Solar System Wave Packet

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Abstract

Pluto, Ceres and all planets of solar system except Neptune, with a high approximation, follow a law called Titius-Bode Law (TBL) or Bode law, which can by no means be considered as a stochastic event. This law shows that the distance of the planets from the sun in Solar system is regulated. If TBL was only a relation between three, four or even five planets, then we could call it a coincidence, but when it is true for seven planets, plus Ceres and Pluto (nine objects), there is definitely a reason for it. Here, we show that, probably, the existence of a standing and cosine wave packet in solar system, with the wavelength $\lambda = 0.6\ AU$ (AU represents the distance of Earth from the sun) and the phase constant $\emptyset_0 = \frac{\pi}{6}$, is the reason for TBL. In this article we obtain the equation of this wave packet. In this article we prove that without Solar System Wave Packet (SSWP), it is not possible to reach to the TBL from the protoplanetary disk of solar system. Here we show that the nebular theory (or nebular hypothesis) without SSWP theory is an incomplete theory.

Keywords: Solar system formation, Titius-Bode law, Nebular theory, Asteroid belt

1. Introduction

The planets of solar system move around the sun in elliptical orbits such that the sun is in one of the focal points of these ellipses. These ellipses are very close to the circle. Pluto, Ceres and all planets of Solar system except Neptune, with a high approximation follow a law known as Bode law or Titius-Bode law (TBL). According to this law, the distance of each planet from the sun is equal to $a = 0.4\ AU + 0.3\ AU \times 2^n$, where $0.4\ AU$ is the distance of Mercury from the sun (or more precisely the length of the semi-major axis of Mercury's orbit) and $n = 0,1,2,3, \ldots$ [1]. It was historically based on this law that Ceres was discovered in 1801 [1]. In this article, we will find the reason for the existence of the TBL. In fact, we prove that, probably, the presence of a cosine and standing wave packet in solar system is the reason for existence of TBL. TBL does not predict the distance of Neptune from the sun but, this article is able to give us the distance of Neptune. In this paper and in the section "SSWP and Nebular theory", we prove that nebular theory, which explains the formation of the solar system, without considering the solar system wave packet is incomplete theory. We prove that without SSWP, it is not possible to reach to the TBL from the protoplanetary disk.

2. A Cosine Standing Wave and Titius-Bode Law

Consider a standing and cosine wave with a wavelength $\lambda = 0.6\ AU$ in solar system; if we assume that the first node of this wave is at a distance of 0.1 AU from the sun the next nodes are at the distances of 0.4 AU, 0.7 AU, 1 AU, 1.3 AU, 1.6 AU, ... 2.8 AU, … from the sun. Each node is 0.3 AU ahead from the previous node. If we consider the planets of solar system in the position of the nodes of this wave, in such a case, there is no planet on the first node (0.1 AU) and Mercury is on the second node, Venus is on the third node, Earth is on the fourth node, Mars is on the sixth node, and the position of fifth node (1.3 AU) is empty. The seventh, eighth, and ninth nodes are empty, and Ceres is on the tenth node. Jupiter is placed on the eighteenth node and Saturn is on the thirty third node, and Uranus, Neptune and Pluto are on the nodes farther from the sun. As you can see, a wave, with the wavelength $\lambda = 0.6\ AU$, easily predicts the position of the planets and therefore, it seems that a huge standing wave plays a role in determination of the position of the planets in solar system. Therefore, we can consider the reason for the TBL to be the existence of a large standing cosine wave in solar system that oscillates along the axis perpendicular to the plane of solar system. We call this wave "Solar system wave packet". In this article, we obtain the equation of this wave packet.

3. Solar System Wave Packet

As mentioned, a cosine **standing** wave, with the wavelength λ = 0.6 AU and $k = \frac{2\pi}{\lambda} = \frac{10\pi}{3}$, can predict the position of the planets in solar system. First, we want to derive the phase constant ($\emptyset_0$) of this wave. Each wave in which the variables x and t are entered as a combination of $kx \pm wt$ is a traveling wave [2]. For example, $\sin(kx - wt + \emptyset_0)$ is a traveling wave. Thus, a standing wave is in the form of $cos(wt)\,cos(kx + \emptyset_0)$ or $sin(wt)\,cos(kx + \emptyset_0)$ or $sin(wt)\,sin(kx + \emptyset_0)$ or $\cos(wt)\sin(kx + \emptyset_0)$. As mentioned, a **cosine** standing wave can predict the positions of planets. Therefore, the form of the standing wave of solar system must be either $sin(wt)\,cos(kx + \emptyset_0)$ or $cos(wt)\,cos(kx + \emptyset_0)$ (In the following, we choose one of these two forms). For the nodes of these two standing waves, we have $cos(kx + \emptyset_0) = 0$. As mentioned previously, Mercury is on the second node of Solar system wave (the second node corresponds to the phase $\frac{3\pi}{2}$ because $Cos\ \frac{3\pi}{2} = 0$). We have:

$$x_{Mercury} = 0.4\ AU \Rightarrow \psi(x_{Mercury}) = 0 \Rightarrow Cos(kx_M + \emptyset_0) = 0 \Rightarrow kx_M + \emptyset_0 = \frac{3\pi}{2} \xrightarrow{k=\frac{10\pi}{3}} \emptyset_0 = \frac{\pi}{6}$$

