Electrostatic quantum dark energy in a seven-dimensional universe

Sasidhar Kesanasetty
kesanasetty@gmail.com

Abstract:

This paper explains the dark energy and acceleration of the universe by quantizing the space in hidden dimensions, which provides the basis and background for the gravitational force through the curvature of space-time. Space-time is considered to be made of a four-dimensional elastic grid in a seven-dimensional universe in which matter also expands along with the universe. Each cube of the grid is considered a quantum of hidden three-dimensional space of Planck volume containing Planck charge, which makes the universe seven-dimensional. The dark energy is explained by the electrostatic repulsion between the Planck charges in each quantum of the hidden space. Mathematically, this electrostatic repulsion is related to the Hubble constant to explain the accelerated expansion, dark energy, and increase in the cosmological potential energy of matter. Expansion of space-time is considered not due to the creation of the new space but due to the stretching of the existing space-time itself like an elastic ruler where the proper length remains constant. As the Hubble constant decreases with time, the rate of acceleration of the universe is considered to be decreasing because of the contraction force of the space-time elastic grid opposing the electrostatic repulsion between Planck charges. The apparent violation of the law of energy conservation in the cosmological redshift is properly explained to show that the cosmological potential energy of matter continues to increase due to the stretching of space-time and hence the redshift without violating the law of energy conservation. The values of the Planck constant, gravitational constant, permittivity of free space, and Boltzmann constant are shown to vary owing to the expansion of space-time and hence provide falsifiable predictions for this theory. This theory builds a framework for the relativistic Newtonian theory of gravity, the relativistic MONDian (Modified Newtonian dynamics) gravity, and identifies a valid reason for the transition of Newtonian gravity to MOND at a_0. Overall, this theory proposes a hidden extradimensional electrostatic background to gravity to explain the curvature of space-time, dark energy, and cosmological redshift and eliminates cosmic inflation, the cosmic event horizon, the cosmic scale factor (a), the critical density (Ω), the cosmological constant (Λ) and singularities.

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1. Introduction

In this theory of electrostatic quantum dark energy in a seven-dimensional universe, Hubble expansion is considered to be due to the stretching of the existing space-time rather than the creation of a new space that is analogous to the markers on an elastic ruler stretching along with the expansion of the ruler, as opposed to the raisin bread model, where matter does not expand along with space. Therefore, the matter is considered to stretch along with the space-time or not become diluted with the expansion of the space-time. As the matter also stretches with space, the expansion is nonobservable locally but can be observed through the redshift of the light coming from a far-off space of a lower stretch. At the outset, we can see that matter exists in different energy states based on the magnitude of the space-time stretch. The matter continues to move to higher energy states as space-time expands, but the energy of the photon remains the same. Therefore, in a lower stretch space-time, blue light has less energy compared to the same blue light in a higher stretch space-time. Therefore, when light travels from a lower stretch of space-time to a higher stretch of space-time, it becomes redshifted without violating the law of energy conservation. From this observation, we can conclude that the energy generated by converting matter based on the mass–energy equivalence formula $E=mc^2$ is time-variant, as the potential energy of the mass increases with the space-time expansion, whereas the energy of the photon does not change with the space-time expansion, and it does not gravitate, as it does not resist the space-time expansion because the mass of the photon is zero but is affected by the gravity of the other objects, which means that the energy of a photon does not curve the space-time around it but takes the path of the curved space around any matter. Thus, electromagnetic fields and electromagnetic-related potential energies whose forces are meditated by virtual photons do not curve the space-time in this model of the universe.

This theory proposes an absolute inertial frame, a way to identify it, and hence revives the concept of relativistic mass. In this theory, only the rest/relativistic mass or the mass density is considered to be responsible for gravitational force (gravitoelectric) and the relativistic mass current density (mass flux) for the gravitomagnetic force. Any other form of energy increases neither the inertial mass nor the gravitational mass in this model of the universe but only the rest/relativistic mass and the relativistic mass flux. Therefore, this theory only partially satisfies the mass–energy equivalence principle but upholds the weak equivalence principle.

The proper distance between any two points in space is considered to remain constant despite the stretching of space-time, similar to the markers on an elastic ruler. Therefore, the volume of the universe does not change with the expansion of space. The expansion of space-time and matter is considered to be due to electrostatic repulsion between the Planck charges present in the Planck volumes and hence discards dark energy. Space-time is considered to be made of a four-dimensional elastic grid in a seven-dimensional universe. Each cube of the grid is considered to be a quantum of three-dimensional space, equivalent to the Planck volume and containing Planck charge. These cubes can become loosely connected to the surrounding cubes for the formation of black holes. The space-time membrane acts as a dielectric material between the Planck charges. Matter only exists as a probability wave function (Ψ) in the four-dimensional space-time grid but not in the hidden three spatial dimensions. The presence of matter in space-time would increase the force required to expand the space-time grid as the matter also expands along with space. The presence of matter increases the permittivity of space-time and hence reduces the electrostatic repulsion within the grid enveloped by the matter compared to the electrostatic repulsion outside the matter.

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As the expansion of the space-time surrounding the matter will be greater than the expansion of the matter due to the permittivity difference and the net compressing force on the matter from the surrounding Planck charges, space-time becomes naturally curved around any mass and hence the gravitational force. Therefore, in this theory, except for the photons/electromagnetic fields, the gravitational force is considered a real force rather than a fictitious force, as in the general theory of relativity, due to the curvature of space-time. In this theory, the electrostatic potential energy between the Planck charges in the three-dimensional space is considered to be the same as the dark energy, which causes accelerated expansion of the universe. The electrostatic dark energy in three-dimensional space does not gravitate because this energy itself is the cause of the gravitational force in the four-dimensional space-time enveloping the three-dimensional Planck charges. The constancy of the speed of light should not limit the apparent velocity of the universe in this model, as it considers it to be applicable only for objects moving through space but not for the expansion of space-time. Beginning from the birth of the universe, the first law of thermodynamics is strictly followed in this model of the universe to uphold the law of conservation of energy, which includes energy conservation in dark energy and cosmological redshift.

This theory proposes a flat or zero-curvature, isotropic, and homogenous universe, where the rate of acceleration will continue to decrease proportionally to the age of the universe, and its velocity only becomes zero after an infinite amount of time. However, the universe will continue to accelerate, but only at a continuously decreasing rate that asymptotes to zero. Invoking the critical density ($\Omega_0 = 1$) or cosmic inflation is not required to explain the flatness in this model of the universe, as the uniform expansion of the whole universe due to electrostatic repulsion explains why the universe is flat rather than closed or open. As there is no increase in the volume of the universe with time, the big bang should be replaced with big repulsion. This theory makes the gravity electrostatic background dependent to explain the curvature of space-time, dark energy, and cosmological redshift.

The electrostatic background provides an additional background to the quantum fields on top of the gravitational background, which mandates an absolute inertial frame and universal time, which is Hubble time in this model of the universe. Therefore, this theory makes the universe three-layered.

1) Electrostatic or the power layer
2) Space-time or the gravitational layer
3) Quantum fields layer

The first layer acts as a power or energy source of the universe. The gravitational layer creates gravity, and the quantum field layer enables matter, energy, and the rest of the fundamental forces of nature to work on top of the space-time or the gravitational layer.

This theory also eliminates the cosmic event horizon and the cosmic scale factor (a) because the proper length and volume remain constant despite the expansion of space-time. Therefore, we should be able to see light from any part of the universe without any distance limit, such as a cosmic event horizon. The cosmological constant Λ in the gravitational field equations to factor in the dark energy is not required in this model of the universe, as dark energy does not gravitate and is isolated to the electrostatic background, which is handled independently from gravity.

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The values of the Planck constant, gravitational constant, Boltzmann constant, and permittivity of free space are shown to vary owing to the expansion of the space-time grid, proving the existence of an absolute frame of reference and hence providing falsifiable predictions for this theory. The aforementioned constants are also shown to vary with the gravitational potential and in all the moving inertial frames and hence enable us to identify the frames through the change in values of the constants compared to the absolute inertial frame or an inertial frame of a different velocity. An absolute inertial frame is one where the values of the constants are minimum or maximum depending on how they vary with the gravitational potential or the velocity of the inertial frame.
Therefore, this theory completely resolves the twin paradox: the person in the frame with the change in physical constants due to velocity will age less than the one in the frame with no change in the physical constants after they meet. However, this theory upholds the invariance of the speed of light in all frames (internal and noninertial) and hence upholds the special theory of relativity.

In this theory, the constants that do not change with the expansion of the universe or the change in gravitational potential or the velocity of the frame are listed below but not limited to.

