Solar System Wave Function

Abolfazl Soltani

Department of Physics, University of Birjand, Birjand, Iran

Email: soltani.a.phyics@gmail.com

Abstract

Pluto, Ceres and all planets of solar system except Neptune, with a high approximation, follow a law called Titius-Bode law (TBL) or Bode law, which can by no means be considered as a stochastic event. This law shows that the distance of the planets from the sun in Solar system is regulated. Here, we prove that the existence of a standing and cosine wave packet in solar system, with the wavelength λ = 0.6 AU (AU represents the distance of earth from the sun) and the phase constant $\emptyset_0 = \frac{\pi}{6}$, is the reason for TBL. Moreover, we prove that this huge wave packet belongs to the sun. In the first part of the article we will obtain the equation of this wave packet empirically; and in the second part we will prove that this equation is exactly in the form of the real part of the solution of the Schrodinger equation and that's why we can attribute it to an object like the sun (Certainly the presence of a standing wave in space is not without a origin). Finally in this article we will prove that without solar system wave function, it is not possible to reach to the TBL from the protoplanetary disk of solar system. Here we prove that the nebular theory without our wave theory is an incomplete theory.

Keywords: Solar system, Titius-Bode law, Quantum mechanics, Schrodinger equation, Nebular theory

1. Introduction

The planets of solar system move around the sun in elliptical orbits such that the sun is in one of the focal points of these ellipses. These ellipses are very close to the circle, and in fact the orbits of the planets of solar system are concentric circles. Pluto, Ceres and all planets of Solar system except Neptune, with a high approximation follow a law known as Bode law or Titius-Bode law (TBL). According to this law, the distance of each planet from the sun is equal to $a = 0.4\,AU + 0.3\,AU \times 2^n$, where 0.4 AU is the distance of Mercury from the sun (or more precisely the length of the semi-major axis of Mercury's orbit) and $n = 0,1,2,3, \ldots.$ [1]. Table. 1 shows the high accuracy of the Bode law. If this law was only true for three or four planets, then we could call it a coincidence, but when it is true for seven planets, plus Ceres and Pluto, there is definitely a reason for it. It was historically based on this law that Ceres was discovered in 1801 [1]. In this article, we will find the reason for the existence of the TBL. In fact, we will prove that the presence of a cosine and standing wave packet in solar system is the reason for existence of TBL; and moreover, we will prove that this wave packet belongs to the sun. In this article we prove that the Schrodinger equation is valid at the astronomical scale and is not limited to subatomic particles. TBL does not predict the distance of Neptune from the sun but, this article is able to give us the distance of Neptune. In this paper and in the section "Elliptical orbits and Isotropic Asteroid Belt", we prove that nebular theory, which explains the formation of the solar system, without considering the solar system wave function (SSWF) is incomplete. We will prove that without solar system wave packet, it is not possible to reach to the TBL from the protoplanetary disk. The distribution of Planetesimals in the asteroid belt is isotropic. In this Article we prove that this isotropy only can explain by our wave theory.

Planet	T–B rule distance (AU)	Semi-major axis (AU)	Deviation from prediction
Mercury	0.4	0.39	−2.5%
Venus	0.7	0.72	+2.8%
Earth	1.0	1.00	0.00%
Mars	1.6	1.52	−4.77%
Ceres	2.8	2.77	−1.16%
Jupiter	5.2	5.20	+0.00%
Saturn	10.0	9.58	−4.45%
Uranus	19.6	19.20	−1.95%
Pluto	38.8	39.48	+1.05%

Table. 1. Planets distances from the sun and the prediction of Bode law. TBL cannot predict the distance of Neptune from the sun.

2. Wave Function and Bode Law

Consider a standing and cosine wave function with a wavelength $\lambda = 0.6\ AU$ in solar system; if we assume that the first node of this wave is at a distance of $0.1\ AU$ from the sun the next nodes are at the distances of $0.4\ AU$, $0.7\ AU$, $1\ AU$, $1.3\ AU$, $1.6\ AU$, ... $2.8\ AU$, … from the sun. Each node is $0.3\ AU$ ahead of the previous node. If we consider the planets of solar system in the position of the nodes of this wave, in such a case, there is no planet on the first node ($0.1\ AU$) and Mercury is on the second node, Venus is on the third node, earth is on the fourth node, Mars is on the sixth node, and the position of fifth node ($1.3\ AU$) is empty. The seventh, eighth, and ninth nodes are empty, and Ceres is on the tenth node. Jupiter is placed on the eighteenth node and Saturn is on the thirty-third node, and Uranus, Neptune, and Pluto are on the nodes farther from the sun. As you can see, a wave function, with the wavelength $\lambda = 0.6\ AU$, easily predicts the position of the planets and it seems that a huge and standing wave plays a role in determining the position of the planets in solar system. Therefore, we can consider the reason for the TBL to be the existence of a large cosine wave in solar system that oscillates along the axis perpendicular to the plane of solar system. We call this wave "Solar system wave function". In this article, we will obtain the equation of this wave function. But what does this wave belong to? We answer this question in this article. The presence of a huge cosine wave in solar system seems strange at first sight, but quantum mechanics eradicates our surprise. Based on quantum mechanics, a wave packet can be attributed to each object which called the "associated wave" of object, and this associated wave is the solution of the Schrodinger equation. In this article, we prove that the above standing and cosine wave function (Solar system wave function) is in the form of a solution of the Schrodinger equation and therefore, based on quantum mechanics, this wave must belong to an object in Solar system; we demonstrate that this object is the sun (We are aware that today Quantum mechanics, the Schrodinger equation, and the de Broglie wavelength relation only use for subatomic scale and subatomic objects. But, in this article, we prove that quantum mechanics is also valid in astronomical scale and we will obtain the shape of Schrodinger equation and de Broglie wavelength relation in astronomical scale).