Having k and $\emptyset_0$, we can easily find the position of the other planets using the equation $kx + \emptyset_0 = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots\ldots, \frac{(2m-1)\pi}{2}$ (Where m is the number of node). For example

$$kx_{Venus} + \emptyset_0 = \frac{5\pi}{2} \Rightarrow \frac{10\pi}{3}x_{Venus} + \frac{\pi}{6} = \frac{5\pi}{2} \Rightarrow x_{Venus} = 0.7\ AU$$

or

$$kx_{Earth} + \emptyset_0 = \frac{7\pi}{2} \Rightarrow \frac{10\pi}{3}x_{Earth} + \frac{\pi}{6} = \frac{7\pi}{2} \Rightarrow x_{Earth} = 1\ AU$$

$$kx_{Mars} + \emptyset_0 = \frac{11\pi}{2} \Rightarrow \frac{10\pi}{3}x_{Mars} + \frac{\pi}{6} = \frac{11\pi}{2} \Rightarrow x_{Mars} = 1.6$$

The distances of the other planets can also be calculated in the same way. According to the above equation (namely $kx + \emptyset_0 = \frac{(2m-1)\pi}{2}$), Neptune is on the one hundred and second node, which corresponds to the phase $\frac{203\pi}{2}$. Contrary to the TBL, which is not able to predict the distance of Neptune, our wave theory predicts the position of Neptune. Therefore, a cosine and standing wave with $\emptyset_0 = \frac{\pi}{6}$ and $k = \frac{10\pi}{3}$ can be attributed to solar system. But what is the general equation of this wave? As mentioned, the equation of solar system wave must contain a component with the equation $cos(\frac{10\pi}{3}x + \frac{\pi}{6})$ and on the other hand, this wave must be a standing wave so that the position of the nodes (planets) does not change. Therefore, as mentioned, the form of solar system wave must be either $cos(\delta wt)\, cos(\frac{10\pi}{3}x + \frac{\pi}{6})$ or $sin(\delta wt)\, cos(\frac{10\pi}{3}x + \frac{\pi}{6})$. There is no difference between $cos(\delta wt)$ and $sin(\delta wt)$ Because we know from trigonometric identities that: $cos(\delta wt) = \sin\left(\delta wt + \frac{\pi}{2}\right)$. Therefore, we choose the function $cos(\delta wt)\, cos(\frac{10\pi}{3}x + \frac{\pi}{6})$. δ is a constant number. Since solar system has a certain size and is not infinitely wide, its wave must be localized (a wave packet). If we consider an expression in the form $e^{-\gamma x^2}$ (which is a Gaussian function and plays the role of the wave envelope) in the final equation of solar system wave packet, in such a case, the final formula is a localized wave or a wave packet[1]. Thus, the primary form of the formula of SSWP is as follows (equation 1) and the planets are on the nodes of this wave packet (Fig. 1):

$$\begin{cases} \psi(x,t) = C\, cos(\delta wt)\, cos(\frac{10\pi}{3}x + \frac{\pi}{6})\; e^{-\gamma x^2} & x \geq 0 \\ \psi(x,t) = C\, cos(\delta wt)\, cos(\frac{10\pi}{3}x - \frac{\pi}{6})\; e^{-\gamma x^2} & x \leq 0 \end{cases} \qquad (1)$$

In equation 1, γ, C and δ are constant values.

[1] Although a Gaussian wave packet is also infinitely wide, but it is very localized.