1) Speed of light
2) Planck length
3) Planck time
4) Planck temperature
5) Electric charge
6) Fine-structure constant (α)
7) Rest mass

Black hole singularities do not exist in this model of the universe as the mass is converted to pure informational entropy at the event horizon and the space-time terminates at the event horizon and cannot be extended beyond the event horizon as in the general theory of relativity as the effective radial length, time and the tangible mass becomes zero at the event horizon and hence discards the Riemannian geometry of space-time. Additionally, charged black holes do not exist in this model of the universe, as the permittivity of free space becomes infinity at the event horizon. Therefore, stationary black holes can only have two properties, namely, informational (entropic) mass and angular momentum, and hence only partially satisfy the no-hair theorem.

This theory also builds a framework for the covariant/relativistic Newtonian theory of gravity. Relativistic Newtonian gravity proposed in this theory produces similar or even better results in closed form than the general theory of relativity without using weak field approximations for the below listed.

1) Black hole radius (Schwarzschild radius)
2) Photon sphere
3) Gravitational lensing
4) Perihelion precession of Mercury
5) Shapiro time delay

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This theory generates the relativistic Poisson equations and Maxwell-like equations for gravity or GEM (Gravitoelectromagnetism) equations to make them work in strong fields as well, which can explain the frame-dragging effect, orbital precession, geodetic effect, and gravitational waves. This theory also generates the equivalent of the Schwarzschild and FLRW metrics.

The absolute reference frame and the type of the expansion of the universe proposed in this theory identify a valid reason for the transition of Newtonian gravity to MOND at a_0 (~1.2×10^{-10} m/s^2) and hence make a case for the absolute reference frame. However, this theory establishes that transitioning to a deep-MOND regime is only possible in the radial space around the black holes but not around the ordinary matter and hence explains the missing gravitational lensing around the gaseous part of the Bullet cluster (1E 0657-56). This theory also builds a framework for relativistic MONDian gravity.

In addition, this new model could act as a precursor to theories explaining baryogenesis and primordial nucleosynthesis based on how the initial extremely high expansion energy of space-time interacted with the quantum fields to create matter. However, it still needs to be seen if the high value of the gravitational constant G proposed in this theory during the emission of CMB (cosmic microwave background) accounts for the observed anisotropy and the angular power spectrum without the dark matter, as high G should produce the same gravitational effect as the equivalent high mass. Additionally, this theory explains the early formation of the galaxies/objects that were recently observed through the JWST (James Webb space telescope) by proving that they were formed much later than calculated.

2. Relativistic acceleration of space-time

Let us consider two points, A and B, on the space-time fabric. We can calculate the apparent outward acceleration of point B when observed from point A based on the redshift of the light coming from point B.

D = Proper distance between points A and B

λ = D (let us consider the wavelength of light to be equal to D)

Thus, by the time light travels from point B to point A, its wavelength would have expanded by D(1+z) based on the cosmological redshift phenomenon. Therefore, point B would have apparently moved from its original location by D(1+z)–D, which is equal to Dz.

The apparent velocity v of point B due to the redshift is given by $v = \frac{Distance}{time} = \frac{Dz}{t}$, where t is the time taken for the light to travel from point B to point A. As the proper distance between points A and B remains constant, the real velocity of point B is zero. Thus, apparent acceleration a is given by $a = \frac{Dz}{t^2}$. As space stretches like an elastic ruler, the proper distance between points A and B should always remain the same, irrespective of the apparent acceleration. Therefore, the light should take the same amount of time t to travel the apparent distance of D(1+z), which is actually D owing to the constancy of the speed of light.

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Therefore, time t is given by $t = D/c$

So, $v = \dfrac{Dz}{\left(\dfrac{D}{c}\right)} = z\text{c}$

$$\boxed{v = zc} \qquad \textbf{(1)}$$

$$and \quad a = \frac{Dz}{\left(\dfrac{D}{c}\right)^2} = \frac{zc^2}{D}$$

As $v = zc$ has already been established in conjunction with the Hubble law, the above derivation proves that space is stretching like an elastic ruler, where the length and volume remain constant, as opposed to the raisin bread model, where length increases and the matter is diluted with the expansion of space-time. Therefore, this theory discards the raisin-bread model of the universe and provides a theoretical basis for $v = zc$ (1) whereas the raisin-bread model does not provide any theoretical reasoning.

Based on the equations $v = H_0 D$ *and* $v = zc$ from Hubble's law, $\dfrac{z}{D} = \dfrac{H_0}{c}$, where v is the apparent receding velocity, H_0 is the bubble's constant, D is the distance, z is the redshift, and c is the speed of light.

Therefore, the apparent acceleration $a = \dfrac{H_0 c^2}{c} = cH_0 =$ **7.549 × 10^-10^ m/s²**, which is the current acceleration of the universe for $H_0 = 77.7\left(km/s\right)/Mpc$ based on the H_0 values within the range mentioned in the references (Chen et al., 2019; de Jaeger et al., 2020; Tully et al., 2016). As the universe is stretching with constant volume, there must be length contraction and time dilation in the past, which are given below.

Cosmological length contraction: $L = \dfrac{L_0}{(1+z)}$

Cosmological time dilation: $T = \dfrac{T_0}{(1+z)}$

Therefore, the relativistic acceleration of the universe is as follows.

$$\boxed{a = cH_0(1+z)} \qquad \textbf{(2)}$$

Therefore, point B will always move from point A with an apparent acceleration equal to the above, although the proper distance between the two points always remains the same. As Hubble's constant decreases over time, the rate of apparent acceleration of the universe or space-time is also considered to decrease over time. As the proper distance between the two points and the volume of the universe always remains constant, acceleration $a = cH_0(1+z)$ is only considered apparent.

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3. Relativistic Hubble law and the cosmological redshift

Based on the relativistic acceleration of space-time $a = cH_0(1+z)$, we can calculate the relativistic velocity v of point B from big repulsion (point A) on the space-time fabric based on the cosmological redshift z.

$$\frac{dv}{dt} = a; \quad dv = a\,dt; \; dv = cH_0(1+z)dt;$$
$$\int_0^v dv = \int_{t_p}^{t} cH_0(1+z)dt$$
$$\int_0^v dv = c\int_{t_p}^{T_0} \frac{1}{T}(1+\frac{v}{c})dT \quad \text{from (1)}$$
$$\int_0^v \frac{1}{(c+v)} dv = \int_{t_p}^{T_0} \frac{1}{T} dT$$
$$\ln(c+v) - \ln(c) = \ln(T_0) - \ln\left(t_p\right)$$

Planck time t_p is the minimum age of the universe at the beginning of the big repulsion owing to the quantization of space and time to Planck units. T is the age of the universe, which is considered to be Hubble time in this model of the universe.

$$\frac{c+v}{c} = \frac{T_0}{t_p} \;\; ; \;\; 1+\frac{v}{c} = \frac{1}{H_0 t_p} \;\; ; \; 1+z = \frac{1}{H_0 t_p}$$

Therefore, the maximum possible cosmological redshift is below.

$$\boxed{z = \frac{1}{H_0 t_p} - 1} \tag{3}$$

which is 7.36614 × 10^{60} for $H_0 = 77.7\left(km/s\right)/Mpc$, and the maximum apparent relativistic recession velocity that is possible is 7.36614 × 10^{60} times the speed of light.

$$\text{For } H_0 < H_D \quad \boxed{1+z = \frac{H_D}{H_0} = \frac{1}{H_0\left(\dfrac{1}{H_0} - \dfrac{D}{c}\right)} = \frac{1}{1-\dfrac{H_0 D}{c}} = \frac{c}{c - H_0 D}} \tag{4}$$

where $\frac{1}{H_d} \geq t_p$ and H_D is Hubble's constant when light is emitted at distance D.

Therefore, the relativistic Hubble law is below based on (1) and (4).

$$\boxed{v = H_0 D(1+z) = \frac{H_0 D}{(1-\dfrac{H_0 D}{c})}} \tag{5}$$

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Acceleration at the beginning of the universe (big repulsion) is below.

$$a = cH_0(1+z) = cH_0 \frac{1}{H_0 t_p} = \frac{c}{t_p}$$ = **5.56 × 10^51 m/s^2** from (2) and (3)

The acceleration of the universe at distance D from the present is below.

$$a = cH_0(1+z) = cH_0\left(\frac{c}{c - H_0 D}\right)$$ from (2) and (4)

$$\boxed{a = cH_0\left(\frac{c}{c - H_0 D}\right)} \quad \textbf{(6)}$$

We can see in the table below that the redshift (z) values match the regular Hubble's formula and the new relativistic formula for low values of D. The new formula restricts the maximum cosmological redshift to 7.36614×10^{60} owing to the model of the universe that is considered. As shown in the table below, the z values differ from each other as D increases or as the time traveled by the light approaches the age of the universe. Therefore, Hubble's law $v = H_0 D$ is only accurate up to moderate distances, as it is the limiting case of the relativistic Hubble law. We can also see that the new z values in Table I are in line with the accelerating model of the universe.