3. Wave Function of Solar System

As mentioned, a cosine and standing wave function, with the wavelength λ = 0.6 AU ($k = \frac{2\pi}{\lambda} = \frac{10\pi}{3}$), can predict the position of the planets in solar system. First we want to derive the phase constant ($\emptyset_0$) of this wave function. Any wave in which the variables x and t are entered as a combination of $kx \pm wt$ is a traveling wave [2]. For example, $\sin(kx - wt + \emptyset_0)$ is a traveling wave. Thus, a standing wave is in the form of $cos(wt)\, cos(kx + \emptyset_0)$ or $sin(wt)\, cos(kx + \emptyset_0)$ or $sin(wt)\, sin(kx + \emptyset_0)$ or $\cos(wt)\sin(kx + \emptyset_0)$. As mentioned, a cosine standing wave can predict the positions of planets. Therefore, the form of the standing wave of solar system must be either $sin(wt)\, cos(kx + \emptyset_0)$ or $cos(wt)\, cos(kx + \emptyset_0)$ (In the following, we choose one of these two forms). For the nodes of these two standing waves we have $cos(kx + \emptyset_0) = 0$. As mentioned previously, Mercury is on the second node of Solar system wave function (the second node corresponds to the phase $\frac{3\pi}{2}$ because $Cos\ \frac{3\pi}{2} = 0$). We have:

$$x_{Mercury} = 0.4\ AU \Rightarrow \psi(x_{Mercury}) = 0 \Rightarrow Cos(kx_M + \emptyset_0) = 0 \Rightarrow kx_M + \emptyset_0 = \frac{3\pi}{2} \xrightarrow{k=\frac{10\pi}{3}} \emptyset_0 = \frac{\pi}{6}$$

Having k and $\emptyset_0$, we can easily find the position of the other planets using the equation $kx + \emptyset_0 = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots\ldots, \frac{(2m-1)\pi}{2}$ (Where m is the node number). For example

$$kx_{Venus} + \emptyset_0 = \frac{5\pi}{2} \Longrightarrow \frac{10\pi}{3} x_{Venus} + \frac{\pi}{6} = \frac{5\pi}{2} \Longrightarrow x_{Venus} = 0.7\ AU$$

or

$$kx_{Earth} + \emptyset_0 = \frac{7\pi}{2} \Longrightarrow \frac{10\pi}{3} x_{Earth} + \frac{\pi}{6} = \frac{7\pi}{2} \Longrightarrow x_{Earth} = 1\ AU$$

$$kx_{Mars} + \emptyset_0 = \frac{11\pi}{2} \Longrightarrow \frac{10\pi}{3} x_{Mars} + \frac{\pi}{6} = \frac{11\pi}{2} \Longrightarrow x_{Mars} = 1.6$$

The distances of the other planets can also be calculated in the same way, which is quite consistent with experience. According to the above equation (namely $kx + \emptyset_0 = \frac{(2m-1)\pi}{2}$), Neptune is on the ninety-eighth node, which corresponds to the phase $\frac{195\pi}{2}$. Contrary to the TBL, which is not able to predict the distance of Neptune, our wave theory predicts the position of Neptune. Therefore, a cosine and standing wave function with $\emptyset_0 = \frac{\pi}{6}$ and $k = \frac{10\pi}{3}$ can be attributed to solar system. But what is the general equation of this wave function? As mentioned, the function of solar system must contain a component with the equation $cos(\frac{10\pi}{3} x + \frac{\pi}{6})$ and on the other hand, this wave must be standing so that the position of the nodes (planets) does not change. Therefore, as mentioned, the form of solar system wave function must be either $cos(\delta wt)\, cos(\frac{10\pi}{3}x + \frac{\pi}{6})$ or $sin(\delta wt)\, cos(\frac{10\pi}{3}x + \frac{\pi}{6})$. There is no difference between $cos(\delta wt)$ and $sin(\delta wt)$ Because we know from trigonometric identities that: $cos(\delta wt) = \sin\left(\delta wt + \frac{\pi}{2}\right)$. Therefore, we choose the function $cos(\delta wt)\, cos(\frac{10\pi}{3}x + \frac{\pi}{6})$ and then we will show that our choice is correct (δ is a constant number that we will derive its value). Since solar system has a certain size and is not infinitely wide, its wave function must be localized (a wave packet). If we consider an expression in the form $e^{-\gamma x^2}$ (which is a Gaussian function and plays the role of a wave envelope) in the final function of solar system, in such a case, the final equation is a localized wave or a wave packet[1]. The value of γ, which is a positive number, will be obtained in the following. Thus, the primary form of the wave function of solar system is as follows (equation 1) and the planets are on the nodes of this wave function (Fig. 1):

$$\begin{cases} \psi(x,t) = C\, cos(\delta wt)\, cos(\frac{10\pi}{3}x + \frac{\pi}{6})\ e^{-\gamma x^2} & x \geq 0 \\ \psi(x,t) = C\, cos(\delta wt)\, cos(\frac{10\pi}{3}x - \frac{\pi}{6})\ e^{-\gamma x^2} & x \leq 0 \end{cases} \tag{1}$$

In equation 1, γ, C and δ are constant values and we obtain their values in this article. This is an empirical equation which we will obtain it from mathematic methods in the next section.

[1] Although a Gaussian wave packet is also infinitely wide, but it is very localized.