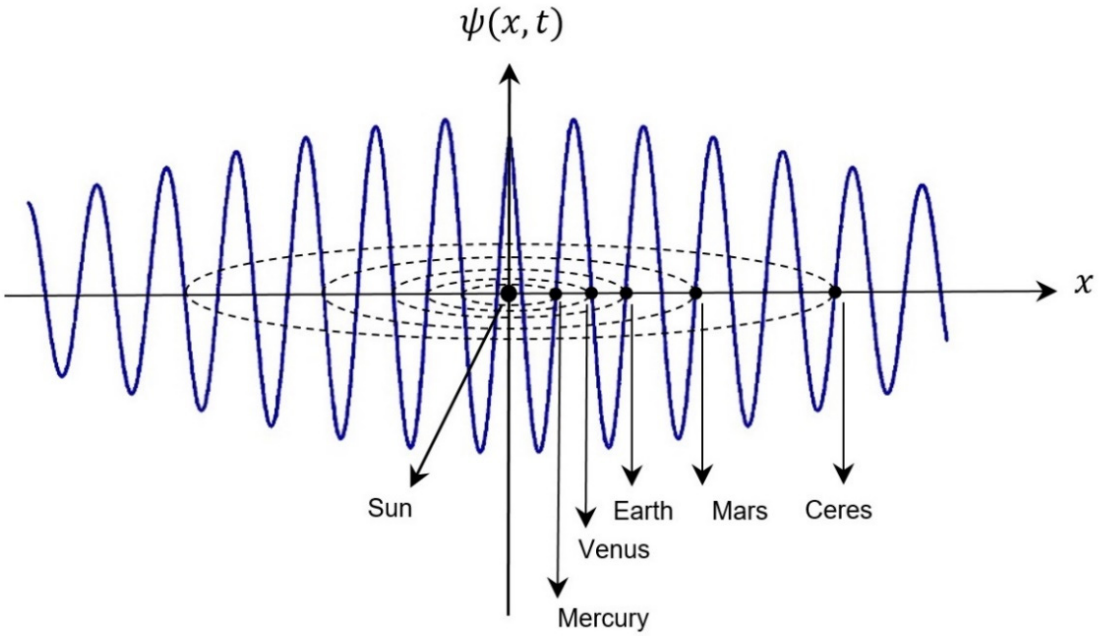

Fig. 1. Standing solar system wave packet with λ = 0.6 *AU* and $\emptyset_0 = \frac{\pi}{6}$. Diagram of $\psi(x,t)$ at the moment t = 0. The value of $\psi(0,0)$ equals $\sqrt{3}C/2$. This diagram is drawn by certain values C and γ in equation 1 ($C = 0.75\ and\ \gamma = 0.03$). As you can see, the planets are on the nodes of the wave packet. Jupiter, Saturn, Uranus, Neptune, and Pluto are on the nodes farther from the sun. The reason why there is no planet in some nodes will explained in the section 4: "SSWP and Nebular theory". This is due to the unbalanced mass distribution in the protoplanetary disk of solar system.

In figure 1, the wave oscillates along the ψ axis over time. Because of SSWP is a standing wave, the nodes and the anti-nodes do not move relative to each other along the x-axis. This does not mean that SSWP is stationary in the space. Solar system wave packet (Fig. 1) rotates, along with solar system, around the center of the Milky Way galaxy.

As you observed, we were able to attribute a wave to the solar system and the TBL. Is this a coincidence? **This is very improbable**.

4. SSWP and Nebular theory

In this section we show that the nebular theory without SSWP theory is an incomplete theory and SSWP theory must be attached to the nebular theory in order to explain the formation of solar system.

The equation 1 in cylindrical coordinate (in cylindrical coordinate we have $x = r\cos\theta$) is:

$$x = r\cos\theta \Rightarrow \psi(r,\theta,t) = C\,cos(\delta wt)\,cos(\frac{10\pi}{3} r\,cos\,\theta + \frac{\pi}{6})\ e^{-\gamma(r\,cos\,\theta)^2} \qquad (2)$$

Figure 2 is the shape of figure 1 in three dimensions based on equation 2 ($C = 0.75$ and $\gamma = 0.03$):

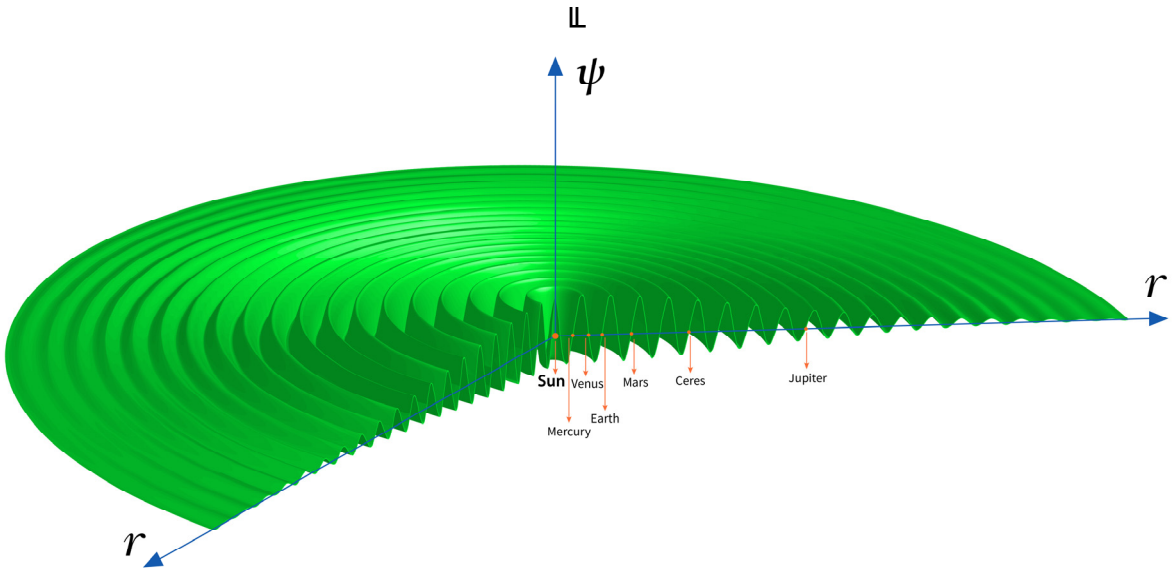

Fig. 2. Solar System Wave Packet in three dimensions. Diagram $\psi(r,\theta,t)$ at the moment t = 0. Here the wave packet of Figure 1 is shown in three dimensions. Fig 2 is drawn in cylindrical coordinates using equation 2. Based on equation 2 because of exponential factor, the amplitude of the wave decreases while we move away from the ψ-axis.