Table I (Cosmological redshift values)

D (meters)	$z = \frac{H_0 D}{c}$	$z = \frac{H_0 D}{c - H_0 D}$
1×10^{10}	$8.3994291420\times10^{-17}$	$8.3994291420\times10^{-17}$
2×10^{15}	$1.6798858284\times10^{-11}$	$1.6798858284\times10^{-11}$
3×10^{20}	$2.5198287426\times10^{-6}$	$2.5198350921\times10^{-6}$
$\sim5.9527854\times10^{25}$	0.5	1
1×10^{26}	0.8399429142	5.2477708815
1.19×10^{26}	0.9995320679	2136.0622240085
$\sim1.1905570\times10^{26}$	1	$7.3661442125\times10^{60}$ (Maximum/Big repulsion)

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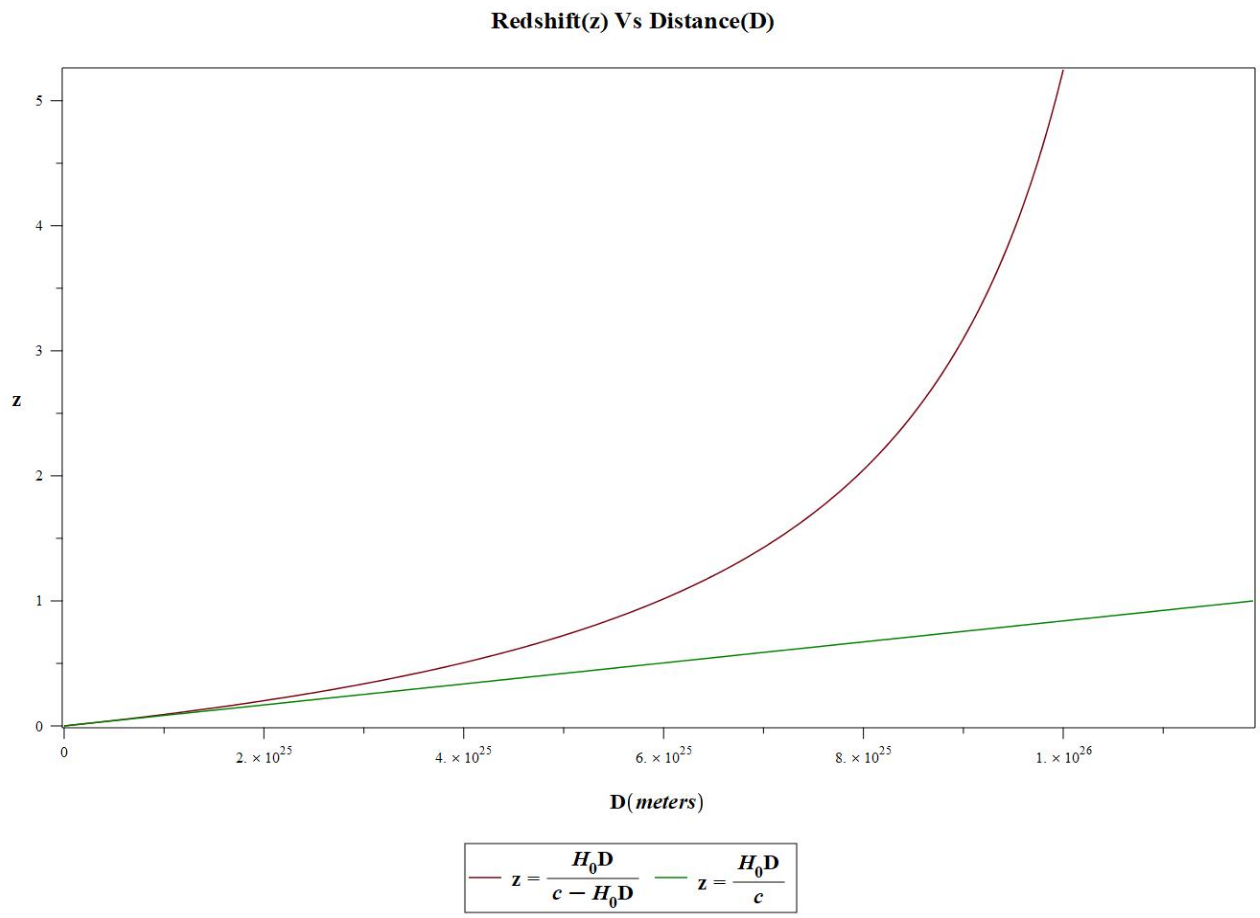

Graph 1

The age of the universe is 12.58 billion years, which is the Hubble time. As the proper distance and the volume remain constant, the maximum observable universe is only 12.58 × 2 billion light years across, which is 25.16 billion light years for $H_0 = 77.7(km/s)/Mpc$.

Important formulas: based on (4)

Time since the big repulsion to the emission of light (T_b)	Time since the emission of light (T_z)	Distance vs Redshift
$T_b = \frac{1}{H_0(1+z)}$	$T_z = \frac{1}{H_0}\frac{z}{(1+z)}$	$D = \frac{zc}{H_0(1+z)}$

Therefore, the light from galaxy HD1 with cosmological redshift z=13.27 (Zhe et al., 2022) should have been emitted after 0.8 billion years since the big repulsion, possibly giving enough time for it to form as a galaxy by also factoring in the high initial gravitational constant G, as mentioned in section 4 of this document, and hence resolving the issue of the early formation of galaxies/objects that were recently observed through the JWST.

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4. Variable constants G, h, ε0, and kB and the mass increase

As the speed of light is constant, the Planck length and Planck time are considered constants. Therefore, the product of the gravitational constant G and Planck constant h is considered to be constant. As the Planck charge $(q_p = \frac{e}{\sqrt{\alpha}})$ is conserved, alpha (α) or the fine-structure constant is considered to be constant. Therefore, the product of the Planck constant h and permittivity of free space ε_0 is considered to be constant. As the Planck temperature is considered to be constant, the product of the gravitational constant G and Boltzmann constant k_B is considered to be constant. Based on the above, we can calculate how constants G, h, ε_0, and k_B change over time. Using the law of conservation of energy, we can calculate the values of G, h, ε_0, and k_B in the past and the future.

Cosmological redshift $z = \frac{\lambda_{obs} - \lambda_{emit}}{\lambda_{emit}}$ or $\lambda_{obs} = \lambda_{emit}(1+z)$ **(7)**

As the energy is conserved in cosmological redshift, $E = \frac{hc}{\lambda_{obs}} = \frac{h_{t_p} c}{\lambda_{emit}}$ **(8)**

Here, h is the current Planck constant, and h_{tp} is the old Planck constant when the age of the universe is Planck's time t_p.

$z = \frac{1}{H_0 t_p} - 1$ from (3), and $h = h_{t_p}(1+z)$ based on (7) and (8) **(9)**

As $hG = h_{t_p} G_{t_p}$, $h\varepsilon_0 = h_{t_p} \varepsilon_{t_p}$ and $k_B G = k_{Bt_p} G_{t_p}$

$$\boxed{h = h_{t_p}(1+z) = h_{t_p}(\frac{1}{H_0 t_p})} \quad \textbf{(10)}$$

$$\boxed{G = \frac{G_{t_p}}{(1+z)} = G_{t_p}(H_0 t_p)} \quad \textbf{(11)}$$

$$\boxed{\varepsilon_0 = \frac{\varepsilon_{t_p}}{(1+z)} = \varepsilon_{t_p}(H_0 t_p)} \quad \textbf{(12)}$$

$$\boxed{k_B = k_{B_{t_p}}(1+z) = k_{B_{t_p}}(\frac{1}{H_0 t_p})} \quad \textbf{(13)}$$

Below are the values of G_{tp}, h_{tp}, ε_{tp}, and k_{Btp} in MKS units when the age of the universe was Planck time t_p, based on the current values of G, h, and ε_0 for H_0 = 77.7. Therefore, these constants vary proportionally to the relativistic Hubble flow.

