Solar System Wave Function

Abolfazl Soltani *

Department of Physics, University of Birjand, Birjand, Iran

Email: soltani.a.phyics@gmail.com

Abstract

Pluto, Ceres and all planets of solar system except Neptune, with a high approximation, follow a law called Titius-Bode law (TBL) or Bode law, which can by no means be considered as a stochastic event. This law shows that the distance of the planets from the sun in Solar system is regulated. Here, we prove that the existence of a standing and cosine wave packet in solar system, with the wavelength $\lambda = 0.6\ AU$ (AU represents the distance of earth from the sun) and the phase constant $\emptyset_0 = \frac{\pi}{6}$, is the reason for TBL. Moreover, we prove that this huge wave packet belongs to the sun. In the first part of the article we will obtain the equation of this wave packet empirically; and in the second part we will prove that this equation is exactly in the form of the real part of the solution of the Schrodinger equation and that's why we can attribute it to an object like the sun (Certainly the presence of a standing wave in space is not without a origin). Finally in this article we will prove that without solar system wave function, it is not possible to reach to the TBL from the protoplanetary disk of solar system.

Keywords: Solar system, Titius-Bode law, Quantum mechanics, Schrodinger equation, Thought experiment,

1. Introduction

The planets of solar system move around the sun in elliptical orbits such that the sun is in one of the focal points of these ellipses. These ellipses are very close to the circle, and in fact the orbits of the planets of solar system are concentric circles. Pluto, Ceres and all planets of Solar system except Neptune, with a high approximation follow a law known as Bode law or Titius-Bode law (TBL). According to this law, the distance of each planet from the sun is equal to $a = 0.4\ AU + 0.3\ AU \times 2^n$, where $0.4\ AU$ is the distance of Mercury from the sun (or more precisely the length of the semi-major axis of Mercury's orbit) and $n = 0,1,2,3, ...$ [1]. Table. 1 shows the high accuracy of the Bode law. If this law was only true for three or four planets, then we could call it a coincidence, but when it is true for seven planets, plus Ceres and Pluto, there is definitely a reason for it. It was historically based on this law that Ceres was discovered in 1801 [1]. In this article, we will find the reason for the existence of the TBL. In fact, we will prove that the presence of a cosine and standing wave packet in solar system is the reason for existence of TBL; and moreover, we will prove that this wave packet belongs to the sun. In this article we prove that the Schrodinger equation is valid at the astronomical scale and is not limited to subatomic particles. TBL does not predict the distance of Neptune from the sun but, this article is able to give us the distance of Neptune. In this paper and in the section "Elliptical orbits and Isotropic Asteroid Belt", we prove that nebular theory, which explains the formation of the solar system, without considering the solar system wave function (SSWF) is incomplete and wrong. We will prove that

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| 39 | without solar system wave packet, it is not possible to reach to the TBL from the protoplanetary |
| 40 | disk. |

Planet	T–B rule distance (AU)	Semi-major axis (AU)	Deviation from prediction
Mercury	0.4	0.39	−2.5%
Venus	0.7	0.72	+2.8%
Earth	1.0	1.00	0.00%
Mars	1.6	1.52	−4.77%
Ceres	2.8	2.77	−1.16%
Jupiter	5.2	5.20	+0.00%
Saturn	10.0	9.58	−4.45%
Uranus	19.6	19.20	−1.95%
Pluto	38.8	39.48	+1.05%

41 **Table. 1. Planets distances from the sun and the prediction of Bode law.** TBL cannot predict the distance of
42 Neptune from the sun.

2. Wave Function and Bode Law

44 Consider a standing and cosine wave function with a wavelength $\lambda = 0.6\ AU$ in solar system; if
45 we assume that the first node of this wave is at a distance of $0.1\ AU$ from the sun the next nodes
46 are at the distances of $0.4\ AU, 0.7\ AU, 1\ AU, 1.3\ AU, 1.6\ AU, \ldots, 2.8\ AU, \ldots$ from the sun. Each
47 node is $0.3\ AU$ ahead of the previous node. If we consider the planets of solar system in the position
48 of the nodes of this wave, in such a case, there is no planet on the first node ($0.1\ AU$) and Mercury
49 is on the second node, Venus is on the third node, earth is on the fourth node, Mars is on the sixth
50 node, and the position of fifth node ($1.3\ AU$) is empty. The seventh, eighth, and ninth nodes are
51 empty, and Ceres is on the tenth node. Jupiter is placed on the eighteenth node and Saturn is on
52 the thirty-third node, and Uranus, Neptune, and Pluto are on the nodes farther from the sun. As
53 you can see, a wave function, with the wavelength $\lambda = 0.6\ AU$, easily predicts the position of the
54 planets and it seems that a huge and standing wave plays a role in determining the position of the
55 planets in solar system. Therefore, we can consider the reason for the TBL to be the existence of
56 a large cosine wave in solar system that oscillates along the axis perpendicular to the plane of solar
57 system. We call this wave "Solar system wave function". In this article, we will obtain the equation
58 of this wave function. But what does this wave belong to? We answer this question in this article.
59 The presence of a huge cosine wave in solar system seems strange at first sight, but quantum
60 mechanics eradicates our surprise. Based on quantum mechanics, a wave packet can be attributed
61 to each object which called the "associated wave" of that object, and this associated wave is the
62 solution of the Schrodinger equation. In this article, we prove that the above standing and cosine
63 wave function (Solar system wave function) is in the form of a solution of the Schrodinger equation
64 and therefore, based on quantum mechanics, this wave must belong to an object in Solar system;
65 we demonstrate that this object is the sun (We are aware that today Quantum mechanics, the
66 Schrodinger equation, and the de Broglie wavelength relation only use for subatomic scale and
67 subatomic objects. But, in this article, we prove that quantum mechanics is also valid in
68 astronomical scale and we will obtain the shape of Schrodinger equation and de Broglie
69 wavelength relation in astronomical scale).