It is unacceptable to imagine that the wave packet of solar system is not related with an object. In my opinion, SSWP belongs to an object in the solar system. I think that object is Sun. As you can see in Fig. 2, the Sun is in the center of SSWP and it is also the heaviest object in Solar System (more than 99 percent of the total mass of the Solar System belongs to the Sun.). Therefore, it is reasonable to assume that SSWP belongs to the Sun. So, we can call SSWP the **associated wave packet of Sun.**

As you can see in Fig. 1 and Fig. 2, because of the symmetry of equations 1 and 2, the orbits, created by the SSWP, are circle. **But the real orbits of the planets are elliptic.** The reason of elliptic orbits is the existence of the inverse_square gravitational force of the sun[2]. As you know, the sun was formed earlier than the planets [3][4][5]. Simultaneously with the formation of the sun, about 4.6 billion years ago [4], its associated wave packet was also formed[3]; and the oscillation of this wave packet arranged and collected the gas-dust particles of Protoplanetary disk in regular orbits (Titius-Bode orbits). Just like the standing wave pattern on the kettledrum head in Fig. 3. As you can observe in Fig.3, by a mechanical oscillator at the upper left of the photograph, the powder particles are collected at the circular nodes [6]. Similarly, because of oscillation of SSWP in the early years of formation of solar system, the gas-dust particles of protoplanetary disk were collected, in some circular nodes of SSWP (like Fig. 3). After this, because of the gravitational force of the sun, these particles were located on elliptical orbits. **Thus, circular orbits were changed to elliptical orbits.** Then, due to collisions with each

[2] We know from classical mechanics that the elliptic orbits of the planets (Kepler's first law) are the result of Newton's law of gravitation, which is an inverse_square relation.

[3] The wave packet of solar system probably was created either when the sun was a protostar or when the newborn sun was entered to the Main-sequence. The distance between these two phases is very short (less than 50 million years) [3] and both phases occurred before the formation of the planets. In both states, we have no idea how and why this wave packet was formed.

other, the gas-dust particles on each orbit were compressed together and large grains (seeds) were formed by electrostatic forces between gas-dust particles[4] [7]. After this, large grains were sticked to each other by electrostatic forces until as the grains grew in mass, gravity began to aid in the accretion process, accelerating their growth into boulders large enough to count as planetesimals, which means "pieces of planets." [7]; then because of gravity force between planetesimals, Protoplanets were formed and in the final stage planets[5]. What we did here was to merge SSWP theory and Nebular theory.

In asteroid belt, the process of planet formation was stopped at planetesimal (or Asteroid) stage; Because of orbital resonances with Jupiter, Protoplanet and subsequently planet were not formed [8]. Orbital resonances with Jupiter disrupted the orbits of asteroids in the asteroid belt, preventing them from accreting into a planet. Many of asteroids were ejected, but some remained and they make up the asteroid belt today [8][9].

Although the heaviest object of the asteroid belt (namely Ceres), which has almost 30 % of the total mass of the belt, is at distance $2.77\ AU$ (which is the distance that Bode law gives us with a high approximation.); But as observations show, the asteroid belt is wide between $2.2\ AU$ and $3.2\ AU$. In my opinion, at the beginning of the formation of solar system, because of oscillation of SSWP, first, the asteroid belt was formed at distance $2.8\ AU$ (The formation stages were explained in the previous paragraph.) and included planetesimals (**primary asteroids**). The orbit was a circle and then, because of gravity force of the sun, was changed to elliptical orbit (with semi major axis $2.77\ AU$). In this elliptical orbit, and in the early years, some of the primary asteroids collided with each other and they formed Ceres, due to gravitational force. But some of these primary asteroids were not absorbed by Ceres. In later years these remaining asteroids collided with each other and become fragmented; and so Asteroid belt was widened and the fragmented asteroids (**secondary asteroids**) were distributed at a distance $2.2\ AU$ to $3.2\ AU$; with various Orbital eccentricities (from zero to above 0.3) and various Orbital inclinations (from zero to >20°). It is obvious that the intensity of the collisions of primary asteroids with each other had been effective in the variety of eccentricities and orbital inclination of secondary asteroids.

In addition to the collision of primary asteroids with each other, the collision of a foreign object could also cause the belt to widen. Imagine that a set of primitive asteroids was formed at a distance of $2.8\ AU$ due to the oscillation of SSWP. A collision of a celestial body with **some of** these early asteroids could disrupt **all or part of** the belt order and cause various collisions. Like a train hitting an obstacle that causes the wagons to crash. These collisions have led to the formation of secondary asteroids and the widening of the asteroid belt.

If SSWP did not exist; the planets might have been formed around the sun but their distance from the sun was random and irregular. Without SSWP, it is not possible to justify the existence of TBL based on nebular hypothesis. Thus, it seems that the existence of a standing wave in

[4] "Collisions" were more like gentle touches. The particles were far too small to attract each other gravitationally at this stage, they were able to stick together through electrostatic forces—the same "static electricity" that makes hair stick to a comb [7].