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$h = 6.62607015 \times 10^{-34}$	$G = 6.67430 \times 10^{-11}$	$\varepsilon_0 = 8.8541878128 \times 10^{-12}$	$k_B = 1.380649 \times 10^{-23}$
$h_{t_p} = 8.99530332 \times 10^{-95}$	$G_{t_p} = 4.91638563 \times 10^{50}$	$\varepsilon_{t_p} = 6.52212243 \times 10^{49}$	$k_{B_{t_p}} = 1.87431709 \times 10^{-84}$

As the factor (1+z) in all the above four constants changes directly in proportion to mass (M) in the dimensional formulas, it follows that $m_0 = m_z(1+z)$, where m_0 is the current mass and m_z is the original mass when the light was emitted in the past at cosmological redshift z. However, the mass increase due to the expansion of space-time should be only seen as an increase in cosmological potential energy U_z of the mass but not the change in rest mass. The rest mass remains constant. This is analogous to the relativistic mass of an object moving with some velocity with constant rest mass.

$$\boxed{|U_z| = m_z z c^2}$$

Changes in mass can be observed by converting m_0 and m_z to energy, which is the observed energy difference in the cosmological redshift. Therefore, the energy of a photon does not increase with the space-time expansion, whereas the potential energy of a mass increases with the expansion, which perfectly explains the observed energy difference in the cosmological redshift. Similarly, for gravitational redshift, the change in mass is manifested as the gravitational potential energy (U). The rest mass remains constant. Similar to cosmological redshift, the change in mass in gravitational redshift is also associated with the change in physical constants G, h, ε_0, and k_B. Therefore, the energy of a photon does not change when moving against gravity, but the gravitational redshift, which is the decrease in the frequency of the photon, is due to the increase in the Planck constant (E=hν). This proves that photons do not curve space-time by themselves but take the path of any curved space.

Therefore, the gravitational potential energy U of mass m is as follows:

$U = -(m(1+z) - m)c^2 = -mzc^2$, where m(1+z) is the mass at infinity **(14)**

$$\boxed{U = -mzc^2} \qquad \textbf{(15)}$$

$$\boxed{\frac{U}{m} = \phi = -zc^2} \qquad \textbf{(16)}$$

where z is the gravitational redshift and ϕ is the gravitational potential.

The change in mass with velocity, which is the relativistic mass, is also associated with changes in the physical constants G, h, ε_0, and k_B. A person moving along with the mass will not observe the change in mass but observes the change in the above physical constants.

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However, a stationary observer with respect to the moving mass will observe the increase in mass with no change in the physical constants. Therefore, both observers will see the same thing in two different ways, enabling the moving observer to know that the inertial frame is moving, although the laws of physics are the same in both frames.

For example, in this theory, a mass M moving with enough relativistic velocity to satisfy the Schwarzschild radius R can become a black hole. A person moving along with the mass will see it becoming a black hole too due to the increase in gravitational constant instead of relativistic mass as the mass remains constant in the moving inertial frame. Therefore, both stationary and moving observers will see the mass M becoming a black hole in two different ways.

From the stationary observer's perspective: $R = \frac{2G(M\gamma)}{c^2}$

From the moving observer's perspective: $R = \frac{2(G\gamma)M}{c^2}$, where γ is the Lorentz factor.

Table II (Change in physical constants due to change in gravitational potential and velocity) **(17)**

Gravitational (from infinity)	$G(1+z)$	$\frac{h}{(1+z)}$	$\varepsilon_0(1+z)$	$\frac{k_B}{(1+z)}$
Relativistic	$G(1+z)$ *or* $G\gamma$	$\frac{h}{(1+z)}$ *or* $\frac{h}{\gamma}$	$\varepsilon_0(1+z)$ *or* $\varepsilon_0\gamma$	$\frac{k_B}{(1+z)}$ *or* $\frac{k_B}{\gamma}$

where z is the gravitational and relativistic redshifts.

For example, the Planck constant due to the cosmological redshift after 10 years from now would be $h(1+\delta z)$ from (10). where δz is the change in the cosmological redshift value after 10 years from now. $(1+\delta z) = \frac{H_0}{H_1}$ using (4). where H_1 is the Hubble constant after 10 years from now.

δz values:

After 10 years: $7.9463103517 \times 10^{-10}$
After 50 years: $3.9731551758 \times 10^{-9}$
After 100 years: $7.9463103517 \times 10^{-9}$

The accuracy of the values given below in Table III depends on the accuracy of the current values of h, G, ε_0, k_B, and H_0.

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Table III (Change in physical constants due to cosmological/space-time expansion) **(18)**

Years	$\frac{G}{(1+\delta z)}$	$h(1+\delta z)$	$\frac{\varepsilon_0}{(1+\delta z)}$	$k_B(1+\delta z)$
0	6.67430×10^{-11}	6.62607015×10^{-34}	$8.8541878128\times10^{-12}$	1.380649×10^{-23}
+10	6.67429999×10^{-11}	6.62607015×10^{-34}	8.85418780×10^{-12}	1.38064900×10^{-23}
+50	6.67429997×10^{-11}	6.62607017×10^{-34}	8.85418777×10^{-12}	1.38064900×10^{-23}
+100	6.67429994×10^{-11}	6.62607020×10^{-34}	8.85418774×10^{-12}	1.38064901×10^{-23}

Therefore, this theory provides falsifiable predictions by predicting the change in Planck's constant, the gravitational constant, the permittivity of free space, and the Boltzmann constant due to space-time expansion. Similar changes in the values of the above physical constants can be observed and calculated in the gravitational field and the relativistic frames using the factor (1+z), where z is the gravitational redshift and the relativistic redshift, respectively.

However, changes in the gravitational constant G in the distant past cannot be observed using the orbital frequencies of objects such as pulsars, as the change in orbital frequency would be compensated by the cosmological time dilation. As the velocity $v=\sqrt{\frac{Gm}{r}}$ of any object in a circular orbit remains constant due to the conservation of kinetic energy and gravitational potential energy, the radius r of the orbit shrinks to $\frac{r}{(1+\delta z)}$ as the universe expands to compensate for the decrease in the value of G to $\frac{G}{(1+\delta z)}$, where δz is the delta cosmological redshift. Therefore, the orbital period would decrease by the same factor, which would exactly be compensated by the delta cosmological time dilation, and hence, the change in G in the distant past cannot be observed. However, as the decrease in the value of G is the equivalent of applying a negative external torque or increase in mass, angular momentum is not conserved. For a circular orbit, angular momentum $mvr \neq mv\frac{r}{(1+\delta z)}$. However, when the increase in the cosmological potential energy or the increase in the mass is factored in, angular momentum is conserved with the expansion of the universe.

$$mvr = m(1+\delta z)v\frac{r}{(1+\delta z)}$$

In the table below, we can see the similarity between the transverse relativistic mass increase, the gravitational mass increase, and the cosmological mass increase. In all three cases, mass increases by a factor of (1+z), proving that there is a change in mass associated with the redshifts.

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Here, m_0 is the observed mass and m_z is the original mass at the point of the emission of the photon. The increase in cosmological and gravitational mass should be understood as an increase in their respective potential energies rather than an increase in their rest masses, which always remains constant.

Table IV (Mass increase formulas)

Transverse relativistic mass increase	Gravitational mass increase	Cosmological mass increase
$m_0 = m_z(1+z) = m_z \dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}}$	$m_0 = m_z(1+z) = m_z \dfrac{1}{\sqrt{1-\dfrac{2GM}{Rc^2}}}$	$m_0 = m_z(1+z) = m_z \dfrac{1}{1-\dfrac{H_0 D}{c}}$

5. Acceleration of the universe due to electrostatic repulsion

In this theory of electrostatic quantum dark energy in a seven-dimensional universe, the expansion of space-time is due to electrostatic repulsion between the Planck charges in the Planck volumes of the seven-dimensional space.

Acceleration at the beginning of the big repulsion is $a = cH_0(1+z) = \dfrac{c}{t_p}$ from (2) and (3), which can be reformulated as $\sqrt{\rho_{t_p} G_{t_p}}$, where ρ_{tp} is the Planck energy density and G_{tp} is the gravitational constant when the age of the universe is t_p. As acceleration a is directly related to the Planck energy density ρ, it proves the proposed model of the universe having Planck charges in Planck volumes, which exactly gives the Planck energy density ρ_{tp} in the hidden three-dimensional space of the seven-dimensional universe.

Therefore, $\dfrac{c}{t_p} = \sqrt{\rho_{t_p} G_{t_p}}$, which can be generalized as follows.

$$\boxed{a^2 = \rho G} \qquad \textbf{(19)}$$

Therefore, the product of the net electrostatic energy density of the universe in the hidden three dimensions and the gravitational constant is equal to the square of the acceleration of the universe.

ρ = Electrostatic energy density of the universe responsible for the acceleration
G = Gravitational constant
a = Current acceleration of the universe, which is cH_0

Planck energy $E_p = \dfrac{1}{4\pi\varepsilon_0}\dfrac{e^2}{\alpha(l_p)} = \sqrt{\dfrac{\hbar c^5}{G}}$, where l_p is the Planck length.

