The Shared Economy

Abstract: The theory of marginal utility describes how consumers choose between goods. However, marginal utility has also found application in a wide range of weightier subjects. For example, marginal utility can be used in the allocation of resources in healthcare programmes. This paper posits that marginal utility is also applicable in the allocation of the national income among corporations, government, and households. Using data from the UK Office for National Statistics, this paper finds that for the most part of the decade, from 2009 to 2018, household disposable income fell short of what might be considered an optimal share of the national income.

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Marginal Utility

National income in the UK grew at a compound annual growth rate of 3.1% over the past decade. Meanwhile, during that same period, household income grew at a rate of 2.9%.

So, did households get a good deal? Should household income have grown more than 2.9%? Or should household income growth have been more modest?

This paper sets out to find some answers to these questions.

The search begins with Benjamin Bentham and John Stuart Mill.[1] To describe their philosophy of utilitarianism would take us too far away from the purpose of this paper. So, I shall instead tell their story through an anecdote.

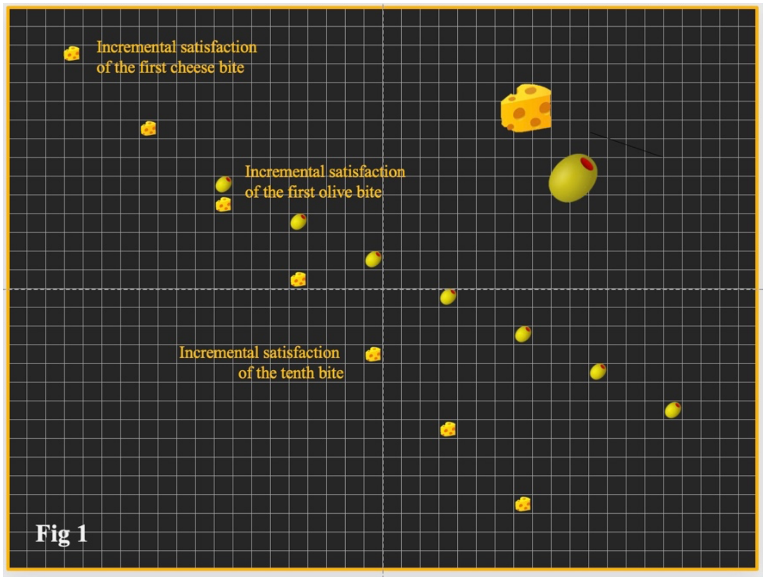
I snack when I write. When I started on this paper, I had two bowls of snacks. But I began only on one bowl, the one that contained tiny cubes of gouda. But as we shall see, it would be too simplistic to conclude from this that I preferred cheese to the snack in the other bowl. The idea of preference needs to be a little more nuanced than saying that one snack is preferred over the other, because sometime later it dawned on me that at some point during the last paragraph, I had unconsciously switched to taking an occasional bite from the other bowl. It seemed that after the initial few cubes of cheese my snacking preference switched to one or two of olives. That was then followed by one or two cheese cubes and so on.

What does my snacking habit say about my preference between cheese and olive? How does one measure preference, anyway? Well, the two nineteenth century philosophers, Bentham, and Mill, offered their interesting explanation. But in the interest of keeping complicated mathematics out of this paper, I shall describe their answers to these questions with the help of a simple diagram, shown as Fig 1.

We start at the top left of the diagram. The level of my preference for cheese or olive is measured by how high up the diagram the particular bite is. So, for example my first preference sits right at the top left corner. Clearly, cheese was my love at first bite. The first olive on the other hand sits lower, about six squares above the half-way dotted line.

Notice that in the diagram each additional cheese bite that I took gave me a little less incremental pleasure than the previous. That is, the second cheese bite was a little lower down the scale than the first, and the third lower than the second. According to Bentham, and Mill, like most things in life, pleasure also follows the law of diminishing returns.

Also note that if we plot a line along the cheese bites, the slope of that line would be steeper than a corresponding line for olives. The diagram is drawn this way to represent the idea that, although cheese was my first preference, with each additional bite its incremental appeal declined faster than the incremental appeal of an additional olive.



In the diagram, after the second cheese bite, an olive now ranked slightly ahead of a third cheese bite. So, at that point, I would mostly likely have forgone cheese and gone for an olive. The fourth bite would be cheese because now the next cheese bite (that is, the cheese that was forgone in favour of an olive) now ranked higher than the second olive. Then it was an olive, followed next with another olive, and then cheese.

