nor fermions. However, they are and can be observed experimentally only on the macroscopic "screen" of the definitive distinguishability of classical mechanics as quantum correlations of the absolutely distinguished apparatuses: i.e. *at least two* apparatuses and therefore representing the relation of *at least two wave* functions in *at least two* well-orderings (or reference frames and "observers") of space-time due to the gradual propagation of electromagnetic interaction.

[1] On the other hand: "Two bound, entangled fermions form a composite boson, which can be treated as an elementary boson as long as the Pauli principle does not affect the behavior of many such composite bosons" (Tichy, Bouvrie, Mølmer 2012: 1).

On the contrary, the meta-distinction of fermions and bosons suggests the intersection of them to be zero therefore forcing the intermediate area between them to be interpreted either only as bosons or only as fermions. In fact, one is to introduce the area of a "qubit" consisting of both Fermi-Dirac and Bose-Einstein statistics as well as their class of equivalence (i.e. the same qubit as "empty") traditionally represented by the Maxwell-Boltzman statistics being the the zero $(\alpha = 0, \beta = 0)$ point of the same qubit.

Rather figuratively, the conceptual mapping of entanglement into both quantum statistics is isomorphic to the mapping of a qubit into a bit in turn representable by the classical Maxwell-Boltzman statistics.

Under the above conditions or considerations, the absence of "supersymmetry" as what the proved availability of "dark matter" and "dark energy" can be interpreted implies a conceptual "equations" meaning an eventual asymmetry of bosons and fermions as their definitive conceptual "decoherence" in relation to "entanglement":

Time reversibility = *Time irreversibility* (in both directions[2]) + *Dark* (matter & energy):

Indistinguishability = *Distinguishability* + *"Darkness"*

II QUANTUM-INFORMATION CONSERVATION AFTER TELEPORTATION AND TWO MISSING BITS FOR DISTINGUISHABILITY

A wide group of phenomena of entanglement can be demonstrated by experiments of quantum teleportation between two space-time points A and B, in which two observers, Alice and Bob correspondingly, are situated in order to transmit quantum information from A to B. All experiments share the following framework:

The transmission of a qubit is representative enough. The velocity of transmission is granted to be infinite since entanglement acts at any distance instantaneously. However, two bits of classical information determining additionally Alice's qubit is impossible to be transferred by entanglement. So, they have to travel with the light speed in a vacuum at best, and Bob can receive them with a delay proportional to the length of the AB trajectory. Consequently, Alice's message can be restored either instantly, but to the nearest two bits, or absolutely, but after the corresponding delay.

On the other hand, the quantity of classical information contained in a qubit of quantum information is infinite (Penchev 2020 July 10) and thus, those additional or missing two bits cannot influence Alice's initial message: i.e. Bob need not wait for them.

The sense of the postponed two bits of classical information can be interpreted naturally as the values of the mapping of a qubit into a bit: entanglement transmits instantly the qubit itself, $Q = \alpha\perp_0 + \beta\perp_1$, but postponed, the binary equivalent, namely the relevant single combination of $\alpha \in (0, 1)$ and $\beta \in (0, 1)$in which the qubit results:

$$Q \to (0, 0);\ Q \to (0, 1);\ Q \to (1, 0);\ Q \to (1, 1).$$

[2] Irreversibility in both directions *separately*.

can exemplify Alice's message to be the qubit describing a certain entanglement statistics between the boson and fermion statistics as in the previous section. Then, the postponed information of two bits means correspondingly that the transferred statistics refers to the following one from those four: Maxwell-Boltzmann, Bose-Einstein, Fermi-Dirac, and entanglement statistics. Their unambiguous distinction from each other is possible only after the projection of the same qubit (or any quantum information) onto the well-ordering of space-time, and thus, does not make sense out of it.

III DISTINGUISHABILITY / INDISTINGUISHABILITY AND CLASSICAL VERSUS QUANTUM STATISTICS

Meaning the last conclusion, the interpretation of a statistics as just one among those four makes sense only if: (1) the space-time screen of classical science and the immediate human experience is established and thus opposed to the "ocean" of all quantum information in the universe, in which space-time is "mounted" and what is visible on it is only a relatively insignificant part (to say, about 5%) of all; (2) on the "screen" itself, still one opposition of a description in classical mechanics versus that in quantum mechanics is valid though the scientific common sense distinguishes them into two areas of experiments: the "tiny", microscopic quantum entities versus the big macroscopic bodies.

The abstract amount of classical information defined by the above two oppositions is exactly those two bits embodied then in the distinction of the four kinds of statistics, which hold only under the condition of the former.

Though the amount of two oppositions, each of which consists of two equally probable alternatives, is defined standardly to be two bits, it refers to a single bit of information in essence: namely, the one opposition means both extremes of indistinguishability versus distinguishability of the two alternatives, and the second one, those of the two alternatives itself. Meaning that, the four statistics are relevant to the single bit of the space-time "screen" to appear among the "ocean" of quantum information and then representable as two successive elementary oppositions.

Being a human being, Alice implies for her experience to be just human, i.e. space-time. However, the phenomenon of entanglement implementing the teleportation of a qubit is out of space-time as far as it is instantaneous. That qubit is to be restored by another observer, Bob, naturally suggested to be also a human being and therefore forced to wait for that single bit of classical information from Alice. Consequently, the sense of that single bil is: Alice is a human being, and thus space-time experience is postulated to be a necessary condition for her.