When the age of the universe is Planck time t_p, the energy density of the hidden three-dimensional space ρ_{t_p} of the universe is $\dfrac{E_{p(t_p)}}{(l_p)^3}$.

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$$\rho_{t_p} = \frac{1}{4\pi\varepsilon_{t_p}} \frac{e^2}{\alpha(l_p)^4} = \frac{\sqrt{\frac{\hbar_{t_p} c^5}{G_{t_p}}}}{(l_p)^3} \tag{20}$$

$$\rho_{t_p} G_{t_p} = \frac{1}{4\pi\varepsilon_{t_p}} \frac{e^2}{\alpha(l_p)^4} G_{t_p} = \left(\frac{\mathrm{c}}{t_p}\right)^2 = a^2$$

$$G_{t_p} = \frac{G}{H_0 t_p} \text{ from (11)}$$

$$\rho_{t_p} G_{t_p} = \frac{\rho_{t_p} G}{H_0 t_p} = \left(\frac{c}{t_p}\right)^2$$

$$\boxed{\rho_{t_p} = \frac{H_0 c^2}{G t_p}} \tag{21}$$

Therefore, ρ_{tp} = **6.29 × 10^52 J/m^3**, which can be generalized below.

$$\rho G(1+z) = (cH_0(1+z))^2$$

$$\rho = \frac{(cH_0)^2(1+z)}{G} = \frac{(cH_0)^2(\frac{c}{c - H_0 D})}{G} \text{ from (4)}$$

The current electrostatic energy density ρ responsible for the acceleration of the universe can be found by setting D = 0 or z = 0.

$\rho = \rho_{t_p} H_0 t_p = \frac{\rho_{t_p}}{1+z} = \frac{(cH_0)^2}{G}$ = **8.54 × 10^-9 J/m^3**, which resolves to (19).

As the universe accelerates due to electrostatic repulsion, the electrostatic potential energy in the hidden three-dimensional space is gradually transferred to the four-dimensional space-time grid and stored as potential energy. As the elastic space-time grid resists expansion, the rate of acceleration gradually decreases to follow $a = cH_0$, which will asymptote to zero. As the total energy is conserved, the potential energy density of the four-dimensional space-time grid ρ_g is below.

$$\boxed{|\rho_g| = \rho_{t_p} - \rho} \tag{22}$$

As ρ decreases with time and ρ_{tp} is constant, the potential energy density of the four-dimensional space-time grid ρ_g will continue to increase until infinity owing to the declining permittivity of free space ε_0.

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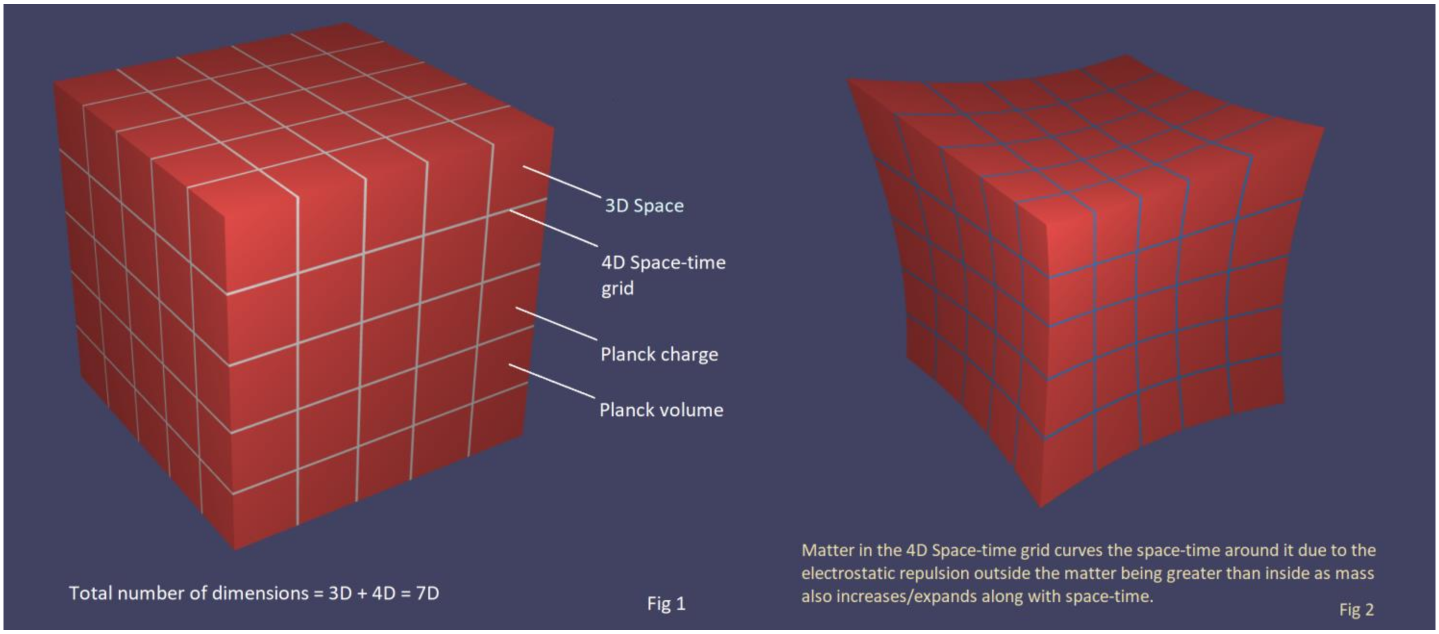

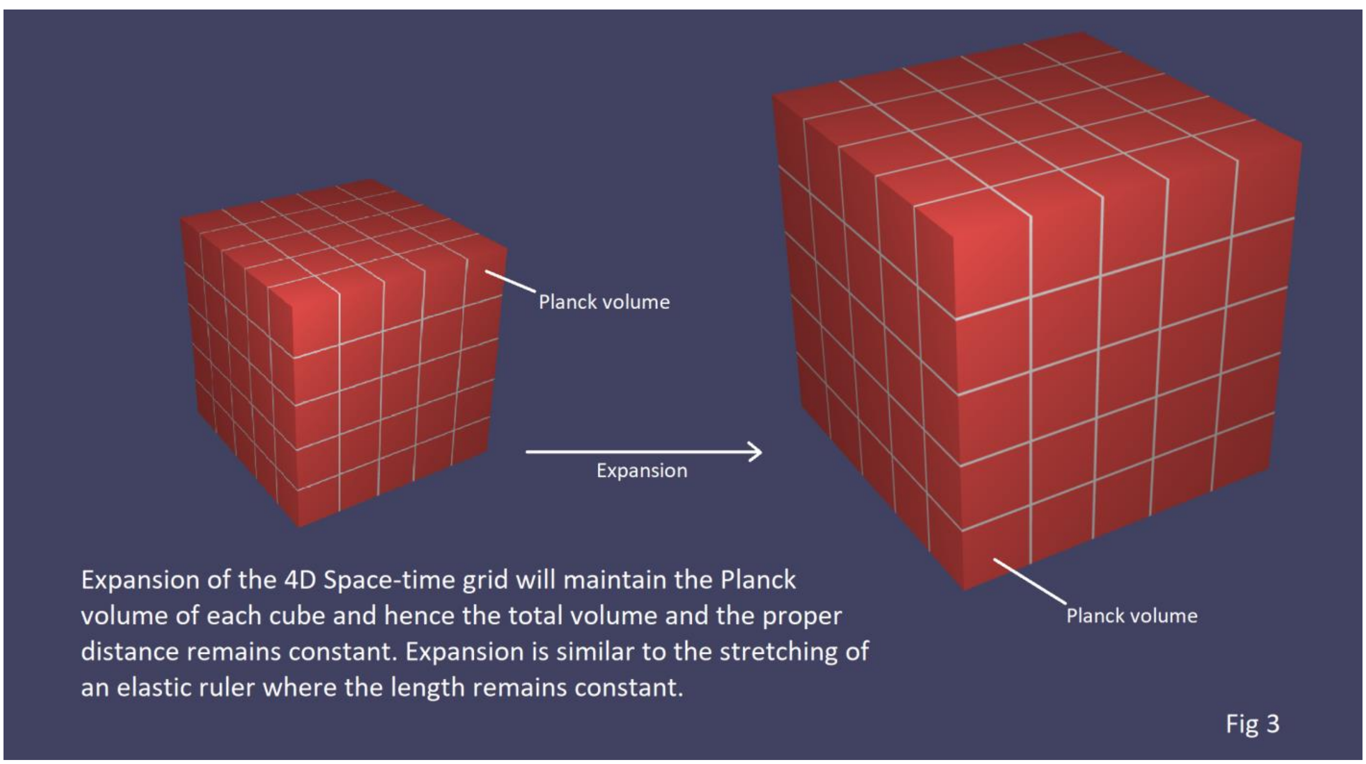

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6. The temperature of the universe

