Studies on Maxwell Equations, Electromagnetic Wave Equations and Functions in Cosmological Special Relativity Theory

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In this paper, we explore various aspects of the Cosmological Special Relativity theory. Our investigations include Maxwell equations, electromagnetic wave equations and functions, energy-momentum relations, the Klein-Gordon equation and wave function, the Yukawa potential in the Cosmological Inertial Frame (CIF), the Schrödinger equation, the Dirac equation, and their respective solutions. We also analyze the interaction between A-type wave function and B-type extended distance.Our particular focus is on the time evolution of the Yukawa potential in the CIF and the quantification of the Klein-Gordon scalar field. Through our analysis, we have established a connection between the newly modified Friedman-Lematre-Robertson-Walker metric, the Dirac equation, and the Klein-Gordon equation.

Keywords: Friedman-Lemaître-Robertson-Walker metric, the Dirac equation,Klein-Gordon equation

1. INTRODUCTION

The Cosmological Special Relativity theory is an important framework for studying the properties of the universe at a fundamental level. It is based on the principles of special relativity and incorporates the effects of cosmological expansion and the large-scale structure of the universe. In this paper, we explore various aspects of this theory, with a focus on the study of Maxwell equations, electromagnetic wave equations and functions, energy-momentum relations, Klein-Gordon equation and wave function, Yukawa potential, Schrödinger equation, Dirac equation and their solutions, and the interaction between the A-type wave function and the B-type extended distance.[1–3]

Maxwell Equations and Electromagnetic Wave Equations: The Maxwell equations describe the behavior of electromagnetic fields in vacuum or matter. They are given by the following equations:

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}, \tag{1}$$

$$\nabla \cdot \mathbf{B} = 0, \tag{2}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \tag{3}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}. \tag{4}$$

where $\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic fields, respectively, ρ is the charge density, $\mathbf{J}$ is the current density, ϵ_0 is the vacuum permittivity, and μ_0 is the vacuum permeability. The electromagnetic wave equations can be derived from the Maxwell equations by taking the curl of the Faraday's law and using the wave equation for the electric field, which gives:

$$\frac{\partial^2 \mathbf{E}}{\partial t^2} - c^2 \nabla^2 \mathbf{E} = 0, \tag{5}$$

where c is the speed of light.

Energy-Momentum Relations: The energy-momentum relation in special relativity is given by $E^2 = p^2c^2 + m^2c^4$, where E is the energy, p is the momentum, c is the speed of light, and m is the mass. For a photon, which has no rest mass, the energy-momentum relation reduces to $E = pc$. In the Cosmological Special Relativity theory, the

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energy-momentum relation is modified by the expansion of the universe and the presence of the cosmic microwave background radiation.

In this paper, when Λ is the cosmological constant about $\left(g^{\theta\theta}\right)^2$[3–6],we focus on the study of the Maxwell equations, electromagnetic wave equations, and functions in the cosmological special relativity theory. We also investigate the energy-momentum relation, Klein-Gordon equation, and wave functions in this theory. One of the main objectives of our research is to understand the variation of the Yukawa potential in the cosmological inertial frame.We have found a connection between the newly modified Friedman-Lemaître-Robertson-Walker metric and the Dirac equation and the Klein-Gordon equation.

2. THE MAXWELL EQUATIONS ARE FUNDAMENTAL EQUATIONS IN ELECTROMAGNETISM

The Maxwell equations are fundamental equations in electromagnetism, describing the behavior of electric and magnetic fields. In the cosmological special relativity theory, these equations take the form:[7–9]

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}, \qquad \nabla \cdot \mathbf{B} = 0, \tag{6}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \qquad \nabla \times \mathbf{B} = \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}, \tag{7}$$

where $\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic fields, respectively, ρ is the charge density, ϵ_0 is the electric permittivity of free space, μ_0 is the magnetic permeability of free space, and t is time.

The electromagnetic wave equation in the cosmological special relativity theory is derived from the Maxwell equations and takes the form:

$$\frac{\partial^2 \mathbf{E}}{\partial t^2} - c^2 \nabla^2 \mathbf{E} = 0, \tag{8}$$

where c is the speed of light.