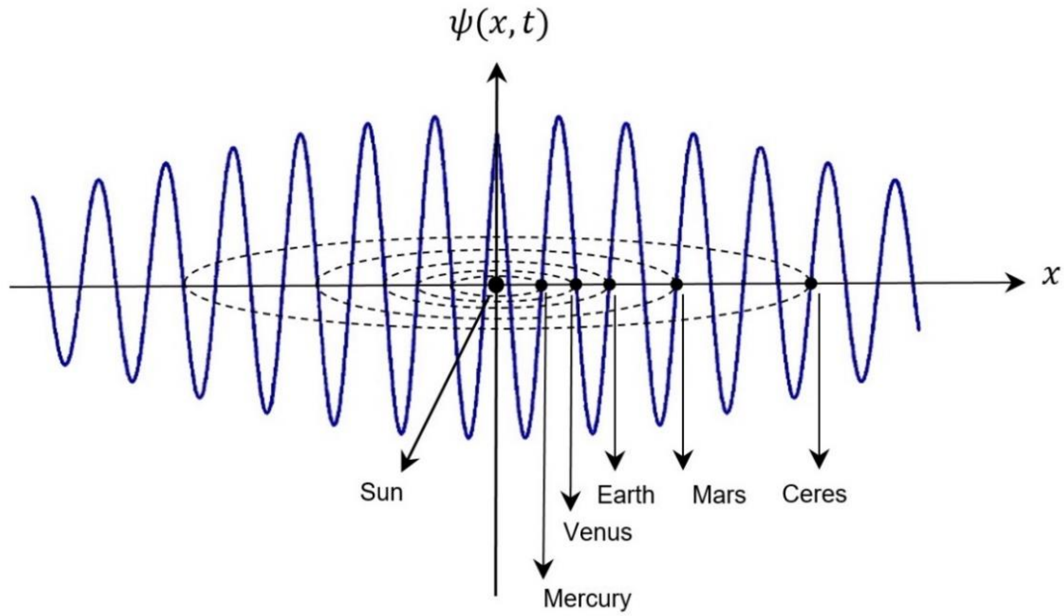

Fig. 1. Solar system standing wave packet with λ = 0.6 AU and $\emptyset_0 = \frac{\pi}{6}$. Diagram of $\psi(x,t)$ at the moment t = 0. The value of $\psi(0,0)$ equals $\sqrt{3}C/_2$. This diagram is drawn by a certain value of C, δ and γ in equation 1, which we will obtain their value in this article. As you can see, the planets are on the nodes of the wave function. Jupiter, Saturn, Uranus, Neptune, and Pluto are on the nodes farther from the sun. The reason why there is no planet in some nodes will explained in the section 5: "Elliptical orbits and Isotropic Asteroid Belt". This is due to the unbalanced mass distribution in the protoplanetary disk of solar system.

In figure 1, the wave oscillates along the ψ axis over time. But the nodes and the anti-nodes do not move relative to each other along the x-axis. This does not mean that the wave packet is stationary in the space; it is just like passengers sitting on a train who do not move relative to each other but the train is moving relative to the rails. In the same way, solar system wave packet (Fig. 1) is a standing wave that rotates, along with solar system, around the center of the galaxy.

As you observed, function 1 could easily predicts the position of planets. In the continuation of the article, we will prove that this function is in the form of the real part of a solution of the Schrodinger equation (As you know, the solutions of the Schrodinger equation have two parts: Real and Imaginary) and that's why we can attribute it to an object like the sun. And in the section "Elliptical orbits and Isotropic Asteroid Belt" we will show that how we can arrive to the TBL from the protoplanetary disk, by oscillation of solar system wave function.

4. Associated Wave Packet of Sun

In this section we will show that the equation 1 is exactly in the form of the real part of the solution of the Schrodinger equation that's why we can attribute it to an object like the sun. Base of calculations in this section is superposition principle. Consider a set of infinite number of flat matter waves $Ae^{i(kx-wt+\emptyset_0)}$, $Ae^{i(kx+wt+\emptyset_0)}$, $Ae^{i(kx-wt-\emptyset_0)}$ and $Ae^{i(kx+wt-\emptyset_0)}$ which move in the positive and negative directions of the x-axis; and assume that all of these waves are under the effect of potential $V(x)$. These four groups of waves are all of possible form of flat matter waves. In such a case the angular frequency (w) of each of these matter waves is equal:

$$w = \frac{E}{\hbar} = \frac{1}{\hbar}\left(\frac{P^2}{2m} + V(x)\right)$$

Because, based on De Broglie theory, the energy of a matter wave is equal to the energy of associated particle of the wave i.e. E=$\frac{P^2}{2m} + V(x)$. As you know these four groups of waves with general equation $Ae^{i(kx \pm wt \pm \emptyset_0)}$ are the solution of the Schrodinger equation namely

$$i\hbar\frac{\partial}{\partial t}\psi(x,t) = -\frac{\hbar^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2} + V(x)\psi(x,t)$$

These waves sum together in space based on superposition principle. From these infinite number of waves we will show that the superposition of a part of these waves, which their angular frequency equals w_0 and their wave number is around the median of k_0 and between $k_0 + {\Delta k}/_2$ and $k_0 - {\Delta k}/_2$ and the amplitude changes of these waves is equal to $A(k) = (\frac{2\alpha}{\pi})^{1/4} e^{-\alpha(k-k_0)^2}$(which is a Gaussian function[2]), is in the form of equation 1.