3. Wave Function of Solar System

As mentioned, a cosine and standing wave function, with the wavelength $\lambda = 0.6\ AU$ ($k = \frac{2\pi}{\lambda} = \frac{10\pi}{3}$), can predict the position of the planets in solar system. First we want to derive the phase constant ($\emptyset_0$) of this wave function. Any wave in which the variables x and t are entered as a combination of $kx \pm wt$ is a traveling wave [2]. For example, $\sin(kx - wt + \emptyset_0)$ is a traveling wave. Thus, a standing wave is in the form of $\cos(wt)\cos(kx + \emptyset_0)$ or $\sin(wt)\cos(kx + \emptyset_0)$ or $\sin(wt)\sin(kx + \emptyset_0)$ or $\cos(wt)\sin(kx + \emptyset_0)$. As mentioned, a cosine standing wave can predict the positions of planets. Therefore, the form of the standing wave of solar system must be either $\sin(wt)\cos(kx + \emptyset_0)$ or $\cos(wt)\cos(kx + \emptyset_0)$ (In the following, we choose one of these two forms). For the nodes of these two standing waves we have $\cos(kx + \emptyset_0) = 0$. As mentioned previously, Mercury is on the second node of Solar system wave function (the second node corresponds to the phase $\frac{3\pi}{2}$ because $Cos\ \frac{3\pi}{2} = 0$). We have:

$$x_{Mercury} = 0.4\ AU \Rightarrow \psi(x_{Mercury}) = 0 \Rightarrow Cos(kx_M + \emptyset_0) = 0 \Rightarrow kx_M + \emptyset_0 = \frac{3\pi}{2} \xRightarrow{k=\frac{10\pi}{3}} \emptyset_0 = \frac{\pi}{6}$$

Having k and $\emptyset_0$, we can easily find the position of the other planets using the equation $kx + \emptyset_0 = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \dots, \frac{(2m-1)\pi}{2}$ (Where m is the node number). For example

$$kx_{Venus} + \emptyset_0 = \frac{5\pi}{2} \Longrightarrow \frac{10\pi}{3}x_{Venus} + \frac{\pi}{6} = \frac{5\pi}{2} \Longrightarrow x_{Venus} = 0.7\ AU$$

or

$$kx_{Earth} + \emptyset_0 = \frac{7\pi}{2} \Longrightarrow \frac{10\pi}{3}x_{Earth} + \frac{\pi}{6} = \frac{7\pi}{2} \Longrightarrow x_{Earth} = 1\ AU$$

$$kx_{Mars} + \emptyset_0 = \frac{11\pi}{2} \Longrightarrow \frac{10\pi}{3}x_{Mars} + \frac{\pi}{6} = \frac{11\pi}{2} \Longrightarrow x_{Mars} = 1.6$$

The distances of the other planets can also be calculated in the same way, which is quite consistent with experience. According to the above equation ($kx + \emptyset_0 = \frac{(2m-1)\pi}{2}$), Neptune is on the ninety-eighth node, which corresponds to the phase $\frac{195\pi}{2}$. Contrary to the TBL, which is not able to predict the distance of Neptune, our wave theory predicts the position of Neptune. Therefore, a cosine and standing wave function with $\emptyset_0 = \frac{\pi}{6}$ and $k = \frac{10\pi}{3}$ can be attributed to solar system. But what is the general equation of this wave function? As mentioned, the function of solar system must contain a component with the equation $\cos(\frac{10\pi}{3}x + \frac{\pi}{6})$ and on the other hand, this wave must be standing so that the position of the nodes (planets) does not change. Therefore, as mentioned, the form of solar system wave function must be either $\cos(\delta wt)\cos(\frac{10\pi}{3}x + \frac{\pi}{6})$ or $\sin(\delta wt)\cos(\frac{10\pi}{3}x + \frac{\pi}{6})$. There is no difference between $\cos(\delta wt)$ and $\sin(\delta wt)$ Because we know from trigonometric identities that: $\cos(\delta wt) = \sin\left(\delta wt + \frac{\pi}{2}\right)$. Therefore, we choose the

100 function $cos(\delta wt)\, cos(\frac{10\pi}{3}x + \frac{\pi}{6})$ and then we will show that our choice is correct (δ is a constant
101 number that we will derive its value). Since solar system has a certain size and is not infinitely
102 wide, its wave function must be localized (a wave packet). If we consider an expression in the
103 form $e^{-\gamma x^2}$ (which is a Gaussian function and plays the role of a wave envelope) in the final
104 function of solar system, in such a case, the final equation is a localized wave or a wave packet[1].
105 The value of γ, which is a positive number, will be obtained in the following. Thus, the primary
106 form of the wave function of solar system is as follows (equation 1) and the planets are on the
107 nodes of this wave function (Fig. 1):

108
$$\begin{cases} \psi(x,t) = C\, cos(\delta wt)\, cos(\frac{10\pi}{3}x + \frac{\pi}{6})\, e^{-\gamma x^2} & x \geq 0 \\ \psi(x,t) = C\, cos(\delta wt)\, cos(\frac{10\pi}{3}x - \frac{\pi}{6})\, e^{-\gamma x^2} & x \leq 0 \end{cases} \tag{1}$$