The internal energy U of black-body photon gas is given by

$U = \left(\frac{8\pi^5 k^4}{15h^3c^3}\right)VT^4$. where k = Boltzmann constant, h = Planck constant, c = Speed of light, V = Volume and T = Temperature. (Leff, 2002)

As $h = h_{t_p}(1+z)$, $k = k_{t_p}(1+z)$, volume V of the universe is constant through length contraction, upholding the law of conservation of energy and CMB being black-body radiation, the maximum possible temperature of the universe, when the age of the universe was t_p, is given by the below based on the above formula for U.

$$U = \left(\frac{8\pi^5 k^4}{15h^3c^3}\right)VT^4 = \left(\frac{8\pi^5\left(\frac{k}{1+z}\right)^4}{15\left(\frac{h}{1+z}\right)^3 c^3}\right)\frac{V}{(1+z)^3}T_{t_p}{}^4$$

Here, $\frac{V}{(1+z)^3}$ should not be seen as a reduction in volume but as a cosmological length contraction. $T_{t_p} = T(1+z)$, which can be generalized as follows.

$$\boxed{T = T_0(1+z)} \quad \textbf{(23)}$$

As $T_0 = 2.725$ K and the maximum cosmological redshift z is 7.4×10^{60} from (3), $T_{tp} = 2 \times 10^{61}$ K. As the T_{tp} value is greater than the Planck temperature, the maximum possible temperature of the universe is the Planck temperature, which is 1.416784×10^{32} K. However, this does not mean that this was the temperature at the time of the big repulsion but only the maximum possible temperature. As the CMB was emitted after the initial t_p of the big repulsion, the original temperature of the CMB when it was first emitted should be less than T_{tp}. The future temperature of CMB radiation can also be calculated using the above formula. For example, the CMB temperature after 10^7 years would be 2.722 K, but the CMB photon density should remain the same as the volume of the universe remains constant.

As the interaction of the electrostatic expansion energy with the quantum vacuum fluctuations to create matter at the time of big repulsion will be the same throughout space-time, the created primordial elementary particles, atoms, and their attributes, such as temperature and density, should be uniform throughout space-time without having the particles interact with one another or without being causally connected, which eliminates the cosmic inflation, horizon problem, and flatness problem and explains the uniformity of CMB radiation. However, minor fluctuations in the density of the created particles due to the randomness of the quantum vacuum fluctuations and the subsequent concentration of the matter due to gravity could explain the temperature anisotropy. However, the angular power spectrum of the CMB still needs to be explained, possibly through BAO (Baryon acoustic oscillations) and a high gravitational constant without dark matter.

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7. The absolute frame of reference

The four-dimensional space-time grid proposed in this theory acts as an absolute inertial frame. As the physical constants (G, h, ε_0, and k_B) change with the change in gravitational potential and velocity, any frame of reference that is not absolute can be easily identified by knowing the values of the physical constants. Physical constants change in the gravitational field and due to velocity according to the formulas given in Table II (17). Therefore, in the gravitational field, the absolute reference frame is the one at an infinite distance from the mass generating the gravitational field, and the values of the physical constants change as we move from infinity toward the mass. For example, the gravitational constant would be higher near a mass than away from the mass, and the Planck constant would be lower near a mass than away from the mass.

As the Hubble constant decreases due to the space-time expansion, the values of the physical constants of the absolute inertial frame need to be updated to keep up with the expansion. Changes in the physical constants of the absolute inertial frame due to cosmological expansion are given in Table III (18). Additionally, the universal time in this theory is the time in the absolute inertial frame, which is the Hubble time.

Similarly, a frame moving with a velocity will have a higher gravitational constant and a lower Planck constant compared to the absolute inertial frame. However, a source of light emitting green photons will still emit green photons in both frames with different Planck constants due to the time dilation in the moving frame.

For example, in this theory, acceleration due to Newtonian gravity is covariant in all inertial frames. Consider two masses each of mass m separated by a distance R moving with velocity v in an inertial frame. The time dilation of the moving masses in the stationary reference frame compensates for the time dilation of the observer moving along with the masses. Therefore, the observers in both the stationary and moving reference frames will observe the same formula for acceleration (a) due to gravity in two different ways, one with relativistic mass γm and the other with relativistic gravitational constant γG, as shown in Table V, except for the time dilation effect observed by the stationary observer, which makes the acceleration due to gravity covariant similar to electromagnetism.

Table V (Covariant acceleration due to Newtonian gravity)

	WRT stationary reference frame	WRT moving reference frame
Masses aligned perpendicular to the direction of motion	$(\gamma m)a = F = \frac{G(\gamma m)(\gamma m)}{R^2}$; $a = \frac{G(\gamma m)}{R^2}$	$a = \frac{(G\gamma)m}{R^2}$
Masses aligned parallel to the direction of motion	$\gamma^3 ma = F = \frac{G(\gamma m)(\gamma m)}{\left(\frac{R}{\gamma}\right)^2}$; $a = \frac{G(\gamma m)}{R^2}$	$a = \frac{(G\gamma)m}{R^2}$

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8. The relativistic Newtonian theory of gravity

In this theory, the presence of mass increases the permittivity of the four-dimensional space-time grid and hence reduces the electrostatic repulsion within the grid enveloped by the mass compared to the electrostatic repulsion outside the mass. As the contraction of the space-time grid within mass will be greater than the contraction outside due to the permittivity difference and the net compressing force on the matter from the surrounding Planck charges, space-time becomes naturally curved around any mass and hence the gravitational force. Therefore, in this theory, the gravitational force is considered a real force rather than a fictitious force, as in the general theory of relativity.

The change in gravitational constant G in the gravitational field is similar to the change in the gravitational constant due to cosmological space-time expansion. Therefore, we can take Newton's law of gravitation and introduce variable G and the gravitational length contraction to derive a relativistic law. Here, the change in gravitational potential energy is not included as part of mass, as it does not cause additional gravitational force; only the rest and relativistic masses cause gravitational force in this theory.

$$F = \frac{G(1+z)Mm}{\left(\frac{R}{1+z}\right)^2} = (1+z)^3 \frac{GMm}{R^2}$$ using (17). where G is the gravitational constant at infinity.

As the space-time grid shrinks as we move from infinity toward the mass, the length contraction is included for R, which is the distance between the masses m and M as observed from the coordinate system at infinity.

$$\frac{dU}{dR} = F = (1+z)^3 \frac{GMm}{R^2} = \left(1 - \frac{U}{mc^2}\right)^3 \frac{GMm}{R^2}$$ using (15)

$$\int_0^U \left(\frac{1}{mc^2 - U}\right)^3 dU = \frac{GMm}{(mc^2)^3} \int_\infty^R \frac{1}{R^2} dR$$

$$\frac{1}{2(mc^2 - U)^2} - \frac{1}{2(mc^2)^2} = -\frac{GMm}{(mc^2)^3} \frac{1}{R}$$, divide the denominator by $(mc^2)^2$ on both sides.

$$\frac{1}{2\left(1 - \frac{U}{mc^2}\right)^2} - \frac{1}{2} = -\frac{GMm}{mc^2} \frac{1}{R}$$

$$\frac{1}{2(1+z)^2} - \frac{1}{2} = -\frac{GMm}{mc^2} \frac{1}{R}$$ using (15); after solving, we obtain the following equation.

$$\boxed{1+z = \sqrt{\frac{1}{1 - \frac{2GM}{Rc^2}}}} \qquad \textbf{(24)}$$

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As the above gravitational redshift matches the redshift from the Schwarzschild solution of the Einstein field equations without using the weak field approximation, it validates this theory, the concept of variable physical constants, and proves the existence of an absolute reference frame. Additionally, the formulas below for the gravitational force and the gravitational potential energy are not approximations but complete solutions that work in both strong and weak gravitational fields for nonrotating stationary spherical masses without using the weak field approximation as in the general theory of relativity. If there is kinetic energy of the particles within the masses (e.g., kinetic energy due to temperature and/or angular momentum), the respective relativistic masses should be used instead of the rest masses M and m to obtain accurate results.

z becomes infinity at $R = \frac{2GM}{c^2}$ in (24), which gives the Schwarzschild radius without using a weak field approximation.