[5] The mass distribution in the Protoplanetary disk had not been uniform. Therefore, during formation of the planets, in some nodes of SSWP, less dusts were collected; and in some nodes, more dusts were placed next to each other. Therefore, in some nodes a planet was formed and in other nodes a planet was not formed; so, the location of these nodes is empty.

the solar system is undeniable. Therefore, the nebular theory becomes more complete by SSWP theory.

The reason why there is no planet in some nodes in figure 1 and 2 is due to the unbalanced mass distribution in the protoplanetary disk of solar system. Since the mass distribution in the Protoplanetary disk had not been uniform. During formation of the planets, in some nodes of SSWP, less dusts were collected; and in some nodes, more dusts were placed next to each other. Therefore, in some nodes a planet was formed and in other nodes a planet was not formed; So, the location of these nodes is empty.

Fig. 3. Standing wave pattern on a kettledrum head. One of many possible standing wave patterns on a kettledrum head, made visible by dark powder sprinkled on the drum head. As the head is set into oscillation at a single frequency by a mechanical oscillator at the upper left of the photograph, the powder particles are collected at the nodes. Similarly, due to the oscillation of the SSWP, the gas-dust particles of protoplanetary disk were collected in certain orbits, in the early years of formation of solar system.

5. What is Waving?

What is the nature of SSWP? In this section we will show that the oscillation of SSWP **probably** is the oscillation of Dark matter.

In this article, we showed that oscillation of a standing wave had been effective in the determination of location of the orbits of solar system planets. In the early years of formation of Solar system, the oscillation of SSWP collected the gas-dust particles in certain orbits. On the other hand, we know that Dark matter interacts with ordinary matters and affects them[6]. Therefore, it can be concluded that the oscillation of SSWP in Fig. 2 and its interaction with gas-dust particles in protoplanetary disk, probably, is the oscillation of Dark matter (or dark matter is one of the candidates for the nature of SSWP). Of course, there is another possibility. This wave may not be dark matter. It might be something we are not familiar with.

[6] In addition to gravitational interactions, dark matter may also has **elastic interaction** with ordinary matter (like collision between drum head and dark powder particles in Fig 3).

6. Extrasolar Planetary Systems and Planets Systems

In recent years, researchers have been able to find mathematical relations similar to TBL law for the planetary systems like Jupiter system, Uranus system and Saturn system and some extrasolar planetary systems. Blagg [10], Richardson [11] and Dermott [12][13] equations are three examples of these formulas. In the planetary systems in the solar system, such as Jupiter and its moons, Saturn and its moons, and Uranus and its moons and in extrasolar systems if we can find the existence of a cosine periodic wave, In such a case, it can be concluded that a wave packet like the wave packet of solar system played a role in the formation process of that planet or star system. We have to consider an important point here. Among the celestial bodies around us, the sun is a special body. The sun is a thousand times heavier than the largest planet in the solar system, Jupiter. In celestial physics, mass is a very decisive factor in determining the characteristics of a celestial body. The difference is the mass, which turns some objects into stars, some into planets, some into black holes, white dwarfs, and neutron stars; each of which has very different characteristics. The difference in mass has turned the sun into a star, and Jupiter, Saturn, and Uranus into planets. Like nuclear fusion and very high temperature and some other features that are only specific to stars and are related to their mass, we can consider SSWP to be specific to stars, not planets. **Therefore, it is not strange thing if a wave packet similar to the wave packet of solar system is not discovered around planets like Jupiter and Saturn.** For example, if you check the distances of the main moons of Jupiter to Jupiter, you will see that no wavelength can be attributed to this system. If we cannot find a wavelength for systems other than Solar system, this does not mean that SSWP theory is invalid (As I said, it is very improbable that the correlation of a wave with wavelength $\lambda = 0.6\,AU$ to the solar system is a coincidence). As I said, if we consider mass as an effective factor in the formation of the wave packet, the SSWP is definitely stronger than Jupiter's wave packet, and therefore the effect of Jupiter's wave packet on the dust disk around it is less than the effect of the SSWP on the protoplanetary disk. Therefore, the non-observation of the effect of the wave packet in the Jupiter system can be attributed to the dominance of the dust disk around Jupiter over the effects of the Jupiter wave packet.

7. Associated Wave Packet of Sun

As we said in section 4 it is unacceptable to imagine that the wave packet of solar system is not related with an object. In this section we will show that the equation 1 is exactly in the form of real part of the solution of Schrodinger equation and therefore, based on what we have learned from Quantum mechanics, we can attribute it to an object like the sun. In fact, in this section we prove that equation 1 is the result of superposition of a set of infinite number of flat matter waves which each of them is a solution of Schrodinger equation; And so we can consider equation 1 as a solution of Schrodinger equation. Base of calculations in this section is superposition principle. Consider a set of infinite number of flat matter waves $Ae^{i(kx-wt+\emptyset_0)}$, $Ae^{i(kx+wt+\emptyset_0)}$, $Ae^{i(kx-wt-\emptyset_0)}$ and $Ae^{i(kx+wt-\emptyset_0)}$ which move in the positive and negative directions of the x-axis[7]; and assume that all of these waves are under the effect of potential $V(x)$[8]. In such a case the angular frequency (w) of each of these matter waves is equal:

[7] These four groups of waves are all of possible form of flat matter waves.