109 In equation 1, γ, C and δ are constant values and we obtain their values in this article. This is an
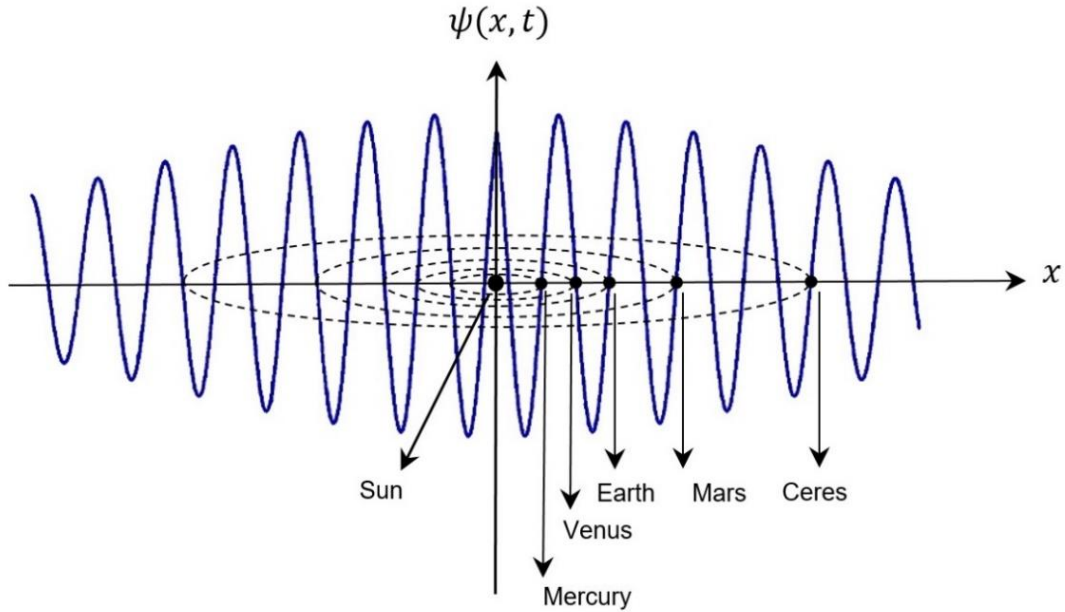110 empirical equation which we will obtain it from mathematic methods in the next section.

111

112 **Fig. 1. Solar system standing wave packet with $\lambda = 0.6$ _AU_ and $\emptyset_0 = \frac{\pi}{6}$.** Diagram of $\psi(x,t)$ at the moment t = 0.
113 The value of $\psi(0,0)$ equals $\sqrt{3}C/2$. This diagram is drawn by a certain value of C, δ and γ in equation 1, which we
114 will obtain their value in this article. As you can see, the planets are on the nodes of the wave function. Jupiter, Saturn,
115 Uranus, Neptune, and Pluto are on the nodes farther from the sun. The reason why there is no planet in some nodes
116 will explained in the section 6: "Elliptical orbits and Isotropic Asteroid Belt". This is due to the unbalanced mass
117 distribution in the protoplanetary disk of solar system.

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[1] Although a Gaussian wave packet is also infinitely wide, but it is very localized.

118   In figure 1, the wave oscillates along the ψ axis over time. But the nodes and the anti-nodes do not
119   move relative to each other along the x-axis. This does not mean that the wave packet is stationary
120   in the space; it is just like passengers sitting on a train who do not move relative to each other but
121   the train is moving relative to the rails. In the same way, solar system wave packet (Fig. 1) is a
122   standing wave that rotates, along with solar system, around the center of the galaxy.

123   As you observed, function 1 could easily predicts the position of planets. In the continuation of the
124   article, we will prove that this function is in the form of the real part of a solution of the Schrodinger
125   equation (As you know, the solutions of the Schrodinger equation have two parts: Real and
126   Imaginary) and that's why we can attribute it to an object like the sun. And in the section "Elliptical
127   orbits and Isotropic Asteroid Belt" we will show that how we can arrive to the TBL from the
128   protoplanetary disk, by oscillation of solar system wave function.

4. Single-Frequency Wave Packet

130   We obtained the equation of a single-frequency wave packet in the previous section, empirically
131   (equation 1). But, a thought experiment [3] arrives us to this conclusion that a single-frequency
132   wave packet cannot be existed. This thought experiment investigates the beat between two single-
133   frequency waves. In this thought experiment, it is proved that only a wave with infinite spreading
134   can be single-frequency. This means that a wave packet, which is localized, cannot be single
135   frequency. This experiment apparently has a logical and unambiguous face. but there is a very
136   important problem about thought experiments. A thought experiment, even if has a logical shape,
137   may lead to wrong results. We prove this with two examples. A well-known example is the famous
138   experiment of Michelson and Morley about the velocity of light [4]. If you do the Michelson and
139   Morley experiment in your mind[2], you will conclude that the velocity of light depends on the
140   velocity of observer and source. This was something that Michelson and Morley also expected.
141   But when their experiments were performed in the real world, they concluded that the velocity of
142   light is independent of the velocity of observer and source. As another example, before the advent
143   of particle accelerators, it was thought that an object could be propelled at any velocity, even
144   beyond the speed of light. But particle accelerators showed us that we could not go beyond the
145   speed of light. This two examples show that we cannot comment on the results of an experiment
146   until it is not performed in the real world. These examples fully prove that a thought experiment
147   is meaningless because you cannot rely on its results and its results may be wrong. Obviously, this
148   rule (Unreliability of the results of a thought experiment) also is true for the mentioned beat thought
149   experiment. And therefore we cannot comment on the results of the beat experiment until it is
150   performed. Which it means that we cannot comment on the existence or nonexistence of single-
151   frequency wave packet. In addition, attention to the empirical equation 1 we have a single-
152   frequency wave packet and therefore the beat thought experiment is contrary to observation and
153   so is wrong.