IV MAXWELL-BOLTZMANN STATISTICS BY QUANTUM-INFORMATION CONSERVATION

As far as one means only relation to the four statistics, Maxwell-Boltzmann statistics can be "bracketed", i.e. to be a conventional unit isomorphic to the unit radius of a qubit ball, or using the standard notations about the logarithm of the average number elements in a state: $ln\, \tilde{n}_i = k_B T(\mu - \varepsilon_i)$ where $k_B$is the Boltzmann constant, $T$is the absolute temperature, μ is the total chemical potential, and ε_i is the energy of the single-particle state *i*. That is: the "unit of the Maxwell-Boltzman statistics" denoted further as "1_{MBS}" is:

$$\{1_{MBS} \Leftrightarrow_{(def)} ln\,\tilde{n}_i = k_B T(\mu - \varepsilon_i)\} \Leftrightarrow \{1_{MBS} = \frac{ln\,\tilde{n}_i}{k_B T(\mu - \varepsilon_i)}\}.$$

Then, the Fermi-Dirac statistics for the same average number elements in a state as a function of the newly introduced "Maxwell-Boltzmann statistics unit" is:

$$FDS(\tilde{n}_i) = \frac{1_{MBS}}{1_{MBS}+1}$$

Respectively, as to to the Bose-Einstein statistics: $EBS(\tilde{n}_i) = \frac{1_{MBS}}{1_{MBS}-1}$.

Then, the condition for both quantum statistics to pass into MBS is the physical equivalent of a qubit to be much greater than the Planck constant (as the arithmetical unit "1" in the two last equations is to be interpreted physically).

This means that one has defined implicitly the quantity of quantum information measured in qubits as absolutely independent of the total physical action of the system, to which it refers. Speaking loosely, the "size of physical action does not matter". Quantum information by itself is a quantity which does not depend on how big an entity is (e.g.whether an atom or a galaxy): it can be the same independently of whether this is a macroscopic body or an elementary particle.

On the contrary, physical action depends on the "size" of energy or matter if the temporal unit is established to be the same. As far as the Planck constant is a natural unit of "counting", the relation of the quantities of action and quantum information is so:

Quantum information is a unit by itself and independently of how great a natural number is, particularly whether it refers to a finite number, to a transfinite ordinal number or to an ordinal number greater or equal to a countable ordinal number. On the contrary, action refers to the magnitude of the number at issue and quite, even qualitatively, different in general.

The "size" of action (respectively energy and matter) is visible only on the screen of space-time where all entities can be well-ordered according to it. However, the quantity and even the notion of "size" or "well-ordering" according to it does not make sense to quantum information by itself. So, the four statistics exist only on that screen and only on it (where Alice and Bob dwell properly and can dwell only). The additional bit serving to distinguish the four statistics means the following two binary questions: (1) is the screen introduced; (2) which of both opposite time arrows on it is chosen?

V HILBERT ARITHMETIC; THE STATISTICS IN ITS TERMS

Hilbert arithmetic is a generalization of Peano arithmetic relying on the qubit Hilbert space where any unit is defined as an "empty qubit" of the latter: i.e. as the class of equivalence of all values which any qubit can acquire (Penchev 2020 August 25).

The definitive condition for both quantum statistics to pass into the classical one is: $1_{MBS} = 1_{MBS} \pm 1$. Mathematically, it is satisfied rigorously if and only if 1_{MBS} refers to an actual infinite set, and quantum statistics, respectively, if it is not.

The sense of the case it does not hold can be visualized by the function defined on natural numbers as $QS(1_{MBS}) = \frac{1_{MBS}}{1_{MBS} \pm 1}$ tending to "1" in infinity and increasing in the finite area as 1_{MBS} is biger. So, if the arithmetical unit "1" relevant physically to quantum information can be alleged to be absolute and independent of how big a natural or ordinal number is, QS is the relative unit depending on and becoming less and less as the natural or ordinal number is bigger and bigger.

Accordingly, the two oppositions of the one bit (sharing the same explanation of the distinction between "oppositions" and "bit" as above) allow for the four statistics to be distinguished from each other on the temporal screen are the following after the reinterpretation by Hilbert arithmetic: (1) is the "screen of finiteness" introduced; (2) which of the two complementary, dual, and anti-isometric Peano arithmetic is chosen on it?

The former opposition can be equivalently reformulated so: which mathematical structure is chosen: either the non-ordered "set" (isomorphic to propositional logic in virtue of the sharing of Boolean algebra) or the well-ordered "natural numbers".

VI CONCLUSIONS

The main conclusion is: the concept of quantum statistics and / or quantum indistinguishability as well as quantum mechanics makes sense only on the space-time "screen" of classical statistics, distinguishability, and classical mechanics. In fact, this is only an extended description of Niels Bohr's fundamental postulate (or "quantum postulate") of what quantum mechanics studies: the system of the researched microscopic quantum entity and measuring apparatus by the reading of the latter (Bohr 1928: 1934).

However, the concept of quantum information allows for going further, to the world of quantum information by itself independently or out of the screen of time, only on which the "size" does make sense.

That general conclusion is exemplified by a series of visualizations referring to quantum and classical statistics, the teleportation and conservation of quantum information and Hilbert arithmetic considered in separate sections successively above.

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