The sequence of preferences, with declining incremental satisfaction, went like this: cheese, cheese, olive, cheese, olive, olive, cheese, olive, olive, cheese, and so on.

That in essence, is the theory of marginal utility. This is how a standard textbook would define marginal utility – marginal utility is the additional utility (e.g., satisfaction) someone gets from acquiring or consuming one more unit of a particular good.[2] Thus, the incremental satisfaction from an additional bite of cheese is the marginal utility of cheese.

Although it is usual to explain marginal utility with examples from consumption of goods, the theory is just as applicable in weightier subjects. So, when it comes to evaluating the merits of different healthcare programmes, the incremental satisfaction from an additional bite of cheese might be equated with the marginal utility of health. And the incremental satisfaction from an additional olive equated with the marginal utility of money.[3] Or when choosing among medical resources in a clinic, the incremental satisfaction of patients could be equated to the marginal utility of an additional bite of cheese, while the incremental satisfaction of staff, and that of the owners of the clinic, could be equated to the marginal utilities of additional olives.[4] Similarly the concept of marginal utility could also be used when making decisions regarding the resource allocation in wireless networks.[5]

The diagram says that if in any particular writing session, I had consumed four bites, then on average I would have taken three cheese bites and one olive. However, if during a session I had consumed six bites, then on average I would have taken three cheese bites and three olives.

Notice that Bentham and Mill's theory of marginal utility is not as simple as saying that I preferred cheese to olive, because at some point during my snacking, the preference switched from cheese to olive. It is not even to say that my preference between cheese and olive was three cheese bites to one olive (i.e., 3.0x), because at the very next instance my preference changed to three cheese to two olives (i.e., 1.5x). That was followed by three cheese bites to three olives (i.e., 1.0x). Then, reversing the steady fall, the ratio rises to four cheese bites to three olives (i.e., 1.3x)!

From these preceding paragraphs, we can draw the following fascinating conclusions about this utility theory of preferences. First, the ratio of incremental preference between cheese and olive is not constant; it changes as the combined number of cheese bites plus olives increases. And second, in the vicinity where the taste preference between cheese and olive is equal, the choice will keep switching from one or two bites of cheese followed by one or two bites of olive and then one or two of cheese. That is, the incremental satisfaction of cheese relative to olive (and, for that matter, also of olive relative to cheese) does not decline in a smooth line, but instead, appears to decrease in steps.

Now if we imagine that both cheese and olive come in the tiniest of bite sizes, then we can surmise that at any point in my snacking, the maximum satisfaction is to be had when the incremental change in satisfaction from a tiny bite of cheese has become the same as the incremental change in satisfaction from a tiny bite of olive. It is important to remember that we are talking about incremental changes. In the theory of marginal utility, this incremental change in satisfaction from an incremental bite of olive (also of cheese) is called marginal utility. We can also surmise that the ratio between cheese and olive depends both on the slopes of the two preference curves as well as on the size of the combined amount consumed.

It should be noted that although the diminishing satisfaction of cheese and olive are represented diagrammatically as sloping straight lines, the theory still holds if these lines were drawn as curves. It hasn't been presented that way simply because the narrative, and the mathematics behind that narrative, would be a lot more complicated. We should expect that in the real world the lines are invariably never straight.

This story about cheese and olives, that is, the theory of marginal utility, forms the foundation, upon which is based my theory about whether the UK households had a good deal, given that the annual income grew at an average rate of 2.9% over ten years.

If one bite of cheese is a metaphor for the growth in household income in one year, what might be the equivalent metaphor of olives? And if taste is the measure for the satisfaction that one derives from snacking on cheese and olive, what might be the equivalent measure for the satisfaction (i.e., utility) that households, and the other sectors of the national accounts, derive from the growth in national income?

In order to answer these questions, one would first have to take into account how the national income of a country is measured. From a practical standpoint, it would not be too useful to formulate a theory for which the data needed to test the theory are not readily available. So, we start by considering what data are relevant and readily available.

The national accounts of every country follow the same international standard format.[6] Fig 2 assembles the relevant data. These data are readily available because they can be extracted from the published national accounts. Notice that the national income is segmented into three major sectors. That is, the national income comprises of the incomes of the corporation sector, the government sector, and the household sector. This is how the national accounts are segmented. And therefore, this is how our analysis should proceed.

In principle, one could of course segment the national income differently, for example, into a different set of sectors, or into many sub-sectors, but that would assume that one is able to collect the necessary data for such sectors or sub-sectors.