154   In the next section, we will obtain equation 1 mathematically and we will show that the equation
155   1 is the real part of the solution of the Schrodinger equation.

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[2] In fact, all experiments are initially a thought experiment. Some of them remain a thought experiment due to the
human inability to perform the experiment in the real world, and some become real physical experiment.

5. Associated Wave Packet of Sun

In this section we will show that the equation 1 is exactly in the form of the real part of the solution of the Schrodinger equation that's why we can attribute it to an object like the sun. Base of calculations in this section is superposition principle. Consider a set of infinite number of flat matter waves $Ae^{i(kx-wt+\emptyset_0)}$, $Ae^{i(kx+wt+\emptyset_0)}$, $Ae^{i(kx-wt-\emptyset_0)}$ and $Ae^{i(kx+wt-\emptyset_0)}$ which move in the positive and negative directions of the x-axis; and assume that all of these waves are under the effect of potential $V(x)$. These four groups of waves are all of possible form of flat matter waves. In such a case the angular frequency (w) of each of these matter waves is equal:

$$w = \frac{E}{\hbar} = \frac{1}{\hbar}(\frac{P^2}{2m} + V(x))$$

Because, based on De Broglie theory, the energy of a matter wave is equal to the energy of associated particle of the wave i.e. $E = \frac{P^2}{2m} + V(x)$. As you know these four groups of waves with general equation $Ae^{i(kx\pm wt\pm\emptyset_0)}$ are the solution of the Schrodinger equation namely

$$i\hbar\frac{\partial}{\partial t}\psi(x,t) = -\frac{\hbar^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2} + V(x)\psi(x,t)$$

These waves sum together in space based on superposition principle. From these infinite number of waves we will show that the superposition of a part of these waves, which their angular frequency equals w_0 and their wave number is around the median of k_0 and between $k_0 + \frac{\Delta k}{2}$ and $k_0 - \frac{\Delta k}{2}$ and the amplitude changes of these waves is equal to $A(k) = (\frac{2\alpha}{\pi})^{1/4}e^{-\alpha(k-k_0)^2}$ (which is a Gaussian function[3]), is in the form of equation 1.

First consider a set of infinite number of matter waves $Ae^{i(kx-wt+\emptyset_0)}$ which move in the positive direction of x-axis and their angular frequency equals w_0 and their wave number is around the median of k_0 and between $k_0 + \frac{\Delta k}{2}$ and $k_0 - \frac{\Delta k}{2}$ and the amplitude changes of these waves is equal to $A(k) = (\frac{2\alpha}{\pi})^{1/4}e^{-\alpha(k-k_0)^2}$. In such a case, the resultant of these waves, using the superposition principle, is a wave packet with equation 2 [5][6][7].

$$\psi_{total}(x,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}A(k)e^{i(kx-w_0t+\emptyset_0)}dk \qquad (2)$$

Where k means k_x. In equation $A(k) = (\frac{2\alpha}{\pi})^{1/4}e^{-\alpha(k-k_0)^2}$, α is a constant with a positive value and shows the width of the bell-shaped function $A(k)$. $(\frac{2\alpha}{\pi})^{1/4}$ is a normalization coefficient which

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[3] In the Electromagnetic (EM) waves we cannot consider one w_0 for two or many waves in which their k is different from each other, because for all of the EM waves we have: $w = ck$ where c is the velocity of light. But for matter waves the issue is different. In the matter waves we have $w = \frac{\hbar k^2}{2m}$ [7]. As you can see w is the function of k and m. Therefore, it is possible to choose one value of w_0 for the waves in which their k is different from each other.

182 is obtained by normalize of $A(k)$. Since equation 2 is derived from the superposition principle, it
183 is the solution of the Schrodinger equation [7].

184 To obtain $\psi_{total}(x,t)$ from equation 2, we calculate the superposition of all of the waves in one
185 moment, which we consider to be the origin of time ($t = 0$), and then we can obtain the net wave
186 at any other time. We have:

187
$$\psi(x,0) = \frac{1}{\sqrt{2\pi}} \int A(k)e^{i(kx+\emptyset_0)}dk \tag{3}$$

188 The above equation is the momentary image of the net wave. Multiply equation 3 by $e^{ik_0x-ik_0x}$.
189 We have:

190
$$\psi(x,0) = \frac{1}{\sqrt{2\pi}} e^{i(k_0x+\emptyset_0)} \int A(k)e^{i(k-k_0)x}dk \tag{4}$$

191 Considering $\acute{k} = k - k_0$, we have:

192
$$\psi(x,0) = (\frac{\alpha}{2\pi^3})^{1/4} e^{i(k_0x+\emptyset_0)} \int e^{-\alpha \acute{k}^2} e^{i\acute{k}x}d\acute{k} \tag{5}$$

193 Using the variable transformation $\acute{k} - \frac{ix}{2\alpha} = q$ [5][6] and the Gaussian integral
194 $\int_{-\infty}^{\infty} dq\, e^{-\alpha q^2} = \sqrt{\frac{\pi}{\alpha}}$, equation 5 can be calculated. After replacement and simplification, we reach
195 the following final solution [5][6]:

196
$$\psi(x,0) = (\frac{\alpha}{2\pi^3})^{1/4} \sqrt{\frac{\pi}{\alpha}}\, e^{i(k_0x+\emptyset_0)}\, e^{-\frac{x^2}{4\alpha}} = (\frac{1}{2\pi\alpha})^{1/4} e^{i(k_0x+\emptyset_0)}\, e^{-\frac{x^2}{4\alpha}} \tag{6}$$