Relativistic gravitational force F for M ≥ m:

$$F = (1+z)^3 \frac{GMm}{R^2} = \left(\sqrt{\frac{1}{1 - \frac{2GM}{Rc^2}}} \right)^3 \frac{GMm}{R^2} \qquad \textbf{(25)}$$

In Table V, we can see that the acceleration due to gravity a is the same in both the stationary and moving inertial frames except for the time dilation effect in the stationary frame. Relativistic acceleration due to gravity in both frames can be obtained by including the $(1+z)^3$ factor (25).

$\frac{d\phi}{dR} = a = (1+z)^3 \frac{G\gamma m}{R^2} = \left(1 - \frac{\phi}{c^2}\right)^3 \frac{G\gamma m}{R^2}$ using (16); $\int_0^U \left(\frac{1}{c^2 - \phi}\right)^3 d\phi = \frac{G\gamma m}{(c^2)^3} \int_\infty^R \frac{1}{R^2} dR$; after solving, we obtain the following equation.

$$1 + z = \sqrt{\frac{1}{1 - \frac{2G\gamma m}{Rc^2}}}$$

Relativistic gravitational potential energy U for M ≥ m:

$$U = -mzc^2 = -m\left(\sqrt{\frac{1}{1 - \frac{2GM}{Rc^2}}} - 1 \right) c^2 \qquad \textbf{(26)}$$

Equating the above relativistic gravitational potential energy U with the relativistic kinetic energy ($m(\gamma-1)c^2$) with escape velocity c directly gives the Schwarzschild radius $R_s = \frac{2GM}{c^2}$.

For weak gravitational fields, (26) reduces to the regular Newtonian gravitational potential energy and hence validates this theory.

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$$U = -m\left(\left(1-\frac{2GM}{Rc^2}\right)^{-\frac{1}{2}} - 1\right)c^2 \approx -m\left(1+\frac{GM}{Rc^2}-1\right)c^2 = -\frac{GMm}{R}$$

Relativistic gravitational potential ϕ for M $\geq$ m:

$$\boxed{\phi = \frac{U}{m} = -zc^2 = -\left(\sqrt{\frac{1}{1-\frac{2GM}{Rc^2}}} - 1\right)c^2} \quad \textbf{(27)}$$

The maximum energy (rest + potential) E_{max} that can be gained by a mass m at radius R from mass M is below.

$$E_{\max} = m(1+z)c^2 = m\left(\sqrt{\frac{1}{1-\frac{2GM}{Rc^2}}}\right)c^2 \text{ from (14)}$$

As the energy is conserved in the freefall motion of mass m in the gravitational field, the velocity of mass m can be derived from the maximum energy E_{max}. Through dimensional analysis, the energy at any point during the freefall must be of the form $m\left(\sqrt{\frac{1}{1-\frac{v^2}{c^2}}}\right)c^2$. Therefore, $v^2 = \frac{2GM}{R}$.

$v = \sqrt{\frac{2GM}{R}}$, which is the velocity of mass m in a freefall motion from the point of maximum potential energy or infinity. This is also equal to the escape velocity v_e. As the freefall motion is inertial, the Lorentz factor γ is directly realized from energy conservation, which is $\sqrt{\frac{1}{1-\frac{v^2}{c^2}}}$, and hence, this theory upholds the special theory of relativity in all inertial frames.

The length contraction, time dilation, and mass change in the gravitational field are given by $L = \frac{L_\infty}{(1+z)}$, $T = \frac{T_\infty}{(1+z)}$, and $m = \frac{m_\infty}{(1+z)}$, where L_∞, T_∞, and m_∞ are the absolute length, absolute time, and maximum mass, respectively, at infinity **(28)**

As the radial length L, T, and m becomes zero at Schwarzschild's radius, space-time terminates at the event horizon of a black hole. Black hole singularities do not exist in this model of the universe as the mass becomes zero and is converted to the equivalent pure informational entropy at the event horizon. Space-time terminates at the event horizon and cannot be extended beyond the event horizon, as in the general theory of relativity, as the radial length, time and tangible mass become zero at the event horizon and hence discards the Riemannian geometry of space-time. Additionally, charged black holes do not exist in this model of the universe as the permittivity of free space becomes infinity (from (17)) at the event horizon. Therefore, stationary black holes can only have two properties, namely, informational (entropic) mass and angular momentum, and hence only partially satisfy the no-hair theorem.

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8.1. Photon sphere

As the photon takes the curved path around the black hole due to the space-time curvature, matching the fictitious acceleration due to gravity for the photon with the centripetal acceleration gives the radius of the photon sphere for a nonspinning black hole, as predicted by the general theory of relativity. Here, the forces are considered to be fictitious, as the photon does not curve space-time by itself but only takes the path of curved space-time due to optical refraction. The path taken by a photon or a particle due to refraction in curved space-time is equivalent to the geodesic path in the general theory of relativity. Here, the refraction of the photon due to the curvature is considered equivalent to the centripetal acceleration. Therefore, the fictitious centripetal acceleration of the photon due to refraction in the curved space-time around mass M is $\frac{c^2}{\left(\frac{R}{1+z}\right)}$.

Here, the factor (1+z) accounts for the length contraction as observed from the coordinate system at infinity or from the absolute reference frame.

$(1+z)^3 \frac{GM}{R^2} = \frac{c^2}{\left(\frac{R}{1+z}\right)}$ using (25), and solving using (24), we obtain the radius of the photon sphere.

$$\left(\sqrt{\frac{1}{1-\frac{2GM}{Rc^2}}}\right)^2 \frac{GM}{Rc^2} = 1 \quad \textbf{(29)}$$

$$\boxed{R = \frac{3GM}{c^2}} \quad \textbf{(30)}$$

The probability wave function (ψ) of any matter particle $(m \ll M)$ also takes the path of the curved space-time and is refracted like a photon but curves the space-time unlike a photon and causes a wobble in mass M; hence, the forces are considered real. A photon does not cause a wobble in mass M, as it is massless. As the matter particles move with velocities less than *c*, the centripetal acceleration experienced by a matter particle due to the refraction of its wave function is $\frac{v^2}{\left(\frac{R}{1+z}\right)}$.

The innermost stable circular orbit (ISCO) is not applicable to this theory, as the relativistic gravitational potential (ϕ) always changes for any radius greater than the Schwarzschild radius.

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8.2. Gravitational lensing

As the angle of deflection θ for the photon sphere is π radians owing to the symmetry of the sphere, as shown in Figure 4, we can calculate the angle of deflection for a nonspinning spherical mass for any radius R within the relativistic Newtonian regime by replacing 1 in (29) with $\mathrm{Tan}\left(\frac{\pi}{4}\right)$, which is the ratio of the gravitational acceleration to the centripetal acceleration of the photon at its closest approach to mass M. As this ratio should be proportional to the angle of deflection θ, we can generalize this equation by replacing $\mathrm{Tan}\left(\frac{\pi}{4}\right)$ with $\mathrm{Tan}\left(\frac{\theta}{4}\right)$ to find the angle of deflection for any radius R.

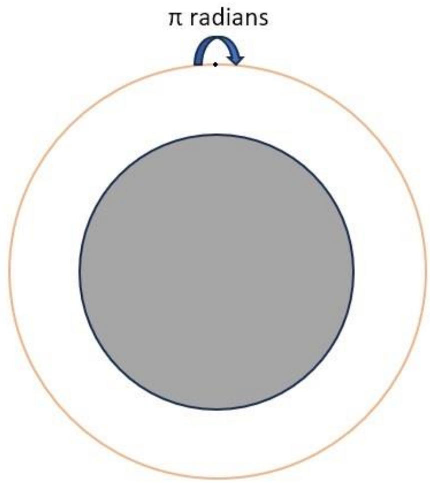

Fig 4

$$\left(\sqrt{\frac{1}{1-\frac{2GM}{Rc^2}}}\right)^2 \frac{GM}{Rc^2} = \mathrm{Tan}\left(\frac{\pi}{4}\right)$$

$$\boxed{\theta = 4\,\mathrm{Tan}^{-1}\left((1+z)^2 \frac{GM}{Rc^2}\right)} \qquad \textbf{(31)}$$

For $\theta = 2\pi$, we obtain $R = \frac{2GM}{c^2}$, which is the Schwarzschild radius and hence validates the above equation. Therefore, light bends onto itself at the event horizon of a black hole and does not travel beyond it as in the general theory of relativity and hence provides another proof that space-time ends at the event horizon and the nonexistence of singularity at the center of a black hole. For weak gravitational fields, the angle of deflection in (31) reduces to $\theta = \frac{4GM}{Rc^2}$, which is the same as the deflection predicted by the general theory of relativity. Therefore, equation (31) predicts the angle of deflection in a closed form that works in both strong and weak gravitational fields better than the general theory of relativity without using any approximations or higher-order terms.