Title	Gross national disposable income GNDI		Household disposable income		Corporation gross disposable income		Government disposable income		Total household population		Capital service based on lease equation		Gross National Income		Cobb Douglas efficiency
		growth		growth		growth		growth		growth		growth		growth	growth
Symbol	*GNDI*	*y_o_y*	*h*	*y_o_y*	*k*	*y_o_y*	*g*	*y_o_y*	*H*	*y_o_y*	*K*	*y_o_y*	*G*	*y_o_y*	*A_cd*
Unit	£ million		£ million		£ million		£ million		'000		£ million		£ million		
2008	1,551,998		1,048,614		207,063		296,321		61,824		384,207		1,565,205		
2009	1,510,910	-2.65%	1,076,258	2.64%	186,512	-9.92%	248,140	-16.26%	62,261	0.71%	380,593	-0.94%	1,525,684	-2.52%	-2.61%
2010	1,568,952	3.842%	1,088,879	1.17%	226,001	21.17%	254,072	2.39%	62,760	0.80%	388,109	1.97%	1,588,548	4.12%	2.31%
2011	1,630,817	3.94%	1,107,842	1.74%	249,831	10.54%	273,144	7.51%	63,285	0.84%	404,236	4.16%	1,651,069	3.94%	2.05%
2012	1,656,168	1.554%	1,163,500	5.02%	220,330	-11.81%	272,338	-0.30%	63,705	0.66%	419,457	3.77%	1,676,620	1.55%	0.33%
2013	1,699,690	2.63%	1,206,015	3.65%	190,646	-13.47%	303,029	11.27%	64,106	0.63%	432,217	3.04%	1,724,971	2.88%	1.31%
2014	1,783,073	4.906%	1,242,751	3.05%	226,167	18.63%	314,155	3.67%	64,597	0.77%	460,509	6.55%	1,806,468	4.72%	2.68%
2015	1,829,658	2.61%	1,313,995	5.73%	176,389	-22.01%	339,274	8.00%	65,110	0.79%	470,239	2.11%	1,852,845	2.57%	1.35%
2016	1,897,621	3.715%	1,332,527	1.41%	197,718	12.09%	367,376	8.28%	65,648	0.83%	491,140	4.44%	1,920,116	3.63%	1.94%
2017	1,998,006	5.29%	1,366,540	2.55%	231,485	17.08%	399,981	8.88%	66,040	0.60%	512,874	4.43%	2,026,058	5.52%	3.22%
2018	2,056,939	2.95%	1,426,389	4.38%	211,908	-8.46%	418,642	4.67%	66,466	0.65%	526,862	2.73%	2,091,074	3.21%	1.55%

Fig 2

The Size of an Economy

When someone talks about the size of a country's economy, we assume that she is talking about the country's gross domestic product, GDP. Indeed, GDP is the one of the most frequently quoted term in the economics vocabulary. But that had not always been the case. GDP was first introduced as a measure of the economy in 1944. Before that, countries measured the size of their economy by their gross national product, GNP.

GDP and GNP, what is the difference?

GDP measures the value of output produced within the country. In other words, GDP includes the value of output produced by foreigners within the country. But it excludes the value of output produced in other countries by its own residents.

GNP on the other hand excludes the value of output produced by foreigners within the country. But it includes the value of output produced by its own residents but produced in other countries.

Since our interest is in the income of households, if we want to relate the size of household income to the size of the economy, GNP would be the better measure than GDP.

But GNP is still not the most appropriate measure for our purpose. GNP, like GDP, measures production output rather than income. An increase in a country's overseas output may not necessarily result in an increase in income flowing into the country. For example, when the profits earned by the country's overseas investments are not disbursed as dividends, but instead the profits are retained in the overseas operations, there is no increased income flowing back to the country. Profits that are not disbursed are not available for households to spend (or to save).

That is, GNP would overstate the income that is at the disposal of households to spend or save.

The obvious candidate against which to compare household income is the gross national income, GNI. That is because GNI measures the sum of the incomes earned by the country's corporations, the government, and the households. And in addition to the income earned within the country, GNI also counts the money that flows back to the country, for example, in the form of dividends paid out by the country's investments overseas. GNI also discounts the money that flows out of the country, as income earned from dividends, or from remuneration earned for services rendered, by foreigners.