197 Lets check the normalization

198
$$\int_{-\infty}^{\infty} |\psi(x)|^2\, dx = \sqrt{\frac{1}{2\pi\alpha}} \int_{-\infty}^{\infty} e^{-\frac{x^2}{2\alpha}}\, dx = \sqrt{\frac{1}{2\pi\alpha}} \sqrt{2\alpha\pi} = 1$$

199 Given a normalized $A(k)$, we get the normalized $\psi(x)$.

200 Now, how is the time variation of equation 6? Let's go back to equation 2:

201
$$\psi(x,t) = \frac{1}{\sqrt{2\pi}} \int A(k)e^{i(kx-w_0t+\emptyset_0)}dk = (\frac{\alpha}{2\pi^3})^{1/4} \int e^{-\alpha(k-k_0)^2} e^{i(kx-w_0t+\emptyset_0)}dk$$

202 Substituting $e^{ik_0x-ik_0x}$ in the equation:

203
$$\psi(x,t) = (\frac{\alpha}{2\pi^3})^{1/4} e^{i(k_0x+\emptyset_0)-iw_0t} \int e^{-\alpha \acute{k}^2} e^{i\acute{k}x}d\acute{k}$$

204 This integral is similar to integral 5, which led to $\psi(x,0)$ (Equation 6). Therefore, we have:

7

205
$$\psi(x,t) = \psi_1(x,t) = (\frac{1}{2\pi\alpha})^{1/4} e^{i(k_0 x - w_0 t + \emptyset_0)} e^{-\frac{x^2}{4\alpha}} \qquad (7)$$

206
$$e^{i\theta} = cos\theta + isin\theta \implies Re\ \psi_1(x,t) = (\frac{1}{2\pi\alpha})^{1/4} cos(k_0 x - w_0 t + \emptyset_0)\ e^{-\frac{x^2}{4\alpha}} \qquad (8)$$

207 Due to the presence of the factor $k_0 x - w_0 t$, equations 7 and 8 represent a traveling wave packet
208 that propagates in the positive direction of the x-axis [2]. This means that the location of the nodes
209 is not known. Due to the absence of t in $e^{-\frac{x^2}{4\alpha}}$ in equations 7 and 8, the wave packets in these
210 equations does not spread.

211 Previous calculations was about superposition of the waves $e^{i(kx - w_0 t + \emptyset_0)}$. Similarly, we use the
212 recent trend to obtain the superposition of flat waves traveling in the negative direction of
213 the x-axis, i.e. $Ae^{i(kx + w_0 t + \emptyset_0)}$. If we do this, we get to equation 9:

214
$$\psi_2(x,t) = (\frac{1}{2\pi\alpha})^{1/4} e^{i(k_0 x + w_0 t + \emptyset_0)} e^{-\frac{x^2}{4\alpha}} \qquad (9)$$

215
$$Re\ \psi_2(x,t) = (\frac{1}{2\pi\alpha})^{1/4} cos(k_0 x + w_0 t + \emptyset_0)\ e^{-\frac{x^2}{4\alpha}} \qquad (10)$$

216 This equation shows a traveling wave packet that propagates in the negative direction of
217 the x-axis.

218 Now we sum up the two equations 10 and 8 together to get the final wave.

219
$$Re\ \psi_{total}(x,t) = Re\ \psi_1 + Re\ \psi_2$$

220 Thus:

221
$$Re\ \psi_{total}(x,t) = (\frac{1}{2\pi\alpha})^{1/4} e^{-\frac{x^2}{4\alpha}}[cos(k_0 x - w_0 t + \emptyset_0) + cos(k_0 x + w_0 t + \emptyset_0)] \qquad (11)$$

222 Using $cos\alpha + cos\beta = 2cos\frac{1}{2}(\alpha + \beta)cos\frac{1}{2}(\alpha - \beta)$ and $cos(\theta) = cos(-\theta)$ we obtain the
223 equation of a standing wave packet.

224
$$\begin{cases} \alpha = k_0 x - w_0 t + \emptyset_0 \\ \beta = k_0 x + w_0 t + \emptyset_0 \end{cases} \implies Re\ \psi_{total}(x,t) = 2(\frac{1}{2\pi\alpha})^{1/4} cos(k_0 x + \emptyset_0)cos(w_0 t)e^{-\frac{x^2}{4\alpha}} \qquad (12)$$

225 There is not the structure of $kx \pm wt$ in equation 12 so the ψ_{total} is a standing wave. As you
226 observe, equation 12, which is the real part of a solution of the Schrodinger equation, is exactly
227 the same as equation 1 for $x \geq 0$, which is solar system wave function. Is this similarity
228 coincidental? No. Therefore equation 1 is the real part of a solution of the Schrodinger equation.
229 It means that the Schrodinger equation and quantum mechanics are valid in astronomical scale. By
230 comparing equation 12 and equation 1, we have

231
$$\delta = 1\ ,\ \gamma = \frac{1}{4\alpha}\ \ and\ \ C = 2(\frac{1}{2\pi\alpha})^{1/4}$$

232 If we put these values in equation 1, then we get the final equation of solar system wave function
233 for $x \geq 0$:

$$Re\, \psi_t(x,t) = 2(\frac{1}{2\pi\alpha})^{1/4} cos(w_0 t) cos(\frac{10\pi}{3}x + \frac{\pi}{6})\, e^{-\frac{x^2}{4\alpha}} \qquad x \geq 0 \qquad (13)$$