[8] Since the protoplanetary disk of solar system was under effect of gravity of new born sun; therefore, here we investigated the superposition of matter waves which they are under the effect of a potential.

$$w = \frac{E}{\hbar} = \frac{1}{\hbar}(\frac{P^2}{2m} + V(x))$$

Because, based on De Broglie theory, the energy of a matter wave is equal to the energy of associated particle of the wave i.e. E= $\frac{P^2}{2m} + V(x)$. As you know these four groups of waves with general equation $Ae^{i(kx \pm wt \pm \emptyset_0)}$ are the solution of the Schrodinger equation namely

$$i\hbar\frac{\partial}{\partial t}\psi(x,t) = -\frac{\hbar^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2} + V(x)\psi(x,t)$$

These waves interact together in space based on superposition principle. From these infinite number of waves we will show that the superposition of **a part of** these waves, which **1)**- their wave number is around the median of k_0 and between $k_0 + {\Delta k}/{2}$ and $k_0 - {\Delta k}/{2}$ and **2)** - their angular frequency equals w_0 and **3)**- the amplitude of these waves is on the bell shaped function $A(k) = (\frac{2\alpha}{\pi})^{1/4}e^{-\alpha(k-k_0)^2}$(which is a Gaussian function), **is in the form of equation 1** [9].

First consider a set of infinite number of these waves with equation: $Ae^{i(kx-wt+\emptyset_0)}$ which move in the **positive direction of x-axis**. In such a case, the resultant of these waves, using the superposition principle, is a wave packet with equation 2 [14][15][16].

$$\psi_{total}(x,t) = \frac{1}{\sqrt{2\pi}}\int_{-\Delta k/2}^{\Delta k/2} A(k)e^{i(kx-w_0t+\emptyset_0)}dk \qquad (2)$$

Where k means k_x. In equation $A(k) = (\frac{2\alpha}{\pi})^{1/4}e^{-\alpha(k-k_0)^2}$, α is a constant with a positive value and shows the width of the bell-shaped function $A(k)$. Coefficient $(\frac{2\alpha}{\pi})^{1/4}$ is a normalization coefficient which is obtained by normalize of $A(k)$. Since equation 2 is derived from the superposition principle, it is the solution of the Schrodinger equation.

To obtain $\psi_{total}(x,t)$ from equation 2, we calculate the superposition of all of the waves in one moment, which we consider to be the origin of time ($t = 0$), and then we can obtain the net wave at any other time. We have:

$$\psi(x,0) = \frac{1}{\sqrt{2\pi}}\int A(k)e^{i(kx+\emptyset_0)}dk \qquad (3)$$

The above equation is the momentary image of the net wave. Multiply equation 3 by $e^{ik_0x-ik_0x}$. We have:

$$\psi(x,0) = \frac{1}{\sqrt{2\pi}}e^{i(k_0x+\emptyset_0)}\int A(k)e^{i(k-k_0)x}dk \qquad (4)$$

[9] In the Electromagnetic (EM) waves we cannot consider one w_0 for two or many waves in which their k is different from each other, because for all of the EM waves we have: $w = ck$ where c is the velocity of light. But for matter waves the issue is different. In the matter waves we have $w = \frac{\hbar k^2}{2m}$ [16]. As you can see w is the **function of k and m**. Therefore, it is possible to choose one value of w_0 for the waves in which their k is different from each other.

Considering $\acute{k} = k - k_0$, we have:

$$\psi(x,0) = (\frac{\alpha}{2\pi^3})^{1/4} e^{i(k_0 x + \emptyset_0)} \int e^{-\alpha \acute{k}^2} e^{i\acute{k}x} d\acute{k} \quad (5)$$

Using the variable transformation $\acute{k} - \frac{ix}{2\alpha} = q$ [14][15] and the Gaussian integral $\int_{-\infty}^{\infty} dq\, e^{-\alpha q^2} = \sqrt{\frac{\pi}{\alpha}}$, equation 5 can be calculated. After replacement and simplification, we reach the following final solution [14][15]:

$$\psi(x,0) = (\frac{\alpha}{2\pi^3})^{1/4} \sqrt{\frac{\pi}{\alpha}}\, e^{i(k_0 x + \emptyset_0)}\, e^{-\frac{x^2}{4\alpha}} = (\frac{1}{2\pi\alpha})^{1/4} e^{i(k_0 x + \emptyset_0)}\, e^{-\frac{x^2}{4\alpha}} \quad (6)$$

Lets check the normalization

$$\int_{-\infty}^{\infty} |\psi(x)|^2\, dx = \sqrt{\frac{1}{2\pi\alpha}} \int_{-\infty}^{\infty} e^{-\frac{x^2}{2\alpha}}\, dx = \sqrt{\frac{1}{2\pi\alpha}} \sqrt{2\alpha\pi} = 1$$

Given a normalized $A(k)$, we get the normalized $\psi(x)$.