Fig 3 shows how GNI differed from GDP during the decade from 2009 and 2018. The data are available from the UK Office for National Statistics.[7] [8]

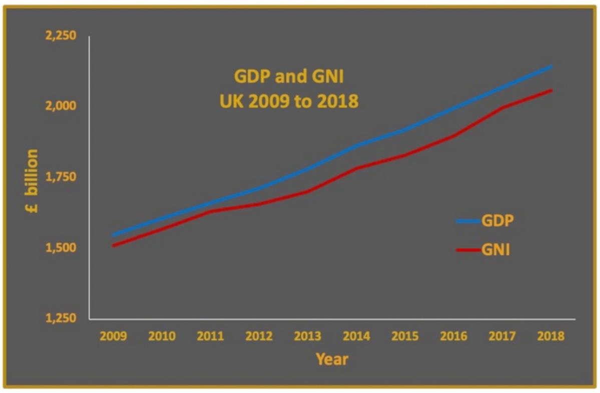


Fig 3

So, perhaps the proper base against which to measure household income is the gross national income. But what goes into household income? Income of the self-employed, earnings of small unincorporated businesses – these are counted as part of household income. So are income that households derive from investments, such as dividends and interests earned. But the largest component of household income comes from wages and salaries. In addition, social benefits such as unemployment benefits are also counted as household income.

However, household income is not entirely at the disposal of households to use for consumption and saving. Households pay interests on loans. Earning members of households pay income tax. Some households also pay wealth tax, and other social contributions. All these expenses act to limit the income available for spending and saving.

What remains after deducting all these non-consumption expenses is called the household disposable income. This household disposable income is the proper measure of households' share of the national income.

If one bite of cheese is a metaphor for the growth in household disposable income in one year, what might olives be metaphors for? Olives can be considered as metaphors for the growth in corporation disposable income and the growth in government disposable income.

Just as household disposable income is the proper measure for households, the corresponding income measures for corporations and government are their respective disposable income.

The sum of corporation, government, and household disposable incomes is called the gross national disposable income, GNDI. This figure may be a little different from the gross national income. The size of the difference depends on how much transfers the country remits or receives. An example of transfers out of the country is migrant workers' remittance to their home countries.

Data for the UK's GDP, and GNI are published every quarter. It goes without saying that any analysis of households' share of the national income must be circumscribed by the availability of such data.

The Efficiency of an Economy

So, one bite of cheese is our metaphor for the growth in household disposable income in one year, and olives our metaphors for growth in corporation disposable income and growth in government disposable income in one year.

If taste is the measure for the incremental satisfaction that one derives from snacking on an additional cheese or olive, what might be a suitable measure for the incremental satisfaction (or marginal utility, to use the proper technical term) that households and other claimants on the national income derive from the growth in the economy?

Gross national disposable income, GNDI, comes to mind.

If taste and satisfaction could be quantified and the cheese or olive came in the tiniest of bite sizes, we might say that each additional bite of cheese or olive represents, say, a 0.1% change in satisfaction.

Similarly, we might say that an incremental increase in household disposable income, or corporation disposable income, or government disposable income, represents an incremental increase in GNDI. Later it will become clear that the preceding sentence should be rephrased as follows: A certain combination of incremental increases in household, corporation, and government disposal incomes represents their best aggregated satisfaction (i.e., maximum utility) from an incremental increase in GNDI.

Did household income grow in step with the national income? To answer that, we extract the necessary data from the UK Office for National Statistics. [9] [10] Fig 4 shows that for the most part of the decade, from 2009 to 2018, household income fell a little shy of the national income, although it tracked the latter fairly closely. The GNDI grew at a compound annual rate of 3.13% while the household disposable income grew at the rate of 2.86%

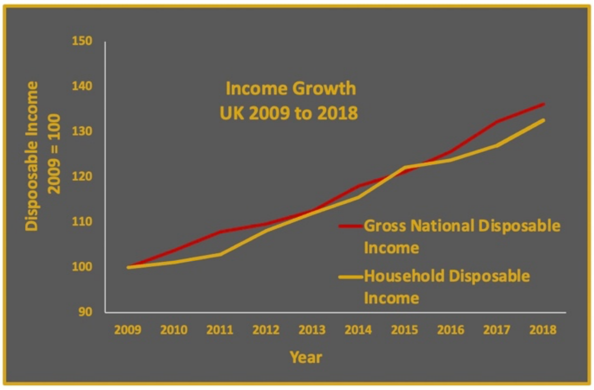


Fig 4

In 2018 GNDI increased by 2.95%. Was 2.95% a good measure for the greater satisfaction that the UK, taken as a whole, enjoyed in 2018? To answer this, we now turn to consider the efficiency of the economy.