235 Equation 13 is obtained by calculating the superposition of a set of infinite number of waves
236 $Ae^{i(kx-w_0 t+\emptyset_0)}$ and $Ae^{i(kx+w_0 t+\emptyset_0)}$ that move in opposite directions to each other (pay attention
237 to the + sign behind $\emptyset_0$). Now if we sum a set of infinite number of flat wave functions with the
238 equations $Ae^{i(kx-w_0 t-\emptyset_0)}$ and $Ae^{i(kx+w_0 t-\emptyset_0)}$ (pay attention to the − sign behind $\emptyset_0$) together, by
239 following the path we have taken from equation 2 to equation 13, we reach the following relation;

$$Re\, \psi_t(x,t) = 2(\frac{1}{2\pi\alpha})^{1/4} cos(w_0 t) cos(\frac{10\pi}{3}x - \frac{\pi}{6})\, e^{-\frac{x^2}{4\alpha}}$$

241 Which is the same as equation 1 for $x \leq 0$. Therefore, the final form of solar system wave function
242 (equation 1) is as follows:

$$\begin{cases} Re\, \psi(x,t) = 2(\frac{1}{2\pi\alpha})^{1/4} cos(w_0 t) cos(\frac{10\pi}{3}x + \frac{\pi}{6})\, e^{-\frac{x^2}{4\alpha}} & x \geq 0 \\ Re\, \psi(x,t) = 2(\frac{1}{2\pi\alpha})^{1/4} cos(w_0 t) cos(\frac{10\pi}{3}x - \frac{\pi}{6})\, e^{-\frac{x^2}{4\alpha}} & x \leq 0 \end{cases} \qquad (14)$$

244 In this equation, the larger the α is, the more the width of wave packet, along the x-axis. We drew
245 Fig. 1 by $\alpha = 10$. In this section we did not do anything strange. Rather, we have used only the
246 superposition principle. We calculated the superposition of infinite number of flat matter waves
247 with general equation $e^{i(kx\pm wt\pm\emptyset_0)}$ which their angular frequency equals w_0 and the wave number
248 of these waves is around the median of k_0 and between $k_0 + \frac{\Delta k}{2}$ and $k_0 - \frac{\Delta k}{2}$ and the
249 amplitude changes of these waves is equal to $A(k) = (\frac{2\alpha}{\pi})^{1/4}e^{-\alpha(k-k_0)^2}$ and they are under the
250 effect of potential $V(x)$[4]. There have been infinite number of flat matter waves in the early solar
251 system which the superposition of a set of them made the wave function in figure 1. We will talk
252 more about this in the section "Elliptical Orbits and Isotropic Asteroid Belt".

253 Here we demonstrated that solar system wave function (equation 1) is the real part of a solution of
254 the Schrodinger equation. So, based on quantum mechanics, we can attribute it to an object in
255 Solar system (Certainly the presence of a standing wave in space is not without a source). The
256 closest star to solar system is at a distance of 4.8 light-years, which is so far. And the biggest and
257 heavyset object in solar system is sun. Therefore, the wave function of solar system can only
258 belong to the sun. In the section "Elliptical orbits and Isotropic Asteroid Belt", we will discuss
259 more about the formation of the solar system wave packet. De Broglie considered the wave nature
260 for subatomic particles, and here we attributed the wave nature to celestial objects. Neither of these
261 two actions is strange. Rather, they are truths that we must become accustomed to.

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[4] Since the solar system is under effect of gravity of Milky Way galaxy center; therefore here we investigated the
superposition of matter waves which they are under the effect of potential.

In this article, we proved that the Schrodinger equation is valid in astronomical scale; on the other hand, as you know, the Schrodinger relation is based on de Broglie equation ($\lambda = \frac{h}{mv}$). Therefore, the de Broglie equation is valid in astronomical scale[5]. But, according to the very large mass of sun, using the de Broglie relation the wavelength 0.6 AU will not obtain. So, instead of Planck constant we must choose another value for celestial objects, which is larger than h. We call this new value the Planck constant in Astronomy ($h_{Astronomy}$) abbreviated as h_A and we have: $\lambda_A = \frac{h_A}{P}$. In such a case, the Schrodinger equation in the astronomical scale can be written as follows:

$$i\hbar_A \frac{\partial}{\partial t}\psi(x,t) = -\frac{\hbar_A^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2} + V(x)\psi(x,t) \tag{15}$$

If we follow the path of proving the Schrodinger equation [8] and put the value $\hbar_A$ instead of $\hbar$, we reach equation 15. The Davisson–Germer experiment [9] is considered as the confirmation of existence of the de Broglie wave at the atomic scale, and the regularity of the distances of the planets from sun (Titius-Bode rule) is the confirmation of the existence of the de Broglie wave in astronomical scale. Moreover, for the celestial wave packet we will have $w = \frac{\hbar_A k^2}{2m}$.

As you will see in the next section, the oscillation of solar system wave function collected the gas and dust particles of Protoplanetary disk, in certain orbits around the newborn sun and then gradually due to collision and gravitational attraction between the dusts and gas particles, Planetesimals, Protoplanets and finally the planets in the orbits were formed.

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[5] The Davisson–Germer experiment [9] is the confirmation of the existence of the de Broglie wave in subatomic scale and the regularity of the distances of the planets from sun (Titius-Bode rule) is the confirmation of the existence of the de Broglie wave in astronomical scale.

6. Elliptical Orbits and Isotropic Asteroid Belt

In this section we will show that the nebular theory without our wave function theory is an incomplete theory and our wave function model must be attached to the nebular theory in order to explain the formation of the solar system.