Now, how is the time variation of equation 6? Let's go back to equation 2:

$$\psi(x,t) = \frac{1}{\sqrt{2\pi}} \int A(k) e^{i(kx - w_0 t + \emptyset_0)} dk = (\frac{\alpha}{2\pi^3})^{1/4} \int e^{-\alpha(k-k_0)^2} e^{i(kx - w_0 t + \emptyset_0)} dk$$

Substituting $e^{ik_0 x - ik_0 x}$; we will have:

$$\psi(x,t) = (\frac{\alpha}{2\pi^3})^{1/4} e^{i(k_0 x + \emptyset_0) - i w_0 t} \int e^{-\alpha \acute{k}^2} e^{i\acute{k}x} d\acute{k}$$

This integral is similar to integral 5, which led to $\psi(x,0)$ (namely equation 6). Therefore, we have:

$$\psi(x,t) = \psi_1(x,t) = (\frac{1}{2\pi\alpha})^{1/4}\, e^{i(k_0 x - w_0 t + \emptyset_0)}\, e^{-\frac{x^2}{4\alpha}} \quad (7)$$

$$e^{i\theta} = cos\theta + isin\theta \Longrightarrow Re\, \psi_1(x,t) = (\frac{1}{2\pi\alpha})^{1/4} cos(k_0 x - w_0 t + \emptyset_0)\, e^{-\frac{x^2}{4\alpha}} \quad (8)$$

Due to the presence of the factor $k_0 x - w_0 t$, equations 7 and 8 represent a traveling wave packet that propagates in the positive direction of the x-axis [2]. This means that the location of the nodes is not known. Due to the absence of t in $e^{-\frac{x^2}{4\alpha}}$ in equations 7 and 8, the wave packets in these equations does not spread.

Previous calculations was about superposition of the waves $e^{i(kx - w_0 t + \emptyset_0)}$. Similarly, we use the recent trend to obtain the superposition of flat waves traveling **in the negative direction of the *x*-axis,** i.e. $Ae^{i(kx + w_0 t + \emptyset_0)}$. If we do this, we get to equation 9:

$$\psi_2(x,t) = (\frac{1}{2\pi\alpha})^{1/4}\, e^{i(k_0 x + w_0 t + \emptyset_0)}\, e^{-\frac{x^2}{4\alpha}} \quad (9)$$

$$Re\, \psi_2(x,t) = (\frac{1}{2\pi\alpha})^{1/4} cos(k_0 x + w_0 t + \emptyset_0)\, e^{-\frac{x^2}{4\alpha}} \quad (10)$$

This equation shows a traveling wave packet that propagates in the negative direction of the x-axis.

Now we sum up the two equations 10 and 8 together to get the final wave.

$$Re\ \psi_{total}(x,t) = Re\ \psi_1 + Re\ \psi_2$$

Thus:

$$Re\ \psi_{total}(x,t) = (\frac{1}{2\pi\alpha})^{1/4}\ e^{-\frac{x^2}{4\alpha}}[cos(k_0x - w_0t + \emptyset_0) + cos(k_0x + w_0t + \emptyset_0)] \qquad (11)$$

Using $cos\alpha + cos\beta = 2cos\frac{1}{2}(\alpha+\beta)cos\frac{1}{2}(\alpha-\beta)$ and $\cos(\theta) = \cos(-\theta)$ we obtain the equation of a standing wave packet.

$$\begin{cases}\alpha = k_0x - w_0t + \emptyset_0 \\ \beta = k_0x + w_0t + \emptyset_0\end{cases} \Longrightarrow Re\ \psi_{total}(x,t) = 2(\frac{1}{2\pi\alpha})^{1/4}\cos(k_0x + \emptyset_0)cos(w_0t)e^{-\frac{x^2}{4\alpha}} \qquad (12)$$

There is not the structure of $kx \pm wt$ in equation 12 so the ψ_{total} is a standing wave. As you observe, equation 12, which is the real part of a solution of the Schrodinger equation, is exactly the same as equation 1 for $x \geq 0$, which is SSWP. Is this similarity coincidental? No. **Therefore, equation 1 is the real part of a solution of the Schrodinger equation. It means that the Schrodinger equation and quantum mechanics are valid in astronomical scale.** By comparing equation 12 and equation 1, we have

$$\delta = 1\ ,\ \gamma = \frac{1}{4\alpha} \quad and \quad C = 2(\frac{1}{2\pi\alpha})^{1/4}$$

If we put these values in equation 1, then we get the final equation of SSWP for $x \geq 0$:

$$Re\ \psi_t(x,t) = 2(\frac{1}{2\pi\alpha})^{1/4}\ cos(w_0t)\ cos(\frac{10\pi}{3}x + \frac{\pi}{6})\ e^{-\frac{x^2}{4\alpha}} \qquad x \geq 0 \qquad (13)$$