Efficiency can be conveniently viewed as the output we get from the resources from which we produced the output. Let's consider this example. In one scenario, it takes ten persons to make one widget. In another scenario, it takes only five persons to make the same widget. (We assume that other factors are the same in both cases). Clearly the second scenario is more efficient (or productive) than the first. Now consider the production of one more widget. If the production of one more widget represents a marginal increase in satisfaction (i.e., marginal utility), clearly this marginal increase in satisfaction should take into account not only the additional satisfaction that one more widget brings, but also the additional persons deployed to make one more widget. And all these presumes that there are these additional persons available to make one more widget. (This example does not consider the scenarios when not only the number of persons, but also the tools used are different between the scenarios. We'll get to that later).

GNDI is like the widgets. The 2.95% rise in GNDI was the outcome. But it says nothing about the changes in various factors prevailing then in the UK. It does not tell us anything about whether number of persons had increased, not only to produce but also to consume more goods, or to earn more income. It does not tell us whether more capital was available, and so on. So, when considering the efficiency in producing the national income, we should take into account the corresponding factors available. For our purpose, since GNDI is distributed among households, and corporations, and government, the corresponding factors that we take into account are the total number of people in households, the gross capital available to corporations, and the public infrastructures, as well as the public services, made available by the government.

From the foregoing paragraphs, it becomes clear that a better measure for the equivalence to an incremental increase in taste satisfaction is not the change in GNDI per se, but rather it is the change in the efficiency in generating that growth in GNDI. Therefore, we need a handle on this efficiency in generating a change in GNDI. Next, we consider what this measure of efficiency might be. And once again we do that with an anecdote.

When it comes to annoying political leaders with irrefutable economic facts, there is no one comparable to Paul Krugman. For example, in 2018 Krugman came up top in President Trump's hate list.[11] But our story goes back two and a half decades earlier.

In 1994 Krugman wrote an article titled, *The Myth of Asia's Economic Miracle*.[12] In that article Krugman claimed that Singapore's miracle was but just "lots of perspiration with little inspiration". Singapore's miraculous economic growth was achieved through massive mobilisation of an otherwise unemployed labour force and through an awesome injection of investments in physical capital.

Krugman's reasoning was of course sound. He was referring to the relationship between an outcome and the factors available to produce that outcome. There are two ways, two mathematical equations, to describe this relationship between output and the inputs. The first equation was developed by Charles Cobb and Paul Douglas.[13] For GNDI, the Cobb Douglas equation would be written as follows:

$$\log(GNDI) = \log(A_{cd}) + k \times \log(K) + g \times \log(G) + h \times \log(H)$$

The second form, developed by Robert Solow, looks quite intimidating. For GNDI, the Solow equation would be written as follows [14]:

$$GNDI = \left(kK^{\frac{\sigma-1}{\sigma}} + gG^{\frac{\sigma-1}{\sigma}} + \ hH^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

The Cobb Douglas equation and the Solow equation are mutually consistent. The former is a special case of the latter. For our purposes, we shall be using the Cobb Douglas equation.

Notice that both equations use the same set of variables, which are:
- k which represents corporation disposable income
- K which represents a proxy measure of the total size of the capital available to all UK corporations
- g which represents government disposable income
- G which represents a proxy measure of the size of the public services and public infrastructures
- h which represents household disposable income and
- H which represents a proxy measure of the total size of all UK households, measured as the number of persons

Data for these variables are published quarterly by the UK Office for National Statistics. From these data, one can then determine the values of A_{cd} and σ. This set of information is all that is needed to determine whether the 2.86% rise in household income was indeed a good deal.

A_{cd} is usually given the label "multi-factor productivity", MFP in short. But let's just name A_{cd} after Charles Cobb and Paul Douglas, and call it the Cobb-Douglas efficiency, which we will define as a ratio. So, the Cobb Douglas efficiency for 2018 would be written as

$$\text{CobbDouglas efficiency 2018} = \frac{A_{cd2018}}{A_{cd2017}}$$

The Maximum Utility of an Economy

Now we are set to find an answer to the question: Was an average 2.86% annual growth in disposable income over a period of one decade really a good deal for the household sector?

The three major sectors that make up a country's total national income are the corporations, the government, and the households. They all vie for a slice of the country's GNDI pie.