The equation 14 in cylindrical coordinate is:

$$x = r \cos\theta \Rightarrow Re\,\psi(r,\theta,t) = 2(\frac{1}{2\pi\alpha})^{1/4} \cos(w_0 t) \cos(\frac{10\pi}{3} r \cos\theta + \frac{\pi}{6})\, e^{-\frac{(r\cos\theta)^2}{4\alpha}} \quad (16)$$

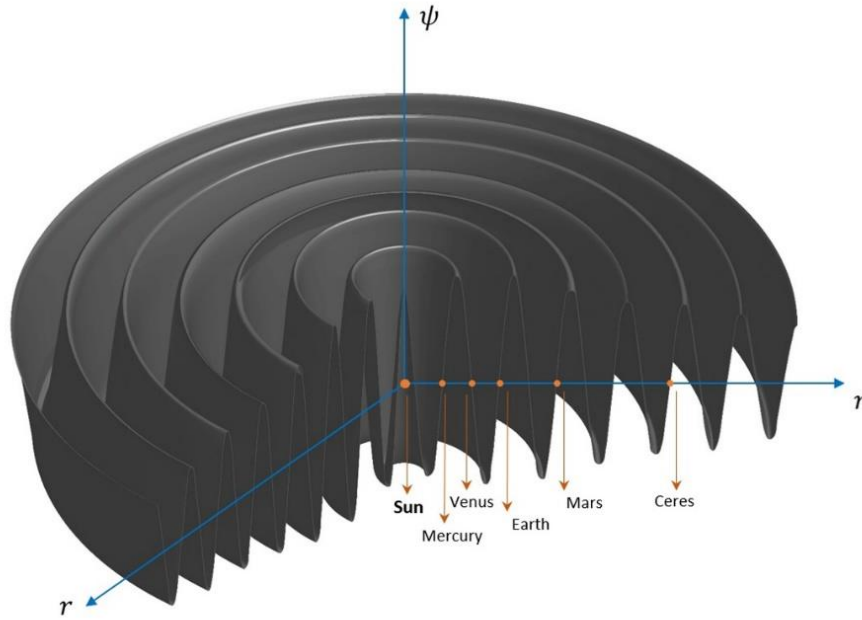Figure 2 is the shape of figure 1 in three dimensions based on equation 16:

Fig. 2. The oscillation of solar system wave function in three dimensions. Here the wave function in Figure 1 is shown in three dimensions. Fig 2 is drawn in cylindrical coordinates using equation 16. Obviously, this wave function continues indefinitely, based on equation 1 (or equation 16). Because of existence the exponential factor in equation 16, the amplitude of the wave gradually decreases while we move away from the ψ-axis.

As you can see in Fig. 1 and Fig. 2, because of symmetry of equation 1, the orbits, created by the solar system wave function, are circle. But the real orbits of the planets are elliptic. The reason for this is the existence of the inverse_square gravitational force of the sun[6]. As you know, the sun formed earlier than the planets [10][11][12]. Simultaneously with the formation of the sun, about 4.6 billion years ago [11], its wave function was also formed[7], and the oscillation of this wave

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[6] We know from classical mechanics that the elliptic orbits of the planets (Kepler's first law) are the result of Newton's law of gravitation, which is an inverse_square relation

[7] The wave function of the solar system probably was formed either when the sun was a protostar or when the newborn sun was entered to the Main-sequence. The distance between these two phases is very short (less than 50 million years) [10] and both phases occurred before the formation of the planets. In both states, we have no idea how or why this wave function formed.

function arranged and collected the gas and dust particles of the Protoplanetary disk in regular orbits (Titius-Bode orbit). Just like the standing wave patterns on the kettledrum head (Fig. 3) As you can observe in Fig.3, by a mechanical oscillator at the upper left of the photograph, the powder collects at the nodes [13]. As the same way, because of oscillation of solar system wave function, in some nodes of the wave function the gas and dust grains of protoplanetary disk agglomerated in the circular nodes (like figure 3) and then compressed due to collisions with each other[8] and formed larger grains, Planetesimals, Protoplanets and finally planets[9]. At the same time, the inverse_square force of the sun was at work, and changed the circular orbits to elliptical orbits. Don't you see resemblance between the powder ring in Figure 3 and the asteroid belt? The mass distribution in the asteroid belt is uniform and isotropic, just like the powder ring in Figure 3, and this is another confirmation for our wave theory (In asteroid belt, Planet formation has stopped at the Planetesimals stage; And because of orbital resonances with Jupiter Protoplanet and planet was not formed [15]). Can a theory other than our wave theory explain the uniform and isotropic distribution of Planetesimals in the asteroid belt?

If solar system wave function did not exist; the planets might have been formed around the sun but the distance of planets from the sun was random and irregular. Without solar system wave packet, it is not possible to reach to the TBL from the protoplanetary disk. Thus, it seems that the existence of a standing wave in the solar system is undeniable. Therefore, the nebular theory becomes more complete by our wave function theory.

The reason why there is no planet in some nodes in figure 1 and 2 is due to the unbalanced mass distribution in the protoplanetary disk of solar system.

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[8] "Collisions" were more like gentle touches. The particles were far too small to attract each other gravitationally at this point, they were able to stick together through electrostatic forces—the same "static electricity" that makes hair stick to a comb [14].

[9] Since the mass distribution in the Protoplanetary disk had not been uniform. During formation of the planets, in some nodes of the solar system wave function, less dusts were collected and in some nodes, more dusts were placed next to each other. Therefore, in some nodes a planet had been formed and in others it had not been formed.

Fig. 3. Standing wave pattern on a kettledrum head. One of many possible standing wave patterns on a kettledrum head, made visible by dark powder sprinkled on the drumhead. As the head is set into oscillation at a single frequency by a mechanical oscillator at the upper left of the photograph, the powder collects at the nodes. By the same way, due to the oscillation of the solar system wave function the dust grains and gas of the solar system protoplanetary disk were grouped in certain orbits.