First consider a set of infinite number of matter waves $Ae^{i(kx - wt + \emptyset_0)}$ which move in the positive direction of x-axis and their angular frequency equals w_0 and their wave number is around the median of k_0 and between $k_0 + {\Delta k}/_2$ and $k_0 - {\Delta k}/_2$ and the amplitude changes of these waves is equal to $A(k) = (\frac{2\alpha}{\pi})^{1/4} e^{-\alpha(k-k_0)^2}$. In such a case, the resultant of these waves, using the superposition principle, is a wave packet with equation 2 [3][4][5].

$$\psi_{total}(x,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} A(k) e^{i(kx - w_0 t + \emptyset_0)} dk \qquad (2)$$

Where k means k_x. In equation $A(k) = (\frac{2\alpha}{\pi})^{1/4} e^{-\alpha(k-k_0)^2}$, α is a constant with a positive value and shows the width of the bell-shaped function $A(k)$. $(\frac{2\alpha}{\pi})^{1/4}$ is a normalization coefficient which is obtained by normalize of $A(k)$. Since equation 2 is derived from the superposition principle, it is the solution of the Schrodinger equation [5].

To obtain $\psi_{total}(x,t)$ from equation 2, we calculate the superposition of all of the waves in one moment, which we consider to be the origin of time ($t = 0$), and then we can obtain the net wave at any other time. We have:

[2] In the Electromagnetic (EM) waves we cannot consider one w_0 for two or many waves in which their k is different from each other, because for all of the EM waves we have: $w = ck$ where c is the velocity of light. But for matter waves the issue is different. In the matter waves we have $w = \frac{\hbar k^2}{2m}$ [5]. As you can see w is the function of k and m. Therefore, it is possible to choose one value of w_0 for the waves in which their k is different from each other.

$$\psi(x,0) = \frac{1}{\sqrt{2\pi}} \int A(k) e^{i(kx+\emptyset_0)} dk \tag{3}$$

The above equation is the momentary image of the net wave. Multiply equation 3 by $e^{ik_0x-ik_0x}$. We have:

$$\psi(x,0) = \frac{1}{\sqrt{2\pi}} e^{i(k_0x+\emptyset_0)} \int A(k) e^{i(k-k_0)x} dk \tag{4}$$

Considering $k' = k - k_0$, we have:

$$\psi(x,0) = (\frac{\alpha}{2\pi^3})^{1/4} e^{i(k_0x+\emptyset_0)} \int e^{-\alpha k'^2} e^{ik'x} dk' \tag{5}$$

Using the variable transformation $k' - \frac{ix}{2\alpha} = q$ [3][4] and the Gaussian integral $\int_{-\infty}^{\infty} dq\, e^{-\alpha q^2} = \sqrt{\frac{\pi}{\alpha}}$, equation 5 can be calculated. After replacement and simplification, we reach the following final solution [3][4]:

$$\psi(x,0) = (\frac{\alpha}{2\pi^3})^{1/4} \sqrt{\frac{\pi}{\alpha}}\, e^{i(k_0x+\emptyset_0)}\, e^{-\frac{x^2}{4\alpha}} = (\frac{1}{2\pi\alpha})^{1/4} e^{i(k_0x+\emptyset_0)}\, e^{-\frac{x^2}{4\alpha}} \tag{6}$$

Lets check the normalization

$$\int_{-\infty}^{\infty} |\psi(x)|^2\, dx = \sqrt{\frac{1}{2\pi\alpha}} \int_{-\infty}^{\infty} e^{-\frac{x^2}{2\alpha}}\, dx = \sqrt{\frac{1}{2\pi\alpha}} \sqrt{2\alpha\pi} = 1$$

Given a normalized $A(k)$, we get the normalized $\psi(x)$.

Now, how is the time variation of equation 6? Let's go back to equation 2:

$$\psi(x,t) = \frac{1}{\sqrt{2\pi}} \int A(k) e^{i(kx-w_0t+\emptyset_0)} dk = (\frac{\alpha}{2\pi^3})^{1/4} \int e^{-\alpha(k-k_0)^2} e^{i(kx-w_0t+\emptyset_0)} dk$$

Substituting $e^{ik_0x-ik_0x}$ in the equation:

$$\psi(x,t) = (\frac{\alpha}{2\pi^3})^{1/4} e^{i(k_0x+\emptyset_0)-iw_0t} \int e^{-\alpha k'^2} e^{ik'x} dk'$$

This integral is similar to integral 5, which led to $\psi(x,0)$ (Equation 6). Therefore, we have:

$$\psi(x,t) = \psi_1(x,t) = (\frac{1}{2\pi\alpha})^{1/4}\, e^{i(k_0x-w_0t+\emptyset_0)}\, e^{-\frac{x^2}{4\alpha}} \tag{7}$$

$$e^{i\theta} = cos\theta + isin\theta \Longrightarrow Re\, \psi_1(x,t) = (\frac{1}{2\pi\alpha})^{1/4} cos(k_0x - w_0t + \emptyset_0)\, e^{-\frac{x^2}{4\alpha}} \tag{8}$$

Due to the presence of the factor $k_0x - w_0t$, equations 7 and 8 represent a traveling wave packet that propagates in the positive direction of the x-axis [2]. This means that the location of the nodes is not known. Due to the absence of t in $e^{-\frac{x^2}{4\alpha}}$ in equations 7 and 8, the wave packets in these equations does not spread.

Previous calculations was about superposition of the waves $e^{i(kx-w_0t+\emptyset_0)}$. Similarly, we use the recent trend to obtain the superposition of flat waves traveling in the negative direction of the x-axis, i.e. $Ae^{i(kx+w_0t+\emptyset_0)}$. If we do this, we get to equation 9:

$$\psi_2(x,t) = (\frac{1}{2\pi\alpha})^{1/4}\, e^{i(k_0x+w_0t+\emptyset_0)}\, e^{-\frac{x^2}{4\alpha}} \tag{9}$$

$$Re\, \psi_2(x,t) = (\frac{1}{2\pi\alpha})^{1/4} cos(k_0x + w_0t + \emptyset_0)\ e^{-\frac{x^2}{4\alpha}} \tag{10}$$

This equation shows a traveling wave packet that propagates in the negative direction of the x-axis.