In 2018 the UK's GNDI increased by 2.95%, from £1.998 trillion to £2.057 trillion. How many bites should each sector have gotten from this larger GNDI pie? Was the household sector's share, at £1.426 trillion, a reflection of the best possible income distribution among the three sectors?

It turns out that the ratio, the Cobb-Douglas efficiency per se, is not helpful here after all. The ratio, derived as it is through the Cobb Douglas equation, favours the factor that has the highest growth rate. In other words if the country's capital stock had grown faster than household population, then A_{cd} would have kept increasing when corporations' share of GNDI increased. An increase in corporations' share of course could only come at the expense of households' and government's shares. Such an outcome of course would not have been tenable.

Note that this is not because of any limitations in the Cobb Douglas equation. It is because the answer we seek is not to be had by simply applying the Cobb Douglas equation per se.

What exactly are we trying to determine? We want to know at what combination of disposable incomes (of households, corporations, and government) the "satisfaction" of the country, taken as a whole, is at its maximum.

Recall that if we imagined that cheese and olive came in the tiniest of bite sizes, then, the maximum satisfaction was to be had when the incremental change in satisfaction from a tiny bite of cheese was the same as the incremental change in satisfaction from a tiny bite of olive. In the lingo of marginal utility theory, we say that the utility is at its maximum when the marginal utilities of cheese and olive are equal.

To make doubly sure that we are on the same page, it is important to reiterate that the sum total of incremental satisfaction is a measure of the total satisfaction. So, for example, the total satisfaction, say, after the second cheese cube, is the sum of the incremental satisfaction from the first cheese cube plus the incremental satisfaction from the second cheese cube.

Similarly, we can surmise that the country, taken as a whole, is best served when the incremental change in A_{cd} for a small rise is household disposable income is equal to the incremental change in A_{cd} from a small rise in corporation disposable income and that these are the same as the incremental change in A_{cd} from a small change in government disposable income.

Note that we are not saying that the changes in disposable incomes of households, corporations, and government, are expected to be the same. What we are saying is that the size of the incremental changes in A_{cd} due to a small change in the disposable income of each of the three sectors would be the same when the national income is shared such that it gives the maximum satisfaction.

In the lingo of marginal utility theory, we say that the utility of GNDI is at its maximum when the marginal utilities of the corporation, the government, and the household sectors are equal.

Mathematically, the tasks boils down to finding the combination of *h*, *k*, and *g*, such that both of the following two conditions are satisfied:

$$\frac{\delta A_{cd}}{\delta h} = \frac{\delta A_{cd}}{\delta k} = \frac{\delta A_{cd}}{\delta g}$$

AND

$$\log(GNDI) = \log(A_{cd}) + k \times \log(K) + g \times \log(G) + h \times \log(H)$$

Recall that

- *k* represents corporation disposable income
- *K* represents a proxy measure of the total size of the capital available to all UK corporations
- *g* represents government disposable income
- *G* represents a proxy measure of the size of the public services and public infrastructures
- *h* represents household disposable income and
- *H* represents a proxy measure of the total size of all UK households, measured as the number of persons.

For practical reasons, we will not calculate the combination of corporation, government, and household disposable incomes such that the marginal utilities, $\frac{\delta A_{cd}}{\delta h}$; $\frac{\delta A_{cd}}{\delta k}$; $\frac{\delta A_{cd}}{\delta g}$ are exactly equal. For the purpose of this paper, I calculated household disposable income to an accuracy of 0.15%. This means that the difference among the three partial derivatives would not be equal exactly to zero, but it would be quite close to zero. In other words, we calculate the combination of disposable incomes such that the differences among the three marginal utilities are as close to zero as possible.

The Shared Economy

The UK gross national disposable income for 2018 was £2,056,939 million (i.e. about £2.06 trillion). Fig 5 tabulates both the actual disposable incomes as well as the combination of disposable incomes that would have given the maximum utility. Recall that the term maximum utility here is taken to mean the sum of satisfaction that is at its greatest.

There are different ways to determine whether the marginal utilities of the three sectors are equal, or at least closest to zero. Since the household disposable income at the point of maximum utility has been calculated only to 0.15% accuracy, the differences among the marginal utilities will not be exactly equal to zero. But we would expect that the *sum of the differences* among the three marginal utilities will be as close to zero as possible.

