Novel Geometric Series for Application of Computational Science

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Abstract: Computational science is a rapidly growing multi-and inter-disciplinary area where science, engineering, information technology, management and its collaboration use advance computing capabilities to understand and solve the most complex real life problems. In this article, a new geometric series is constituted for application of computational science and engineering.

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1. Introduction

Geometric series played a vital role in differential and integral calculus at the earlier stage of development and still continues as an important part of the study in science, engineering, management and its applications. In this article, a new geometric series [1-6] is constructed for application [7] of computational science and engineering.

2. Novel Geometric Series

In today's world, the growing complexity of mathematical modelling demands the simplicity of mathematical equations for solving today's scientific problems. For this purpose, the following theorem [1-6] is introduced.

Theorem : Let R be a system of real numbers and $p \in R$.

$$\sum_{j=k}^{n} x^{pj} = \frac{x^{p(n+1)} - x^{pk}}{x^p - 1}, x^p \neq 1 \text{ and } x^p \neq 0^0 \ (i.e., x \neq 0 \text{ and } p \neq 0).$$

Proof. Let us prove this theorem by the equality given below:

$$y^{n+1} = y^{n+1} \Rightarrow y^{n+1} = (y-1)y^n + y^n \Rightarrow y^{n+1} = (y-1)y^n + (y-1)y^{n-1} + y^{n-1}.$$

If we continue this expansion again and again, we can obtain the following series.

$$y^{n+1} = (y-1)y^n + (y-1)y^{n-1} + (y-1)y^{n-2} + (y-1)y^{n-3} + \cdots + (y-1)y^k + y^k.$$

$$i.e., \quad y^{n+1} - y^k = (y-1)(y^n + y^{n-1} + y^{n-2} + y^{n-3} + \cdots + y^k). \quad (1)$$

From the expansion (1), we obtain the series $\sum_{j=k}^{n} y^j = \frac{y^{n+1} - y^k}{y-1}, y \neq 1.$ (2)

Now, let us prove the theorem shown below:

$$1 + x^p + x^{2p} + x^{3p} + \cdots + x^{kp} = \frac{x^{p(n+1)} - x^{p(k)}}{x^p - 1}, x^p \neq 1 \text{ and } x^p \neq 0^0.$$

Let y be x^p and substitute it in the series (2).

$$\text{Then,} \sum_{j=k}^{n}(x^p)^j = \frac{(x^p)^{n+1} - (x^p)^k}{x^p - 1}, x^p \neq 1 \text{ and } x^p \neq 0^0. \quad (3)$$

From the Equation (3), we conclude that

$$\sum_{j=k}^{n} x^{pj} = \frac{x^{p(n+1)} - x^{pk}}{x^p - 1}, x^p \neq 1 \text{ and } x^p \neq 0^0.$$

Hence, theorem is proved.

Note that the novel geometric series is: $\sum_{j=0}^{n} x^{pj} = \frac{x^{p(n+1)} - 1}{x^p - 1}, x^p \neq 1 \text{ and } x^p \neq 0^0.$

For example,

$1 + x^{0.25} + x^{0.5} + x^{0.75} + x^{1.0} + x^{1.25} + x^{1.5}.$

$= 1 + x^{1(0.25)} + x^{2(0.25)} + x^{3(0.25)} + x^{4(0.25)} + x^{5(0.25)} + x^{6(0.25)} = \frac{x^{0.25(6+1)} - 1}{x^{0.25} - 1}.$

$\therefore 1 + x^{0.25} + x^{0.5} + x^{0.75} + x^{1.0} + x^{1.25} + x^{1.5} = \frac{x^{1.75} - 1}{x^{0.25} - 1}.$

Also, $x^{0.5} + x^{0.75} + x^{1.0} + x^{1.25} + x^{1.5} = \frac{x^{1.75} - x^{0.5}}{x^{0.25} - 1}.$

Corollary 1: $\sum_{j=k}^{n}(ax)^{pj} = \frac{(ax)^{p(n+1)} - (ax)^{pk}}{(ax)^p - 1}, (ax)^p \neq 1 \text{ and } (ax)^p \neq 0^0.$

Corollary 2: $\sum_{j=0}^{n}(ax)^{pj} = \frac{(ax)^{p(n+1)} - 1}{(ax)^p - 1}, (ax)^p \neq 1 \text{ and } (ax)^p \neq 0^0.$

Corollary 3: $\sum_{j=-k}^{n}(ax)^{pj} = \frac{(ax)^{p(n+1)} - (ax)^{-pk}}{(ax)^p - 1}, (ax)^p \neq 1 \text{ and } (ax)^p \neq 0^0.$

Note that $(ax)^p \neq 0^0$ denotes $ax \neq 0$ and $p \neq 0$.

3. Conclusion

Nowadays, the growing complexity of mathematical and computational modelling demands the simplicity of mathematical and combinatorial equations for solving today's scientific problems and challenges. This article provides a novel geometric series and its theorem. This idea can enable the scientific researchers for further involvement in research and development.

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