Now we sum up the two equations 10 and 8 together to get the final wave.

$$Re\, \psi_{total}(x,t) = Re\, \psi_1 + Re\, \psi_2$$

Thus:

$$Re\, \psi_{total}(x,t) = (\frac{1}{2\pi\alpha})^{1/4}\, e^{-\frac{x^2}{4\alpha}}[cos(k_0x - w_0t + \emptyset_0) + cos(k_0x + w_0t + \emptyset_0)] \tag{11}$$

Using $cos\alpha + cos\beta = 2cos\frac{1}{2}(\alpha+\beta)cos\frac{1}{2}(\alpha-\beta)$ and $\cos(\theta) = \cos(-\theta)$ we obtain the equation of a standing wave packet.

$$\begin{cases}\alpha = k_0x - w_0t + \emptyset_0\\ \beta = k_0x + w_0t + \emptyset_0\end{cases} \Rightarrow Re\, \psi_{total}(x,t) = 2(\frac{1}{2\pi\alpha})^{1/4}\cos(k_0x + \emptyset_0)cos(w_0t)e^{-\frac{x^2}{4\alpha}} \tag{12}$$

There is not the structure of $kx \pm wt$ in equation 12 so the ψ_{total} is a standing wave. As you observe, equation 12, which is the real part of a solution of the Schrodinger equation, is exactly the same as equation 1 for $x \geq 0$, which is solar system wave function. Is this similarity coincidental? No. Therefore equation 1 is the real part of a solution of the Schrodinger equation. It means that the Schrodinger equation and quantum mechanics are valid in astronomical scale. By comparing equation 12 and equation 1, we have

$$\delta = 1\ \ ,\ \gamma = \frac{1}{4\alpha}\quad and\quad C = 2(\frac{1}{2\pi\alpha})^{1/4}$$

If we put these values in equation 1, then we get the final equation of solar system wave function for $x \geq 0$:

$$Re\, \psi_t(x,t) = 2(\frac{1}{2\pi\alpha})^{1/4} cos(w_0t)\, cos(\frac{10\pi}{3}x + \frac{\pi}{6})\ e^{-\frac{x^2}{4\alpha}} \qquad x \geq 0 \tag{13}$$

Equation 13 is obtained by calculating the superposition of a set of infinite number of waves $Ae^{i(kx-w_0t+\emptyset_0)}$ and $Ae^{i(kx+w_0t+\emptyset_0)}$ that move in opposite directions to each other (pay attention to the + sign behind $\emptyset_0$). Now if we sum a set of infinite number of flat wave functions with the equations $Ae^{i(kx-w_0t-\emptyset_0)}$ and $Ae^{i(kx+w_0t-\emptyset_0)}$ (pay attention to the $-$ sign behind $\emptyset_0$) together, by following the path we have taken from equation 2 to equation 13, we reach the following relation;

$$Re\,\psi_t(x,t) = 2(\frac{1}{2\pi\alpha})^{1/4}\,cos(w_0t)\,cos(\frac{10\pi}{3}x - \frac{\pi}{6})\;e^{-\frac{x^2}{4\alpha}}$$

Which is the same as equation 1 for $x \leq 0$. Therefore, the final form of solar system wave function (equation 1) is as follows:

$$\begin{cases} Re\,\psi(x,t) = 2(\frac{1}{2\pi\alpha})^{1/4}\,cos(w_0t)\,cos(\frac{10\pi}{3}x + \frac{\pi}{6})\;e^{-\frac{x^2}{4\alpha}} & x \geq 0 \\ Re\,\psi(x,t) = 2(\frac{1}{2\pi\alpha})^{1/4}\,cos(w_0t)\,cos(\frac{10\pi}{3}x - \frac{\pi}{6})\;e^{-\frac{x^2}{4\alpha}} & x \leq 0 \end{cases} \qquad (14)$$

In this equation, the larger the α is, the more the width of wave packet, along the x-axis. We drew Fig. 1 by $\alpha = 10$. In this section we did not do anything strange. Rather, we have used only the superposition principle. We calculated the superposition of infinite number of flat matter waves with general equation $e^{i(kx\pm wt\pm\emptyset_0)}$ which their angular frequency equals w_0 and the wave number of these waves is around the median of k_0 and between $k_0 + {}^{\Delta k}/_2$ and $k_0 - {}^{\Delta k}/_2$ and the amplitude changes of these waves is equal to $A(k) = (\frac{2\alpha}{\pi})^{1/4}e^{-\alpha(k-k_0)^2}$ and they are under the effect of potential $V(x)$[3]. There have been infinite number of flat matter waves in the early solar system which the superposition of a set of them made the wave function in figure 1. We will talk more about this in the section "Elliptical Orbits and Isotropic Asteroid Belt".

Here we demonstrated that solar system wave function (equation 1) is the real part of a solution of the Schrodinger equation. So, based on quantum mechanics, we can attribute it to an object in Solar system (Certainly the presence of a standing wave in space is not without a source). The closest star to solar system is at a distance of 4.8 light-years, which is so far. And the biggest and heavyset object in solar system is sun. Therefore, the wave function of solar system can only belong to the sun. In the section "Elliptical orbits and Isotropic Asteroid Belt", we will discuss more about the formation of solar system wave packet. De Broglie considered the wave nature for subatomic particles, and here we attributed the wave nature to celestial objects. Neither of these two actions is strange. Rather, they are truths that we must become accustomed to.