The *sum of difference* among the three marginal utilities is represented by the following equation

$$Sum\ of\ difference = \frac{1}{3} \times \sqrt{\left(\frac{\delta A}{\delta h}\right)^2 + \left(\frac{\delta A}{\delta k}\right)^2 + \left(\frac{\delta A}{\delta g}\right)^2}$$

Notice that in Fig 5 the *sum of difference* of the actual income distribution was 0.1329 whereas the sum of difference at maximum utility was lower, at 0.1200. In other words, if the household disposable income was at £1,591,137 million (approximately £1.59 trillion), the combination of disposable incomes would have been at its maximum utility. That is, the combination would have been the most economic efficient distribution of the national income.

UK 2018	Income Distribution Efficiency				
	Gross national disposable income	Household disposable income	Corporation disposable income	Government disposable income	Sum of difference of marginal utitilities
Actual	2,056,939	1,426,389	211,908	418,642	0.1329
Maximum utility	2,056,939	1,591,137	159,461	306,341	0.1200

Fig 5

How can we demonstrate that £1.59 trillion represents the most economic efficient distribution of the national income? To find out, Fig 6 plots the expected *sum of difference* against different household disposable incomes. The chart shows that when the household disposable income was at £1,591,137 million, indicated by a red dot, the *sum of difference* was closest to zero. That is, when household disposable income was at £1.59 trillion, the distribution of the national income was at its most economic efficient. This is the combination where a small increase in household disposable income, *h*, (analogous to a small bite of cheese), increases the Cobb Douglas efficiency, A_{cd}, to the same extent as would a small increase in corporation disposable income, *k*, or a small increase in government disposable income, *g* (analogous to small bites of olive).

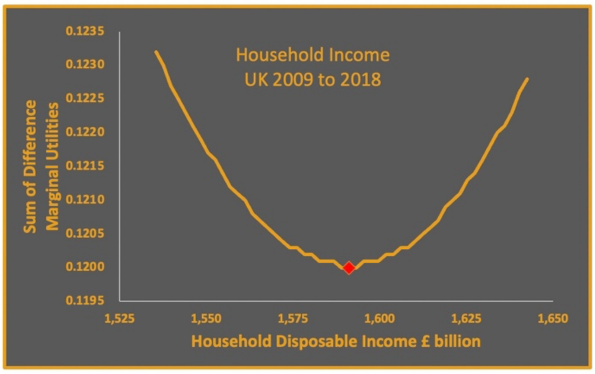


Fig 6

In other words, the satisfaction of the country is at the maximum when the country's gross national disposable income is shared according to the theory of marginal utility.

Does the theory of marginal utility always favour an increase in household disposable income? Not necessarily. One way to find out if the theory is biased toward households is to compare the actual household disposable income with what the theory of marginal utility recommends, not for just 2018, but for a whole decade, from 2009 to 2018.

Fig 7 compares the household disposable income at maximum utility (brown curve) with the actual household disposable income (yellow curve), for the decade from 2009 to 2018. Notice that although the theory of marginal utility would have recommended a household income higher than the actual for the most part of the decade, there was one year when the recommended fell below the actual. To get a feel on the percentage of the gap between the two, Fig 8 redraws the chart, with the 2009 actual income set to a base of 100.

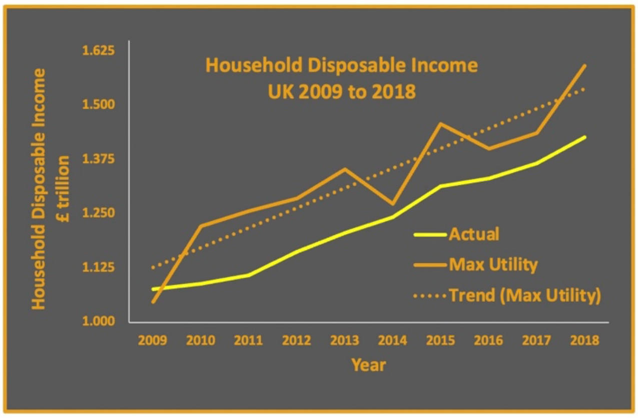


Fig 7

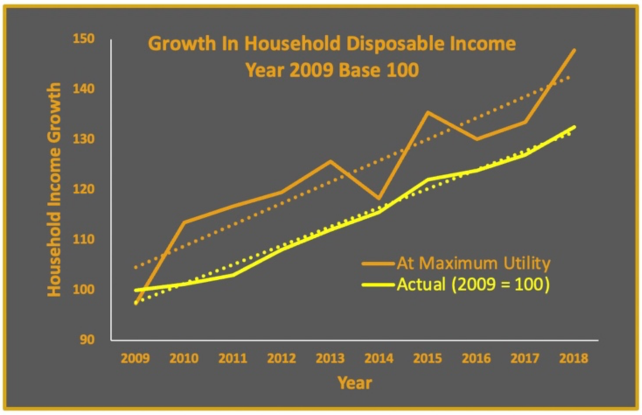


Fig 8

From Fig 7 and Fig 8, we conclude that it is not necessarily true that the marginal utility theory biases the results towards households. If the UK household sector's share of the national income fell short of the recommended share, it must have been because exogenous forces had pushed the UK income distribution away from maximum utility.