The Klein-Gordon equation is a relativistic wave equation that describes the behavior of spinless particles with mass. In the cosmological special relativity theory, the Klein-Gordon equation takes the form:

$$\left(\frac{\partial^2}{\partial t^2} - c^2 \nabla^2 + \frac{m^2 c^4}{\hbar^2} \right) \phi(x) = 0, \tag{9}$$

where m is the mass of the particle, $\hbar$ is the reduced Planck constant, and $\phi(x)$ is the wave function.

The Yukawa potential is an important quantity in particle physics and describes the behavior of the strong nuclear force. In the cosmological special relativity theory, the Yukawa potential in the inertial frame is given by:

$$V(r,t) = \frac{g^2}{4\pi r} e^{-\lambda(t) r}, \tag{10}$$

where g is the coupling constant, r is the distance between two particles, $\lambda(t)$ is the time-dependent range of the interaction, and t is time.

By solving the Klein-Gordon equation, we can obtain the time dependence of the Yukawa potential in the cosmological inertial frame.

The Schrödinger equation is a wave equation that describes the behavior of quantum mechanical systems. In the cosmological special relativity theory, we can derive the Schrödinger equation from the free-particle wave function of the Klein-Gordon equation:

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi, \tag{11}$$

where ψ is the wave function of the system and m is the mass of the particle.

3. THE DIRAC EQUATION IS A FIRST-ORDER WAVE EQUATION THAT DESCRIBES THE BEHAVIOR OF PARTICLES WITH SPIN

In order to investigate the Yukawa potential in the cosmological inertial frame, we begin by solving the Klein-Gordon equation. The Klein-Gordon equation is given by:[7–9]

$$\Box\phi + m^2\phi = 0, \tag{12}$$

where $\Box = \partial_\mu\partial^\mu$ is the d'Alembertian operator, ϕ is the field, and m is the mass of the field.

The purpose of this theory is that we find that the modified Einstein gravitational equation has a Reissner-Nodstrom solution in vacuum. First, we can consider the following equation (modified Einstein's gravitational equation). The proper time of spherical coordinates is [7–10]

$$\begin{aligned} ds^2 =&G(t,r)dt^2 - \frac{1}{G(t,r)}dr^2 + \left[r^2 d\theta^2 + r^2\sin^2\theta d\varphi^2\right] \\ R_{\mu\nu} - &\frac{1}{2}g_{\mu\nu}R + \Lambda\left(\left(g^{\theta\theta}\right)^2\right)g_{\mu\nu} = -\frac{8\pi G}{C^4}T_{\mu v}. \end{aligned} \tag{13}$$

According to the cosmological principle, one of the most important features of astronomical systems on the cosmological scale is their uniformity and isotropy. Howard P. Robertson and Arthur Walker proved in 1935 and 1936, respectively, that there are only three types of four-dimensional spacetimes that satisfy the above requirements of uniformity and isotropy, and their spacetime metrics have the following form:

$$\mathrm{d}s^2 = R^2(t)\left(\frac{\mathrm{d}r^2}{1-kr^2} + r^2\,\mathrm{d}\theta^2 + r^2\sin^2\theta\mathrm{d}\phi^2\right) - c^2\,\mathrm{d}t^2. \tag{14}$$

Here, $R(t)$ is the cosmic scale factor, r, θ, and ϕ are spherical coordinate variables, t is the cosmic time, and k represents the spatial curvature. Let k as $\Lambda\left(\left(g^{\theta\theta}\right)^2\right)$.

The d'Alembertian operator is then given by:

$$\Box = -\frac{\partial^2}{\partial t^2} + \nabla^2, \tag{15}$$

where ∇^2 is the Laplacian operator. Substituting this into the Klein-Gordon equation and using the separation of variables method, we obtain the following solutions:

$$\phi(t,\vec{x}) = \int \frac{d^3k}{(2\pi)^{3/2}}\left[a(\vec{k})e^{-i\omega_k t + i\vec{k}\cdot\vec{x}} + a^\dagger(\vec{k})e^{i\omega_k t - i\vec{k}\cdot\vec{x}}\right], \tag{16}$$

where $\omega_k = \sqrt{k^2+m^2}$ is the energy of the field, and $a(\vec{k})$ and $a^\dagger(\vec{k})$ are the annihilation and creation operators respectively.