Equation 13 is obtained by calculating the superposition of a set of infinite number of waves $Ae^{i(kx-w_0t+\emptyset_0)}$ and $Ae^{i(kx+w_0t+\emptyset_0)}$ that move in opposite directions to each other (pay attention to the + sign behind $\emptyset_0$). Now if we sum a set of infinite number of flat wave functions with the equations $Ae^{i(kx-w_0t-\emptyset_0)}$ and $Ae^{i(kx+w_0t-\emptyset_0)}$ (pay attention to the $-$ sign behind $\emptyset_0$) together, by following the path we have taken from equation 2 to equation 13, we reach the following relation;

$$Re\ \psi_t(x,t) = 2(\frac{1}{2\pi\alpha})^{1/4}\ cos(w_0t)\ cos(\frac{10\pi}{3}x - \frac{\pi}{6})\ e^{-\frac{x^2}{4\alpha}}$$

Which is the same as equation 1 for $x \leq 0$. Therefore, the final form of SSWP (equation 1) is as follows:

$$\begin{cases} Re\ \psi(x,t) = 2(\frac{1}{2\pi\alpha})^{1/4}\ cos(w_0t)\ cos(\frac{10\pi}{3}x + \frac{\pi}{6})\ e^{-\frac{x^2}{4\alpha}} & x \geq 0 \\ Re\ \psi(x,t) = 2(\frac{1}{2\pi\alpha})^{1/4}\ cos(w_0t)\ cos(\frac{10\pi}{3}x - \frac{\pi}{6})\ e^{-\frac{x^2}{4\alpha}} & x \leq 0 \end{cases} \qquad (14)$$

In this equation, the larger the α is, the more the width of wave packet, along the x-axis. We drew Fig. 1 by $\alpha = 10$. In this section we did not do anything strange. Rather, we have used only the superposition principle. We calculated the superposition of infinite number of flat

matter waves with general equation $e^{i(kx\pm wt\pm\emptyset_0)}$ which their angular frequency equals w_0 and the wave number of these waves is around the median of k_0 and between $k_0+{\Delta k}/{2}$ and $k_0-{\Delta k}/{2}$ and the amplitude changes of these waves is equal to $A(k)=(\frac{2\alpha}{\pi})^{1/4}e^{-\alpha(k-k_0)^2}$ and they are under the effect of potential $V(x)$. There have been infinite number of flat matter waves in the early solar system which the superposition of a set of them made the wave function in figure 1. Our calculations showed that only the interference of this group of waves is constructive. In the protoplanetary disk and **before the formation of the sun**, there were countless flat matter waves with the general equation $e^{i(kx\pm wt\pm\emptyset_0)}$. **After the formation of sun** because of gravitational collapse, a special group of these waves interfered constructively with each other and created the wave function of the solar system.

Here we demonstrated that equation of SSWP (equation 1) is the real part of a solution of the Schrodinger equation. So, based on quantum mechanics, we can attribute it to an object in Solar system. The closest star to solar system is at a distance of 4.8 light-years, which is so far. And the biggest and heavyset object in solar system is sun. Therefore, the wave function of solar system can only belong to the sun. De Broglie considered the wave nature for subatomic particles, and here we attributed the wave nature to celestial objects. Neither of these two actions is strange. Rather, they are truths that we must become accustomed to.

In this article, we proved that the Schrodinger equation is valid in astronomical scale; on the other hand, as you know, the Schrodinger relation is based on de Broglie equation ($\lambda=\frac{h}{mv}$). Therefore, the de Broglie equation is valid in astronomical scale[10]. But, according to the very large mass of sun, using the de Broglie relation the wavelength 0.6 AU will not obtain. So, instead of Planck constant we must choose another value for celestial objects, which is larger than h. We call this new value the Planck constant in Astronomy ($h_{Astronomy}$) abbreviated as h_A and we have: $\lambda_A=\frac{h_A}{P}$. In such a case, the Schrodinger equation in the astronomical scale can be written as follows:

$$i\hbar_A\frac{\partial}{\partial t}\psi(x,t)=-\frac{{\hbar_A}^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2}+V(x)\psi(x,t) \qquad (15)$$

If we follow the path of proving the Schrodinger equation [17] and put the value $\hbar_A$ instead of $\hbar$, we reach equation 15. The Davisson_Germer experiment [18] is considered as the confirmation of existence of the de Broglie wave at the atomic scale, and the regularity of the distances of the planets from sun (Titius-Bode law) is the confirmation of the existence of the de Broglie wave in astronomical scale.

Conclusion

In this article, first, in the abstract, we showed that it is very improbable for TBL to be a coincidence; because it is valid for nine objects, not two or three objects. Here, we showed that, probably, in the early years of the formation of the solar system, there was a wave packet in solar system. Does this wave packet still exist? I do not know and I have no idea about it yet.

[10] The Davisson–Germer experiment [18] is the confirmation of the existence of the de Broglie wave in subatomic scale and the regularity of the distances of the planets from sun (Titius-Bode law) is the confirmation of the existence of the de Broglie wave in astronomical scale.

In this article and in section 4, we merged Nebular theory with SSWP theory and we proved that nebular theory without SSWP theory is an incomplete theory. SSWP theory must be attached to the nebular theory to explain the formation of the solar system. Without SSWP, it is not possible to reach to the TBL from the protoplanetary disk. Is the description of TBL by a wave a coincidence? This is very improbable. De Broglie considered the wave nature for subatomic particles, and here we attributed the wave nature to celestial objects. Neither of these two actions is strange. Rather, they are truths that we must become accustomed to.

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