Sum of Successive Partitions of Binomial Coefficient

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Abstract: This paper focuses on the successive partition method applied to a binomial coefficient in combinatorial geometric series. The coefficient for each term in combinatorial geometric series refers to a binomial coefficient. These ideas can enable the scientific researchers to solve the real life problems.

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1. Introduction

When the author of this article was trying to develop the multiple summations of geometric series, a new idea stimulated his mind to create a combinatorial geometric series [1-13]. The combinatorial geometric series is a geometric series whose coefficient of each term of the geometric series denotes the binomial coefficient V_n^r. In this article, binomial identities and multinomial theorem is provided using the binomial coefficients for combinatorial geometric series.

2. Combinatorial Geometric Series

The combinatorial geometric series [1-13] is derived from the multiple summations of geometric series. The coefficient of each term in the combinatorial refers to the binomial coefficient V_n^r.

$$\sum_{i_1=0}^{n}\sum_{i_2=i_1}^{n}\sum_{i_3=i_2}^{n}\cdots\sum_{i_r=i_{r-1}}^{n} x^{i_r} = \sum_{i=0}^{n} V_i^r x^i \;\;\&\;\; V_n^r = \frac{(n+1)(n+2)(n+3)\cdots(n+r-1)(n+r)}{r!},$$

where $n \geq 0, r \geq 1 \;\; and \;\; n, r \in N = \{0, 1, 2, 3, \cdots\}$.

Here, $\sum_{i=0}^{n} V_i^r x^i$ refers to the combinatorial geometric series and

V_n^r *is* the binomial coefficient for combinatorial geometric series.

Here, $V_0^1 = 1;\; V_1^1 = 2;\; V_2^1 = 3;\; V_3^1 = 4;\; V_4^1 = 5;\; V_5^1 = 6;\; \cdots$

$N = \{V_0^1, V_1^1, V_2^1, V_3^1,\; V_4^1, V_4^1, \cdots\}$ is a set of natural numbers .

$V_n^r = V_r^n, (n, r > 0$ and $n, r \in N)$ is an important binomial identity.

3. Partition of Binomial Coefficient

Let $Q = \{V_1^1, V_2^1, V_3^1,\; V_4^1, V_4^1, \cdots\}$ and Q is contained in N, i.e. $Q \subseteq$ N.

The sum of partition of V_n^r is give below:

$$V_n^r = V_n^{r-1} + V_r^{n-1}, (n, r \geq 1 \;\&\; n, r \in N).$$

Prove that V_n^r belongs to Q by successive partitions, i.e. $V_n^r \in$ Q, $(n, r \geq 1 \;\&\; n, r \in N)$.

$Proof$. Here, $Q = \{V_1^1, V_2^1, V_3^1,\ V_4^1, V_4^1, \cdots\}$ and Q is contained in N, i.e. $Q \subseteq$ N.

$V_2^2 = V_2^1 + V_2^1 = 2V_2^1$. $Since\ V_2^1 \in Q, V_2^2 \in Q$.

$V_3^2 = V_3^1 + V_2^2 = V_3^1 + 2V_2^1$. Since $V_3^1 \in Q$ and $V_2^1 \in Q, V_3^2 \in Q$.

$V_3^3 = V_3^2 + V_3^2 = 2V_3^1 + 4V_2^1$. Since $V_3^1 \in Q$ and $V_2^1 \in Q, V_3^2 \in Q$.

Similarly, if we can continue these partitions up to V_n^r. Then,

the partitions of V_n^r are in the form of $mV_k^1\ for\ m, k = 1, 2, 3, 4, \cdots$
Here, $V_n^r \in Q$.

Hence, $V_n^r \in$ Q, $(n, r \geq 1 \;\&\; n, r \in N)$ is proved.

4. Conclusion

In this article, partition of a binomial coefficient has been introduced and this idea can enable the scientific researchers to solve the real life problems.

References

[1] Annamalai, C. (2022) Binomial Coefficients and Identities in Combinatorial Geometric Series. OSF Preprints. http://dx.doi.org/10.31219/osf.io/4ha3c.

[2] Annamalai, C. (2022) Multinomial Theorem on the Binomial Coefficients for Combinatorial Geometric Series. Zenodo. http://dx.doi.org/10.5281/zenodo.7029149.

[3] Annamalai, C. (2020) Optimized Computing Technique for Combination in Combinatorics. *hal*-0286583. https://doi.org/10.31219/osf.io/9p4ek.

[4] Annamalai, C. (2020) Novel Computing Technique in Combinatorics. *hal*-02862222. https://doi.org/10.31219/osf.io/m9re5.

[5] Annamalai, C. (2022) Computation of Combinatorial Geometric Series and its Combinatorial Identities for Machine Learning and Cybersecurity. *COE*, *Cambridge University Press*. https://doi.org/10.33774/coe-2022-b6mks.

[6] Annamalai, C. (2022) Annamalai's Binomial Identity and Theorem, *SSRN Electronic Journal*. http://dx.doi.org/10.2139/ssrn.4097907.

[7] Annamalai, C. (2022) Computation Method for Combinatorial Geometric Series and its Applications. *COE*, *Cambridge University Press*. https://doi.org/10.33774/coe-2022-pnx53-v22.

[8] Annamalai, C. (2022) Computing Method for Combinatorial Geometric Series and Binomial Expansion. *SSRN Electronic Journal*. http://dx.doi.org/10.2139/ssrn.4168016.

[9] Annamalai, C. (2022) Factorials and Integers for Applications in Computing and Cryptography. *COE, Cambridge University Press.* https://doi.org/10.33774/coe-2022-b6mks.

[10] Annamalai, C. (2022) Factorial and Multinomial Theorem on the Binomial Coefficients for Combinatorial Geometric Series. *Zenodo*. https://doi.org/10.5281/zenodo.7032292.

[11] Annamalai, C. (2022) Computation of Multinomial and Factorial Theorems for Cryptography and Machine Learning. *COE, Cambridge University Press*. https://doi.org/10.33774/coe-2022-b6mks-v9.

[12] Annamalai, C. (2022) Computation of Factorial and Multinomial Theorems for Machine Leaning and Cybersecurity. *COE*, *Cambridge University Press*. https://doi.org/10.33774/coe-2022-b6mks-v10.

[13] Annamalai, C. (2022) Computation of Binomial, Factorial and Multinomial Theorems for Machine Leaning and Cybersecurity. *COE*, *Cambridge University Press*. https://doi.org/10.33774/coe-2022-b6mks-v11.