Abstract: Several professors of mathematics from the renowned universities in Australia, Canada, Europe, India, USA, etc. argue with me that the gamma function is not related to the factorial function. For them, this paper describes the derivation of gamma function from the factorial function.

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Euler’s Factorial Function and Gamma Function
In general, the factorial for positive integer n is the product of all positive integers less than or equal to n. Symbolically, n!. For example, 5!=1×2×3×4×5=720 and 0!=1.

The Pi (Π) function is given below:

\[ \Pi(x) = x! = x(x - 1)(x - 2) \cdots 1, \forall x \in W, \]  \hspace{1cm} (1)

where \( W \) denotes the system of whole numbers.

Euler’s factorial function, also known as Pi (Π) function, is the basis for gamma function [1-6].

Swiss mathematician Leonhard Euler was defined the pi function by integral as follows:

\[ \Pi(x) = \int_{0}^{1} (-\ln t)^n \, dt, \forall n \in W, \] \hspace{1cm} (2)

where \( \ln \) denotes the logarithm to the base of the mathematical constant \( e \).

By substituting \( t = e^{-x} \) in (3), we get

\[ \Pi(x) = \int_{0}^{\infty} x^n e^{-x} \, dx, \forall n \in W. \] \hspace{1cm} (3)

By integrating (3), we obtain

\[ \Pi(x) = \left[ -x^n e^{-x} \right]_0^\infty - \int_{0}^{\infty} -nx^{n-1} e^{-x} \, dx. \]

\[ = 0 - \int_{0}^{\infty} -nx^{n-1} e^{-x} \, dx. \]

Now, \( \Pi(x) = n \int_{0}^{\infty} x^{n-1} e^{-x} \, dx. \) \hspace{1cm} (4)

By substituting (3) in (4), we obtain

\[ \Pi(x) = n\Pi(x - 1). \] \hspace{1cm} (5)
The gamma function $\Gamma(x)$ is obtained as follows.

$$\Pi(x) = n\Pi(x - 1) \Rightarrow \Gamma(x + 1) = n\Gamma(x). \quad (6)$$

$$n\Gamma(x) = n \int_0^\infty x^{n-1} e^{-x} dx \Rightarrow \Gamma(x) = \int_0^\infty x^{n-1} e^{-x} dx. \quad (7)$$

The gamma function [1-6], therefore, is derived from the Euler’s factorial function that uses the actual factorial function.

**References**