However, it is premature, and unwise, to see the chart as a definitive recommendation to close the gap between what should have been and what was the household disposable income. This paper is only a first attempt, a tentative attempt, to apply the theory of marginal utility to the question of income distribution. It is, as yet, an untested approach.

There are many sound, and time-tested, ways to measure income distribution and income inequality. So, it is not the objective that this paper be seen as a call for action. The objective will be amply met if this paper could convince readers that more research should be undertaken on the application of marginal utility as a means of measuring income distribution.

Income Distribution and Inequality

There are countless ways to describe inequality. Here we just consider two.

The inequality measure that is most frequently bandied about is the GINI index.[15] This is surprising because what GINI really measures is probably also the least understood.

The GINI index is defined as a set of data's relative mean absolute difference divided by the number two. Mathematically it can be expressed as

$$GINI = \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} \sqrt{\left(x_i - x_j\right)^2}}{2 \times \sum_{j=1}^{n} x_j}$$

For example, in 2018 the GINI index for UK's household disposable income was 0.332.[16] Singapore's GINI was 0.404.[17] But what does the difference between 0.332 and 0.404 really mean? Do the two numbers measure the same thing? It is hard to get an intuitive sense of what is being represented by these two numbers.

Nonetheless, the GINI index is the most popular measure of inequality. It has been in use since it was first formulated by Corrado Gini in 1912. Like GDP, GINI is regularly reported in the press. It doesn't seem to matter that most readers, and probably the reporters too, do not really know how these two numbers, 0.332 and 0.404, are derived.

Of a more recent vintage is Thomas Piketty's exposition on inequality. It is impossible to summarise his 700-page book in a few paragraphs.[18] Instead, I shall briefly describe his three fundamental equations.

Of the three, perhaps it is this equation of inequality that Piketty is most famous for. It is a charmingly simple inequality equation:

$$r > g$$

For most of human history, at least with regard to countries where income records had been kept, the return on capital had been greater than the rate of growth of the economies. True, for a few decades during the first half of the twentieth century, the trend seemed to have reversed. Many economies grew faster than the rate of return on capital during the first half of the twentieth century. But, in reality, during that period, it was as much that the rate of return on capital had dropped sharply as it was that the economies had grown rapidly. The drop in the rate of return on capital, r, was just a reflection of the massive destruction of capital as a result of two world wars. And the rapid growth, g, reflected the reconstruction and recovery of the economies after the world wars.

Less written about by book reviewers, but probably more noteworthy from the standpoint of understanding the drivers of inequality, is Piketty's *second law*, that over a long enough period, the stock of capital, K, as a ratio to the country's national income, Y, is a function of the size of the country's saving rate, s, over its growth rate, g.

$$\frac{K}{Y} = \frac{s}{g}$$

Piketty's *first law* may be nothing more than an accounting identity, but nonetheless it is arguably Piketty's crowning achievement in formulating an insight into the nature of inequality.

$$\alpha = r \times \frac{K}{Y}$$

This inequality equation says that capital's share, α, of the national income is determined by the rate of return on capital and the ratio of capital to national income.

Now, for a given size of the national income, if capital's share is larger, wage earners' share must necessarily be smaller. That in essence is why Piketty's *first law* is important for an understanding of the nature of inequality.

So, three simple equations to explain the nature of inequality.

Conclusion

This paper introduced the theory of marginal utility through an anecdote of cheese and olive. However, the application of the theory is not confined only to how consumers choose between goods. Marginal utility has found application in a wide variety of weightier subjects. It has found uses in the allocation of resources, such as in wireless networks. The marginal utility of medical resources has been used in a healthcare clinic setting as a methodology to improve both operating efficiency and patient satisfaction. This paper posited that marginal utility was also applicable in the allocation of the national income among corporations, government, and households. Using data from the UK Office for National Statistics, this paper found that household disposable income fell short of the optimal the for most part of the decade from 2009 to 2018.

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