7. What is Waving?

What is the nature of solar system wave function? As you know, since the solution of the Schrodinger equation is a complex function; It is meaningless to talk about what is waving [16][17]. But in this section we will show that the real part of the solution of the Schrodinger equation **probably** is the oscillation of Dark matter.

In this article, we showed that a standing wave oscillation had been effective in the formation of orbits of solar system planets and created the Bode rule. On the other hand, we know that the Dark matter interacts with the matters and affects them[10]. Therefore, it can be concluded that the standing wave oscillation in Fig. 1, with the equation 1, probably is the oscillation of Dark matter (Or dark matter is one of the candidate for the nature of solar system wave function). On the other hand, the equation 1 is the real part of a solution of the Schrodinger equation. Therefore, the real part of the solution of the Schrodinger equation is the oscillation of Dark matter. Of course, the Spacetime also interacts with matter and affects it. But we cannot consider the oscillation of Spacetime as the wave of the solar system. Because in such a case the curvature of Spacetime lines in figure 2 causes the planets to move along the r-axis. I think this article could give scholars new ideas about dark matter. Of course, there is another possibility. This wave may not be dark matter. It might be something we are not familiar with.

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[10] In addition to gravitational interactions, dark matter may also have elastic collision (like collision between drumhead and dark powder particles in fig 3) with baryonic matter (namely objects made with protons, neutrons, and electrons like gas-dust particles in protoplanetary disk)

8. Solar System Wave Function and the Other Theories

Up-to-date astronomy books, such as the very famous book: ''Astronomy'' by Professor Michael Zeilik (9th edition, 2002), usually list seven observational features for the solar system, which one of them is the TBL [18]. To compile our article, we read a lot about the attempt and work of previous scholars. We explain here the features of the existing theories in the language of a good book namely '' The Titius-Bode Law of Planetary Distances'' by Professor Michael Martin Nieto (1972) [19]. Due to curiosity of Mr. Nieto about the reason for TBL (As he says in the Foreword of his book), he carried out a comprehensive study about theories that had been formed until 1972. In this book, he investigates about 13 theories, which are about reason of TBL. We read in this book at the beginning of chapter 11: ((. . . the theories about Bode law can roughly be categorized as either: a) electromagnetism theories b) gravitational theories c) nebular theories . . . Unfortunately, many of the theories that we will review are "theories" and "Laws" in large part due to assumptions or arguments of possibility rather than by calculations that predict definite and unambiguous results . . . Too often the Titius-Bode Law has been discussed in terms of some mechanism which could conceivably produce the Law if all other physical effects could be ignored. Sad to say, that is not the type of physical universe with which we are dealing. Thus, one should be aware of the fact that any claim that a particular theory of the solar system is correct since it explains the Titius-Bode Law is not valid because, as of yet, no theory has properly explained this Rule. What can be said of some theories is that they are compatible with and indicative of a Titius-Bode Law, but not very much more.)) [20]. For example, In his three article, Hannes Alfven hypothesized that if he consider the original solar dipole moment 100000 times larger than at present and ignore the angular momentum in the early solar system [21] He could explain the mass distribution in the solar system. Or Hendrick Petrus Berlage, in his theory, assumed that the Sun emitted positive ions and negatively charged, solid particles that had condensed in the solar atmosphere. Berlage then further hypothesized that all the planet rings were formed a constant distance $d = 0.4$ AU further out from the ion rings (because of the addition of momentum to the growing Planetesimals via radiation pressure) [22]. But in our article is not any simplification or assumption. Our paper, without eliminating the physical parameters of solar system, justifies the Bode law and elliptical orbits. The oscillation of solar system wave function put up the gas-dust particles of protoplanetary disk in the circular orbits (nodes) just like Figure 3. In the first four pages of the article, we simply arrived to the TBL using wave theory. The important point is that none of the 13 theories examined in Professor Nieto's book easily reach the TBL. And most of them give this law or the other shape of it by approximating. Of course using a lot of assumptions and simplifications. None of these theories is satisfactory.

It seems that no specific work has been done on the cause of the TBL since 1970. In Professor Peter Lynch's 2003 article, we read:'' Despite the distinguished part the Titius–Bode law has played in the evolution of our knowledge of planetary dynamics, no theoretical explanation of it has been advanced that has found general acceptance.''[23]

In addition, some calculations and attempts were made which showed that TBL was a matter of chance. But in 2003, Professor Peter Lynch denied that it was a coincidence: '' . . . this conclusion

is unsafe, and that the possibility that the observed regularity in the patterns of the planetary and satellite systems has some physical explanation is still open.''[23].

Conclusion

In this article we proved that almost there is no doubt about the existence of a wave function in the solar system. In this article we proved that the nebular theory without our wave function theory is an incomplete theory and our wave function model must be attached to the nebular theory in order to explain the formation of the solar system. Without the solar system wave packet, it is not possible to reach to the TBL from the protoplanetary disk. Can a theory other than wave theory explain the uniform and Isotropic distribution of mass in the asteroid belt? De Broglie considered the wave nature for subatomic particles, and here we attributed the wave nature to celestial objects. Neither of these two actions is strange. Rather, they are truths that we must become accustomed to. In this article we proved that the Schrodinger equation is also valid for celestial objects and in astronomical scale we need a larger Planck constant instead of present Planck constant.

Based on this article, the associated wave packet of an object (such as electron, proton or sun) is one of the candidates for the nature of dark matter. I think this article could give scholars new ideas about dark matter.

Here, we proved that the word of " thought experiment " is meaningless and an experiment must be done in the real world so that we can talk about its results.

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