Angular Velocity-Addition Formula

Abolfazl Soltani *

Department of Physics, University of Birjand, Birjand, Iran

***Email**: soltani.a.physics@gmail.com*

Abstract

We are all familiar with Galileo's velocity-addition formula, which is about the sum of linear velocities. Our investigations show that there is also a formula for the sum of angular velocities. Here we obtain this formula.

Keywords: Classical mechanic, Angular velocity

Angular Velocity-Addition Formula

Consider an object moving with velocity $v_{S'}$ relative to reference frame S'. If the frame S' moves with the velocity $v_{S'S}$ relative to the S, then the velocity of the object relative to the S is equal to

$$v_S = v_{S'} + v_{S'S}$$

This law is Galileo's velocity-addition formula [1][2]. Our investigations show that there is also a formula for the sum of angular velocities. Consider two coordinate systems S and S', which their centres and axes are coinciding (Fig. 1a). The object A is separated from the y and y' axes at the moment $t = 0$, with a value of angular velocity w_{AS} relative to S, and moves on a circle counter-clockwise around z and z' axes and, after travelling a full circle, reaches again to the y-axis in time Δt. **Simultaneously**, in the length of Δt, the reference frame S' has moved in the clockwise direction, with a value of angular velocity $w_{S'S}$ relative to S, as much as $\Delta\emptyset_{S'S}$. Now, for example, if object A is an inkjet one, it will draw a complete circle on the plane xy in Δt ($\Delta\emptyset_{AS} = 360^{\circ}$). And on the plane $x'y'$ will draw: $\Delta\emptyset_{AS'} = 360 + \Delta\emptyset_{S'S}$. This means that from the point of view of observer S', object A has a greater angular velocity than from the point of view of observer S.

$$\Delta\emptyset_{AS'} = \Delta\emptyset_{AS} + \Delta\emptyset_{S'S} \overset{\times\frac{1}{\Delta t}}{\Longrightarrow} w_{AS'} = w_{AS} + w_{S'S}$$

Formula 1 is *angular velocity-addition formula* that we have managed to discover. For example if $w_{AS} = \pi \ {}^{Rad}/_{S}$ and $w_{S'S} = \frac{\pi}{3} \ {}^{Rad}/_{S}$ So $w_{AS'} = \frac{4\pi}{3} \ {}^{Rad}/_{S}$.

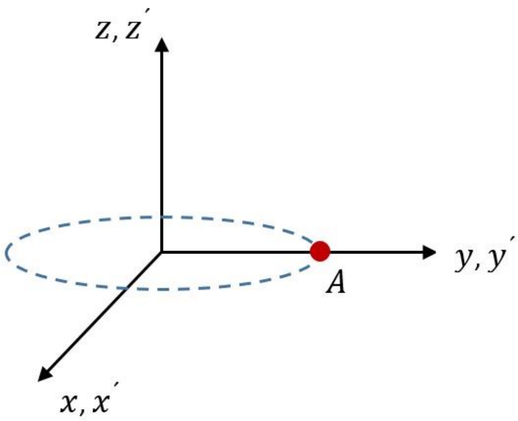

Fig. 1. The two reference frames S and $S^{'}$ are coincident.

If the direction of the vector of angular velocity $\overrightarrow{w_{S^{'}S}}$ is in the positive direction of the z and $z^{'}$ axes (i.e. counter-clockwise rotation of $S^{'}$ around z and $z^{'}$ axes); In such a case, the form of the equation is as follows: $w_{AS^{'}} = w_{AS} - w_{S^{'}S}$. Therefore, the general form of the angular velocity-addition formula is as follows:

$$w_{AS^{'}} = w_{AS} \pm w_{S^{'}S}$$

When the direction of $\overrightarrow{w_{AS}}$ and $\overrightarrow{w_{S^{'}S}}$ is opposite, we use the positive sign; and when they are in the same direction, we use the negative sign.

References:

[1]. Resnick, R. *Introduction to Special Relativity*, Chapter 1, (John Wiley and Sons, Inc, 1968), pp 3-9

[2]. Weidner, R. Sells, R. Elementary Modern Physics, Chapter 2, (Allyn and Bacon, ed. 2, 1973), pp 25-28