Abelian Group on the Binomial Coefficients of Combinatorial Geometric Series

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Abstract: This paper discusses an abelian group, also called a commutative group, under addition and multiplication of the binomial coefficients of combinatorial geometric series. The coefficient for each term in combinatorial geometric series refers to a binomial coefficient. This idea can enable the scientific researchers to solve the real life problems.

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1. Introduction

When the author of this article was trying to develop the multiple summations of geometric series, a new idea was stimulated his mind to create a combinatorial geometric series [1-11]. The combinatorial geometric series is a geometric series whose coefficient of each term of the geometric series denotes the binomial coefficient V_n^r. In this article, binomial identities and multinomial theorem is provided using the binomial coefficients for combinatorial geometric series.

2. Combinatorial Geometric Series

The combinatorial geometric series [1-11] is derived from the multiple summations of geometric series. The coefficient of each term in the combinatorial refers to the binomial coefficient V_n^r.

$$\sum_{i_1=0}^{n}\sum_{i_2=i_1}^{n}\sum_{i_3=i_2}^{n}\cdots\sum_{i_r=i_{r-1}}^{n} x^{i_r} = \sum_{i=0}^{n} V_i^r x^i \ \ \& \ \ V_n^r = \frac{(n+1)(n+2)(n+3)\cdots(n+r-1)(n+r)}{r!},$$

where $n \geq 0, r \geq 1 \ \ and \ \ n, r \in N = \{1, 2, 3, \cdots\}$.

Here, $\sum_{i=0}^{n} V_i^r x^i$ refers to the combinatorial geometric series and

$V_n^r \ is$ the binomial coefficient for combinatorial geometric series.

$V_0^1 = 1;\ V_1^1 = 2;\ \ V_2^1 = 3;\ V_3^1 = 4;\ V_4^1 = 5;\ V_5^1 = 6;\ \cdots$

$N = \{V_0^1, V_1^1, V_2^1, V_3^1,\ V_4^1, V_4^1, \cdots\}$ is a set of natirual umbers .

$Z = \{\cdots, -V_2^1, -V_1^1, -V_0^1, 0, V_0^1, V_1^1, V_2^1, \cdots\}$ is a set of integers.

Any binomial coefficient belongs to N such that $V_n^r \in N,$ where $n \geq 0;\ r \geq 1 \ \ and \ \ n, r \in N$.

Note that $V_0^r + V_1^r + V_2^r + V_3^r + \cdots + V_{n-1}^r + V_n^r = V_n^{r+1} \ \ \Longrightarrow V_{n-1}^{r+1} + V_n^r = V_n^{r+1},$
where $V_n^r = V_0^{r-1} + V_1^{r-1} + V_2^{r-1} + \cdots + V_n^{r-1}$ and $V_0^r + V_1^r + V_2^r + \cdots + V_{n-1}^r = V_{n-1}^{r+1}$.

3. Abelian Group

$Z = \{-V_r^n, 0, V_r^n \mid n \geq 1, r \geq 0 \;\&\; n, r \in N\}$ is a set of integers.

Closure property: Addition of any two binomial coefficients is also a binomial coefficient.
$(V_m^n + V_p^q) \in Z$ for all $V_m^n, V_p^q \in Z$.

Associativity: For all $V_m^n, V_p^q, V_r^s \in Z,\; V_m^n + (V_p^q + V_r^s) = (V_m^n + V_p^q) + V_r^s$.

Identity element: $0 + V_r^n = V_r^n + 0 = V_r^n$, where 0 is an identity element.

Inverse element: $V_r^n + (-V_r^n) = (-V_r^n) + V_r^n = 0$, where $-V_r^n$ is an additive inverse.

Commutative law: $V_m^n + V_p^q = V_p^q + V_m^n$ is alway true.

$Z = \{-V_r^n, 0, V_r^n \mid n, r \geq 1 \;\&\; n, r \in N\}$ is an abelian group under addition.

4. Conclusion

In this article, an abelian group was formed on the binomial coefficients of combinatorial geometric series under addition. This new idea can enable the scientific researchers for research and development further.

References

[1] Annamalai, C. (2020) Optimized Computing Technique for Combination in Combinatorics. *hal*-0286583. https://doi.org/10.31219/osf.io/9p4ek.

[2] Annamalai, C. (2020) Novel Computing Technique in Combinatorics. *hal*-02862222. https://doi.org/10.31219/osf.io/m9re5.

[3] Annamalai, C. (2022) Computation of Combinatorial Geometric Series and its Combinatorial Identities for Machine Learning and Cybersecurity. *COE, Cambridge University Press*. https://doi.org/10.33774/coe-2022-b6mks.

[4] Annamalai, C. (2022) Annamalai's Binomial Identity and Theorem, *SSRN Electronic Journal*. http://dx.doi.org/10.2139/ssrn.4097907

[5] Annamalai, C. (2022) Novel Binomial Series and its Summations. *SSRN Electronic Journal*. http://dx.doi.org/10.2139/ssrn.4078523.

[6] Annamalai, C. (2022) Successive Partition Method for Binomial Coefficient in Combinatorial Geometric Series. *Zenodo*. https://doi.org/10.5281/zenodo.7060159.

[7] Annamalai, C. (2022) Partition of Multinomial Coefficient. *Zenodo*. https://doi.org/10.5281/zenodo.7063004.

[8] Annamalai, C. (2022) A Generalized Method for proving the Theorem derived from the Binomial Coefficients in Combinatorial Geometric Series. *Zenodo*. https://doi.org/10.5281/zenodo.7047548.

[9] Annamalai, C. (2022) Computation Method for Combinatorial Geometric Series and its Applications. *COE, Cambridge University Press*. https://doi.org/10.33774/coe-2022-pnx53-v22.

[10] Annamalai, C. (2022) Computing Method for Combinatorial Geometric Series and Binomial Expansion. *SSRN Electronic Journal*. http://dx.doi.org/10.2139/ssrn.4168016.

[11] Annamalai, C. (2022) Factorials and Integers for Applications in Computing and Cryptography. *COE, Cambridge University Press*. https://doi.org/10.33774/coe-2022-b6mks.