In this article, we proved that the Schrodinger equation is valid in astronomical scale; on the other hand, as you know, the Schrodinger relation is based on de Broglie equation ($\lambda = \frac{h}{mv}$). Therefore, the de Broglie equation is valid in astronomical scale[4]. But, according to the very large mass of sun, using the de Broglie relation the wavelength 0.6 AU will not obtain. So, instead of Planck

[3] Since the solar system is under effect of gravity of Milky Way galaxy center; therefore here we investigated the superposition of matter waves which they are under the effect of potential.

[4] The Davisson–Germer experiment [7] is the confirmation of the existence of the de Broglie wave in subatomic scale and the regularity of the distances of the planets from sun (Titius-Bode rule) is the confirmation of the existence of the de Broglie wave in astronomical scale.

constant we must choose another value for celestial objects, which is larger than h. We call this new value the Planck constant in Astronomy ($h_{Astronomy}$) abbreviated as h_A and we have: $\lambda_A = \frac{h_A}{P}$. In such a case, the Schrodinger equation in the astronomical scale can be written as follows:

$$i\hbar_A \frac{\partial}{\partial t}\psi(x,t) = -\frac{{\hbar_A}^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2} + V(x)\psi(x,t) \tag{15}$$

If we follow the path of proving the Schrodinger equation [6] and put the value $\hbar_A$ instead of $\hbar$, we reach equation 15. The Davisson–Germer experiment [7] is considered as the confirmation of existence of the de Broglie wave at the atomic scale, and the regularity of the distances of the planets from sun (Titius-Bode rule) is the confirmation of the existence of the de Broglie wave in astronomical scale. Moreover, for the celestial wave packet we will have $w = \frac{\hbar_A k^2}{2m}$.

As you will see in the next section, the oscillation of solar system wave function collected the gas and dust particles of Protoplanetary disk, in certain orbits around the newborn sun and then gradually due to collision and gravitational attraction between the dusts and gas particles, Planetesimals, Protoplanets and finally the planets in the orbits were formed.

5. Elliptical Orbits and Isotropic Asteroid Belt

In this section we will show that the nebular theory without our wave function theory is an incomplete theory and our wave function model must be attached to the nebular theory in order to explain the formation of the solar system.

The equation 14 in cylindrical coordinate is:

$$x = r\cos\theta \Rightarrow \; Re\,\psi(r,\theta,t) = 2(\frac{1}{2\pi\alpha})^{1/4}\, cos(w_0 t)\, cos(\frac{10\pi}{3} r\cos\theta + \frac{\pi}{6})\; e^{-\frac{(r\cos\theta)^2}{4\alpha}} \tag{16}$$

Figure 2 is the shape of figure 1 in three dimensions based on equation 16 at $t = 0$ ($\alpha = 10$):

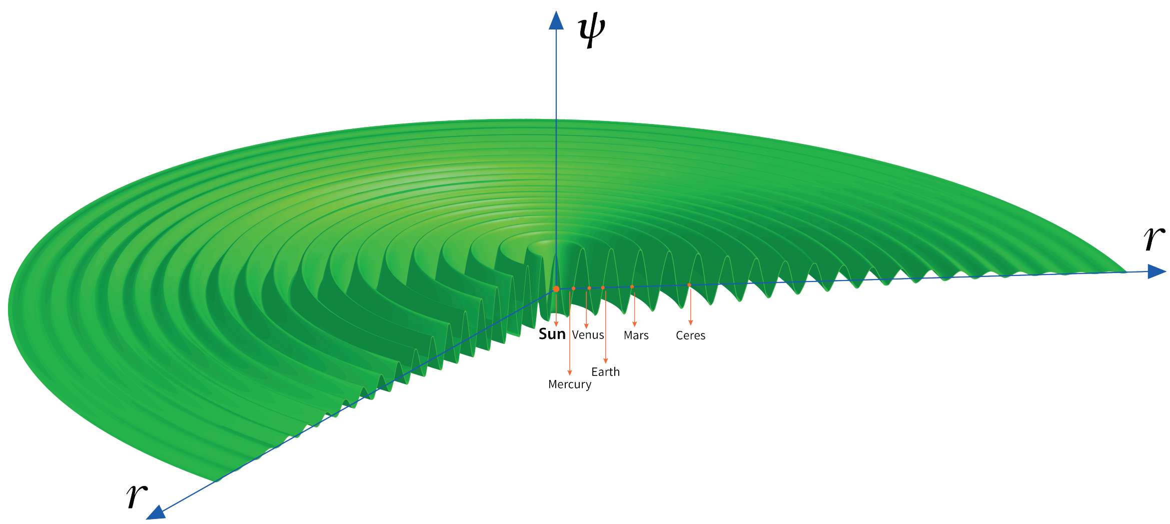

Fig. 2. The oscillation of solar system wave function in three dimensions. Here the wave function in Figure 1 is shown in three dimensions. Fig 2 is drawn in cylindrical coordinates using equation 16. Obviously, this wave function continues indefinitely, based on equation 1 (or equation 16). Because of existence the exponential factor in equation 16, the amplitude of the wave gradually decreases while we move away from the ψ-axis.

As you can see in Fig. 1 and Fig. 2, because of symmetry of equations 1 and 16, the orbits, created by the solar system wave function, are circle. But the real orbits of the planets are elliptic. The reason for this is the existence of the inverse_square gravitational force of the sun[5]. As you know, the sun formed earlier than the planets [8][9][10]. Simultaneously with the formation of the sun, about 4.6 billion years ago [9], its wave function was also formed[6], and the oscillation of this wave function arranged and collected the gas and dust particles of the Protoplanetary disk in regular orbits (Titius-Bode orbits). Just like the standing wave patterns on the kettledrum head (Fig. 3) As you can observe in Fig.3, by a mechanical oscillator at the upper left of the photograph, the powder collects at the circular nodes [11]. As the same way, because of oscillation of solar system wave function, in some nodes of the wave function the gas and dust grains of protoplanetary disk agglomerated (like figure 3) and then compressed due to collisions with each other[7] and formed larger grains, Planetesimals, Protoplanets and finally planets[8]. At the same time, the

[5] We know from classical mechanics that the elliptic orbits of the planets (Kepler's first law) are the result of Newton's law of gravitation, which is an inverse_square relation.