Next, we derive the Schrödinger equation from the Klein-Gordon equation. The Schrödinger equation is given by:

$$i\hbar\frac{\partial}{\partial t}\Psi(\vec{x},t) = H\Psi(\vec{x},t), \tag{17}$$

where $\Psi(\vec{x},t)$ is the wave function and H is the Hamiltonian. In order to derive the Schrödinger equation from the Klein-Gordon equation, we first write the field ϕ as:

$$\phi(t,\vec{x}) = e^{-imt/\hbar}\psi(\vec{x},t), \tag{18}$$

where $\psi(\vec{x},t)$ is the wave function. Substituting this into the Klein-Gordon equation and using the chain rule, we obtain the following equation for $\psi(\vec{x},t)$:

$$i\hbar\frac{\partial}{\partial t}\psi(\vec{x},t) = \left[\frac{-\hbar^2}{2m}\nabla^2 + V(\vec{x},t)\right]\psi(\vec{x},t), \tag{19}$$

where $V(\vec{x},t)$ is the potential energy of the field. This is the Schrödinger equation for a free particle in the presence of a time-dependent potential. In the cosmological inertial frame, the potential energy is given by the Yukawa potential:

$$V(\vec{x},t) = -\frac{g^2}{4\pi}\frac{e^{-m\sqrt{-g}(t-t_0)}}{\sqrt{-g}|\vec{x}|}, \tag{20}$$

where g is the coupling constant, t_0 is the time at which the potential is evaluated, and $\sqrt{-g}$ is the determinant of the metric.To study the Yukawa potential in an inertial frame in cosmology, we need to solve the Klein-Gordon equation. The Klein-Gordon equation describes spin-0 particles and is given by:

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 + \frac{m^2c^2}{\hbar^2}\right)\phi = 0. \tag{21}$$

Here, ϕ is the wave function, m is the mass of the particle, and $\hbar$ is the reduced Planck constant. The wave function ϕ is a complex scalar field that describes the properties of the particle.

The Schrödinger equation is a wave equation that describes the behavior of particles in quantum mechanics. It is given by:

$$i\hbar\frac{\partial}{\partial t}\psi = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi. \tag{22}$$

Here, ψ is the wave function, m is the mass of the particle, V is the potential energy, and $\hbar$ is the reduced Planck constant. The wave function ψ is a complex scalar field that describes the probability amplitude of the particle.

The Dirac equation is a relativistic wave equation that describes the behavior of particles with spin-1/2 in quantum mechanics. It is given by:

$$i\hbar\gamma^\mu\partial_\mu\psi - mc\psi = 0. \tag{23}$$

Here, ψ is the wave function, m is the mass of the particle, c is the speed of light, and γ^μ are the Dirac matrices. The wave function ψ is a four-component complex field that describes the probability amplitude of the particle.

In the cosmological inertial frame, the Dirac equation satisfies the Klein-Gordon equation. This means that the wave function ψ can be related to the wave function ϕ of the Klein-Gordon equation. This relationship can be used to derive the Yukawa potential in the cosmological inertial frame.

In the Klein-Gordon-Maxwell theory, we have discovered the equations for the complex scalar field and electromagnetic field from the interaction of the A-type wave function and B-type extended distance in the cosmological inertial frame. We have also quantized the Klein-Gordon scalar field in the cosmological inertial frame and derived the Lagrangian density and Hamiltonian for the quantized field.

4. CONCLUSION

In conclusion, in this paper, we have studied various equations and functions in the cosmological special relativity theory, including the Maxwell equations, electromagnetic wave equations, Klein-Gordon equation, wave function, Dirac equation, Yukawa potential, Schrödinger equation, and quantized Klein-Gordon scalar field. We have also discussed the forces and kinetic energy of particles in the cosmological special relativity theory. Our research will provide insights into the behavior of particles in the cosmological inertial frame and their interactions with electromagnetic fields. We have found a connection between the newly modified Friedman-Lemaître-Robertson-Walker metric and the Dirac equation, and the Klein-Gordon equation.

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