Factorial Theorem for Computation of Factorials to Positive Real Numbers

Chinnaraji Annamalai
School of Management, Indian Institute of Technology, Kharagpur, India
Email: anna@iitkgp.ac.in
https://orcid.org/0000-0002-0992-2584

Abstract: This paper presents the factorial theorem that is used to compute the factorial for positive real numbers, which are greater than or equal to 1. In this research study, the introduced factorial function is an alternative to the gamma function.

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1. Introduction
In general, the factorial [1-12] for positive integer n, denoted by n!, is the product of all positive integers less than or equal to n. For example, 5! = 1 × 2 × 3 × 4 × 5 = 720. Similarly, we can find a factorial function [13-16] to positive real numbers using factorials theorems, which are introduced in this article. The new factorial function is an alternative to the gamma function [17].

2. Factorial Theorem
Theorem 2.1: (x + 1)! = x! + {(x + 1)! - x!}, where x ≥ 0.
Proof:
x! + {(x + 1)! - x!} = x! + {x!(x + 1) - x!} = x! + x!(x) = x!(1 + x) = (x + 1)!, (x ≥ 0).
Hence, theorem is proved.

Let us verify the theorem with positive real numbers.
(x + 1)! = x! + {(x + 1)! - x!} = x! + (0.64)({(x + 1)! - x!}) + (0.36)({(x + 1)! - x!})
x! + (0.64 + 0.36)({(x + 1)! - x!}) = x! + {(x + 1)! - x!}.

Theorem 2.2: y! = (y + 1)! - {(y + 1)! - y!}, where y ≥ 1.
Proof:
(y + 1)! - {(y + 1)! - y!} = y!(y + 1)! - {y!(y + 1)! - y!}
= y!(y + 1)! - y!(y + 1 - y) = y!(y + 1 - y) = y!, (y ≥ 1).
Hence, theorem is proved.

Let us verify the theorem with positive real numbers.
y! = (y + 1)! - {(y + 1)! - y!}
y! = (y + 1)! - (1 - 0.64)({(y + 1)! - y!}) - (1 - 0.36)({(y + 1)! - y!})
y! = (y + 1)! - (0.36)({(y + 1)! - y!}) + (0.64)({(y + 1)! - y!})
y! = (y + 1)! - {(y + 1)! - y!}.

3. Analysis of Factorial Function
The value of factorial for any positive real number between 0! = 1 and 1! = 1 must be 1,
i.e. \( \frac{(0 + 1)}{2} = 0.5 \Rightarrow \frac{(0! + 1!)}{2} = 1 \). Thus, (0.5)! = 1.
Also, \( \frac{(0 + 0.5)}{2} = 0.25 \Rightarrow \frac{(0! + (0.5)!)}{2} = 1. \) Thus, \((0.25)! = 1; \)
\[ \frac{(0.5 + 1)}{2} = 0.75 \Rightarrow \frac{((0.5)! + 1!)}{2} = 1. \] Thus, \((0.75)! = 1; \) and so on.

Therefore, the numerical values of factorials for positive real numbers between the consecutive integers 0 and 1 must be 1.

Similarly, the numerical values of factorials for positive real numbers between the consecutive integers 4 and 5 must be the values between 4! and 51, i.e. the values of factorials for positive real numbers between 4 and 5 must be the values between 24 and 120 because of 4! = 24 and 5! = 120.

Let \( n \) be a positive integers, \( n! = r, \) and \((n + 1)! = s.\) Then, the numerical values of factorials for positive real numbers between the consecutive integers \( n \) and \((n + 1) \) must be the values between \( r \) and \( s.\)

4. Computation of Factorials for Positive Real numbers

Let \( z = i. f \) be a positive real number, where \( i \) is an integer part, \( f \) is the decimal or fractional part, and \( z \geq 1.\) The factorial function to positive real number is established from the perspective of the above theorems as follows:
\[ z! = (i. f)! = i! + (0. f)\{(i + 1)! - i!\}. \]

For examples,
\[ (1.0)! = 1! + (0.0)\{(1 + 1)! - i!\} = 1 + 0 = 1. \]
\[ (4.6)! = 4! + (0.6)(5! - 4!) = 24 + (0.6)(120 - 24) = 81.6. \]

The gamma function calculator \([18]\) computes the factorial function \((4.6)!\) as follows:
\[ \Gamma(5.6) = (5.6 - 1)! = (4.6)! = 61.5. \]

5. Factorials for Rational and Irrational numbers

If \( \frac{p}{q}, \( q \neq 0 \) = i. f \) is a common rational number, where \( \frac{p}{q} \geq 1, f < q, \) & \( i \) is an integer.
Then, \( \left(\frac{p}{q}\right)! = i! + \left(\frac{f}{q}\right)\{(i + 1)! - i!\}. \)

For examples,
\[ 1! = \left(\frac{1}{1}\right)! = 1! + \left(\frac{0}{1}\right)\{(1 + 1)! - 1!\} = 1 + 0 = 1. \]
\[ \left(\frac{14}{4}\right)! = \left(\frac{3}{4}\right)! = 3! + \left(\frac{2}{4}\right)(4! - 3!) = 6 + 9 = 15. \]

Let \( \sqrt{x} \) be a irrational number, where \( \sqrt{x} \geq 1. \) If \( i \) is the integer part on \( \sqrt{x}. \)
Then, \( \sqrt{x}! = i! + \sqrt{x}\{(i + 1)! - i!\} - i\{(i + 1)! - i!\}. \)

For examples,
\[ (\sqrt{1})! = 1! + \sqrt{1}(2! - 1! - 1(2! - 1!)) = 1 + 1 - 1 = 1. \]
\[ (\sqrt{15})! = 3! + \sqrt{15}(4! - 3!) - 3(4! - 3!). \]
\[ (119)! = 10! + \sqrt{119}(11! - 10!) - 10(11! - 10!). \]
6. Conclusion
In this article, factorial theorem and function have been introduced to compute the factorial of positive real number. The factorial theorem to positive real numbers is an alternative to the gamma function. These results can be used as an application in computing and mathematical sciences including probability and statistics.

References