[6] The wave function of the solar system probably was formed either when the sun was a protostar or when the newborn sun was entered to the Main-sequence. The distance between these two phases is very short (less than 50 million years) [8] and both phases occurred before the formation of the planets. In both states, we have no idea how or why this wave function formed.

[7] "Collisions" were more like gentle touches. The particles were far too small to attract each other gravitationally at this point, they were able to stick together through electrostatic forces—the same "static electricity" that makes hair stick to a comb [12].

[8] Since the mass distribution in the Protoplanetary disk had not been uniform. During formation of the planets, in some nodes of the solar system wave function, less dusts were collected and in some nodes, more dusts were placed next to each other. Therefore, in some nodes a planet had been formed and in others it had not been formed.

inverse_square force of the sun was at work, and changed the circular orbits to elliptical orbits. If you look closely, you will see that the mass distribution in the asteroid belt is similar to the powder ring in Figure 3. The Planetesimals distribution in the asteroid belt is uniform and isotropic, just like the powder ring in Figure 3, and this is another confirmation for our wave theory (In asteroid belt, Planet formation has stopped at the Planetesimals stage. Because of orbital resonances with Jupiter, Protoplanet and planet was not formed [13]). Can a theory other than our wave theory explain the uniform and isotropic distribution of Planetesimals in the asteroid belt?

If solar system wave function did not exist; the planets might have been formed around the sun but the distance of planets from the sun was random and irregular. Without solar system wave packet, it is not possible to reach to the TBL from the protoplanetary disk. Thus, it seems that the existence of a standing wave in the solar system is undeniable. Therefore, the nebular theory becomes more complete by our wave function theory.

The reason why there is no planet in some nodes in figure 1 and 2 is due to the unbalanced mass distribution in the protoplanetary disk of solar system.

Fig. 3. Standing wave pattern on a kettledrum head. One of many possible standing wave patterns on a kettledrum head, made visible by dark powder sprinkled on the drumhead. As the head is set into oscillation at a single frequency by a mechanical oscillator at the upper left of the photograph, the powder collects at the nodes. By the same way, due to the oscillation of the solar system wave function the dust grains and gas of the solar system protoplanetary disk were collected in certain orbits.

6. What is Waving?

What is the nature of solar system wave function? As you know, since the solution of the Schrodinger equation is a complex function; It is meaningless to talk about what is waving [14][15]. But in this section we will show that the real part of the solution of the Schrodinger equation **probably** is the oscillation of Dark matter.

In this article, we showed that a standing wave oscillation had been effective in the formation of orbits of solar system planets and created the Bode rule. On the other hand, we know that the Dark matter interacts with the matters and affects them[9]. Therefore, it can be concluded that the standing wave oscillation in Fig. 1, with the equation 1, probably is the oscillation of Dark matter (Or dark matter is one of the candidate for the nature of solar system wave function). On the other hand, the equation 1 is the real part of a solution of the Schrodinger equation. Therefore, the real part of the solution of the Schrodinger equation is the oscillation of Dark matter. Of course, the Spacetime also interacts with matter and affects it. But we cannot consider the oscillation of Spacetime as the wave of the solar system. Because in such a case the curvature of Spacetime lines in figure 2 causes the planets to move along the r-axis. I think this article could give scholars new ideas about dark matter. Of course, there is another possibility. This wave may not be dark matter. It might be something we are not familiar with.

7. Solar System Wave Function and the Other Theories

Up-to-date astronomy books, such as the very famous book: ‘’Astronomy’’ by Professor Michael Zeilik (9th edition, 2002), usually list seven observational features for the solar system, which one of them is the TBL [16]. To compile our article, we read a lot about the attempt and work of previous scholars. We explain here the features of the existing theories in the language of a good book namely ‘’ The Titius-Bode Law of Planetary Distances’’ by Professor Michael Martin Nieto (1972) [17]. Due to curiosity of Mr. Nieto about the reason for TBL (As he says in the Foreword of his book), he carried out a comprehensive study about theories that had been formed until 1972. In this book, he investigates about 13 theories, which are about reason of TBL. We read in this book at the beginning of chapter 11: ((. . . the theories about Bode law can roughly be categorized as either: a) electromagnetism theories b) gravitational theories c) nebular theories . . . Unfortunately, many of the theories that we will review are "theories" and "Laws" in large part due to assumptions or arguments of possibility rather than by calculations that predict definite and unambiguous results . . . Too often the Titius-Bode Law has been discussed in terms of some mechanism which could conceivably produce the Law if all other physical effects could be ignored. Sad to say, that is not the type of physical universe with which we are dealing. Thus, one should be aware of the fact that any claim that a particular theory of the solar system is correct since it explains the Titius-Bode Law is not valid because, as of yet, no theory has properly explained this Rule. What can be said of some theories is that they are compatible with and indicative of a Titius-Bode Law, but not very much more.)) [18]. For example, In his three article, Hannes Alfven hypothesized that if he consider the original solar dipole moment 100000 times larger than at present and ignore the angular momentum in the early solar system [19] He could explain the mass distribution in the solar system. Or Hendrick Petrus Berlage, in his theory, assumed that the Sun emitted positive ions and negatively charged, solid particles that had condensed in the solar atmosphere. Berlage then further hypothesized that all the planet rings were formed a constant distance $d = 0.4\ AU$ further out from the ion rings (because of the addition of momentum

[9] In addition to gravitational interactions, dark matter may also have elastic collision (like collision between drumhead and dark powder particles in fig 3) with baryonic matter (namely objects made with protons, neutrons, and electrons like gas-dust particles in protoplanetary disk)

to the growing Planetesimals via radiation pressure) [20]. But in our article is not any simplification or assumption. Our paper, without eliminating the physical parameters of solar system, justifies the Bode law and elliptical orbits. The oscillation of solar system wave function put up the gas-dust particles of protoplanetary disk in the circular orbits (nodes) just like Figure 3. In the first four pages of the article, we simply arrived to the TBL using wave theory. The important point is that none of the 13 theories examined in Professor Nieto's book easily reach the TBL. And most of them give this law or the other shape of it by approximating. Of course using a lot of assumptions and simplifications. None of these theories is satisfactory.

It seems that no specific work has been done on the cause of the TBL since 1970. In Professor Peter Lynch's 2003 article, we read:'' Despite the distinguished part the Titius–Bode law has played in the evolution of our knowledge of planetary dynamics, no theoretical explanation of it has been advanced that has found general acceptance.''[21]

In addition, some calculations and attempts were made which showed that TBL was a matter of chance. But in 2003, Professor Peter Lynch denied that it was a coincidence: '' . . . this conclusion is unsafe, and that the possibility that the observed regularity in the patterns of the planetary and satellite systems has some physical explanation is still open.''[21].

Conclusion

In this article we proved that almost there is no doubt about the existence of a wave function in the solar system. In this article we proved that the nebular theory without our wave function theory is an incomplete theory and our wave function model must be attached to the nebular theory in order to explain the formation of the solar system. Without the solar system wave packet, it is not possible to reach to the TBL from the protoplanetary disk. Can a theory other than wave theory explain the uniform and Isotropic distribution of Planetesimals in the asteroid belt? The isotropy of Planetesimals distribution and Titius-Bode Law are two reasons for existence of Solar System Wave Function. De Broglie considered the wave nature for subatomic particles, and in this article we attributed the wave nature to celestial objects. Neither of these two actions is strange. Rather, they are truths that we must become accustomed to. In this article we proved that the Schrodinger equation is also valid for celestial objects and in astronomical scale we need a larger Planck constant instead of present Planck constant.

I think the discussions in the section 6 could give scholars new ideas about the nature of dark matter.

Finally, our theory seems to be more plausible than other theories for explaining reason of TBL.

References:

[1]. Carrol, B. Ostlie, D. *An Introduction to Modern Astrophysics* (Cambridge University Press, Cambridge, ed. 2, 2017), pp. 716-717

[2]. Walker, J. Halliday, D. *Fundamentals of Physics.* (Wiley Ltd, ed. 10, 2007), pp. 449-450

[3]. Branson, Jim. "Fourier Transform of Gaussian", *Quantum Physics* (April 2003). URL: https://quantummechanics.ucsd.edu/ph130a/130_notes/node88.html.

[4]. Gasiorowicz, S. *Quantum Physics* (John Wiley & Sons Inc, ed. 1, 1974), pp. 27-32
[5]. Cohen-tannoudji, C. Diu, B. *Quantum Mechanics.* (John Wiley & Sons Inc, Vol. 1, 1977), pp. 21-22
[6]. Ohanian, H. *Modern Physics.* (Prentice-Hall, Inc, 1987), pp. 198-200
[7]. Davisson, C. Germer, L. Diffraction of Electrons by a Crystal of Nickel. *Phys. Rev*. 30, 705. (1927).
[8]. Zeilik, M. *Astronomy. Ch. 14* (Cambridge University Press, ed. 9, 2002).
[9]. Bonanno, A. Schlattl, H. Paternò, L. The age of the Sun and the relativistic corrections in the EOS. *Astronomy and Astrophysics.* **390** (3). 1115–1118 (2002)
[10]. Williams, J. Cieza, L. Protoplanetary Disks and Their Evolution. *Annual Review of Astronomy and Astrophysics.* **49**: 67–117 (2011)
[11]. Rossing, T. The Physics of Kettledrums. *Scientific American,* **247***,* No. 5, 172-179 (1982)
[12]. Bennett, J & et al. *The Essential Cosmic Perspective* (Addison-Wesley, ed. 6, 2012), page 165
[13]. Bennett, J & et al. *The Essential Cosmic Perspective* (Addison-Wesley, ed. 6, 2012), page 263
[14]. Eisberg, R. *Quantum Physics.* (John Wiley & Sons Inc, New York, ed. 2, 1974), pp. 134-135
[15]. Hoffmann, B. *The strange story of the Quantum.* (Dover Publications, 2011)
[16]. Zeilik, M. *Astronomy.* (Cambridge University Press, ed. 9, 2002)
[17]. Nieto, M. *The Titius-Bode Law of Planetary Distances: Its History and Theory.* (Pergamon Press, ed. 1, 1972).
[18]. Nieto, M. *The Titius-Bode Law of Planetary Distances: Its History and Theory.* (Pergamon Press, ed. 1, 1972) pp 62-63
[19]. Nieto, M. *The Titius-Bode Law of Planetary Distances: Its History and Theory.* (Pergamon Press, ed. 1, 1972) pp 71-75.
[20]. Nieto, M. *The Titius-Bode Law of Planetary Distances: Its History and Theory.* (Pergamon Press, ed. 1, 1972) pp 68-71.
[21]. Lynch, P. On the significance of the Titius–Bode law for the distribution of the planets, *Monthly Notices of the Royal Astronomical Society*, Volume 341, Issue 4, June 2003, Pages 1174–1178,