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8.3. Perihelion precession of Mercury

From Kepler's second law, $dA = \frac{1}{2}r^2 d\theta$, where dA is the change in the area swept out by the orbiting mass, $d\theta$ is the change in angle and r is the radius.

Let dA' and $d\theta'$ be the change in area and change in angle, respectively, from the absolute reference frame perspective, where the length and time are absolute. Divide the above equation on both sides by dt' and apply length contraction (using (28)).

$$\frac{dA'}{dt'} = \frac{1}{2}\left(\frac{r}{1+z}\right)^2 \frac{d\theta'}{dt'}$$

Apply time dilation using (28), where dt' is the absolute time and dt is the local time.

$$\frac{dA'}{dt'} = \frac{1}{2}\left(\frac{r}{1+z}\right)^2 \frac{d\theta'}{dt(1+z)} = \frac{1}{2}r^2 \frac{d\theta'}{dt}\frac{1}{(1+z)^3}$$

As the angular momentum is conserved, the above equation can be compared with the nonrelativistic equation to obtain the following equation.

$$d\theta' = d\theta(1+z)^3 = d\theta\left(1-\frac{2GM}{Rc^2}\right)^{-\frac{3}{2}} \approx \left(1+\frac{3GM}{Rc^2}\right)$$ by ignoring the higher-order terms.

The polar equation of the ellipse is $R = \frac{a(1-\varepsilon^2)}{1-\varepsilon\cos\theta}$.

$$\theta' = \int_0^{2\pi}\left(1+\left(\frac{3GM}{c^2}\frac{1-\varepsilon\cos\theta}{a(1-\varepsilon^2)}\right)\right)d\theta$$

$\theta' = 2\pi + \frac{6\pi GM}{c^2 a(1-\varepsilon^2)}$, the second term gives the precession angle $\Delta\phi$ of the perihelion per revolution. where M is the mass of the sun, a is the semimajor axis of Mercury and ε is the orbital eccentricity.

$$\boxed{\Delta\phi = \frac{6\pi GM}{c^2 a(1-\varepsilon^2)}}$$

Solving the above, we obtain a perihelion precession of 43"/century, which is the same as that predicted by the general theory of relativity (Park et al., 2017).

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8.4. Shapiro time delay

We can calculate the gravitational time delay of light passing by a mass such as the sun. As the gravitational length contraction and gravitational time dilation go together in this theory, we can calculate the effective speed of light as observed by an observer on Earth as the light from a distant object passes nearby the sun and reaches the Earth.

The speed of light is constant at every point in space in the gravitational field, as the length contraction is compensated by the time dilation. Therefore, $\dfrac{\frac{\text{Distance}}{(1+z)}}{\frac{Time}{(1+z)}} = \dfrac{\text{Distance}}{Time} = c$ (the speed of light) remains constant. However, a stationary observer would see a change in the speed of light w.r.t. his reference frame as the light goes through the gravitational field. Here, the time expansion is applied instead of contraction, as the time delay is observed from the observer's frame of reference whose local time is faster than the time near the sun. Therefore, the length contraction and time expansion do not cancel out in the observer's reference frame and hence produce a time delay according to this theory, which is called Shapiro time delay.

The effective speed of light as experienced by the stationary observer is $\dfrac{\frac{\text{Distance}}{(1+z)}}{\text{Time}(1+z)} = \dfrac{c}{(1+z)^2}$.

$\dfrac{dx}{dt} = \dfrac{c}{(1+z)^2} = c\left(1 - \dfrac{2GM}{Rc^2}\right)$ using (24), where R is the distance between the sun and the traveling photon. Neglecting the deflection of the light near the sun, the path of the light from A to B is a straight line.

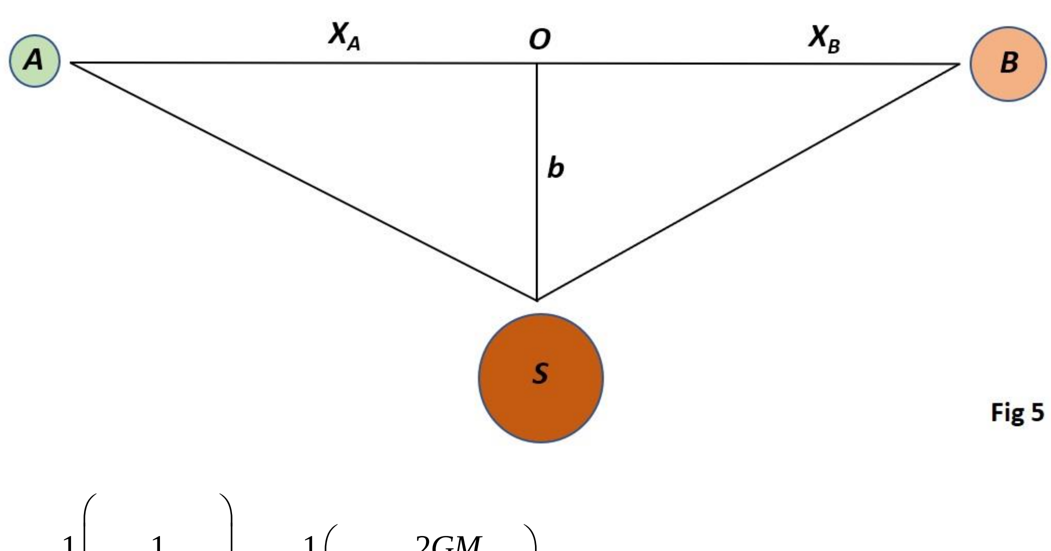

Fig 5

$dt = \dfrac{1}{c}\left(\dfrac{1}{1 - \dfrac{2GM}{Rc^2}}\right)dx \approx \dfrac{1}{c}\left(1 + \dfrac{2GM}{\sqrt{x^2 + b^2}\,c^2}\right)dx$ by ignoring the higher-order terms, where b is the impact parameter. Integrating on the left side from T_A to T_B and on the right side from X_A to X_B.

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$$\int_{T_A}^{T_B} dt = \int_{X_A}^{X_B} \frac{1}{c}\left(1+\frac{2GM}{\sqrt{x^2+b^2}c^2}\right)dx$$

$$T_B - T_A = \frac{X_B - X_A}{c} + \frac{2GM}{c^3}\ln\left(\frac{X_B+\sqrt{X_B{}^2+b^2}}{X_A+\sqrt{X_A{}^2+b^2}}\right)$$

The second term gives the additional one-way time delay Δt of the light coming from point A as observed by an observer at point B, which better fits the experimental data than the Schwarzschild metric using Schwarzschild coordinates (Pössel, 2019, Page 8).

$$\boxed{\Delta t = \frac{2GM}{c^3}\ln\left(\frac{X_B+\sqrt{X_B{}^2+b^2}}{X_A+\sqrt{X_A{}^2+b^2}}\right)}$$

8.5. Relativistic Poisson and gravitoelectromagnetism (GEM) equations

Relativistic Poisson and Maxwell-like GEM equations can be generated to make them work in both strong and weak gravitational fields by applying the Laplace operator (∇^2) on the relativistic gravitational potential. Relativistic GEM equations can explain the frame-dragging effect, orbital precession, geodetic effect, and gravitational waves. Both the relativistic GEM and Poisson equations together can form the equivalent of Einstein field equations for this theory with the scope for further generalization.

As $z = \frac{-\phi}{c^2}$ (16), $(1+z) = \left(1-\frac{\phi}{c^2}\right)$.

8.5.1. Relativistic Poisson equations

The relativistic gravitational potential in Cartesian coordinates based on (27) is given below.

$$\phi = -\left(\frac{1}{\sqrt{1-\frac{2GM}{\left(\sqrt{x^2+y^2+z^2}\right)c^2}}}-1\right)c^2$$

The relativistic Poisson equation outside the point mass can be generated by applying the Laplace operator to the above gravitational potential.

$$\nabla^2\phi = -\frac{3G^2M^2x^2(1+z)^5}{(x^2+y^2+z^2)^3c^2} - \frac{3G^2M^2y^2(1+z)^5}{(x^2+y^2+z^2)^3c^2} - \frac{3G^2M^2z^2(1+z)^5}{(x^2+y^2+z^2)^3c^2}$$
$$-\frac{3GMx^2(1+z)^3}{(x^2+y^2+z^2)^{\frac{5}{2}}} - \frac{3GMy^2(1+z)^3}{(x^2+y^2+z^2)^{\frac{5}{2}}} - \frac{3GMz^2(1+z)^3}{(x^2+y^2+z^2)^{\frac{5}{2}}} + \frac{3GM(1+z)^3}{(x^2+y^2+z^2)^{\frac{3}{2}}}$$

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$\nabla^2\phi = -2\pi G\rho\left[(1+z)^5-(1+z)^3\right]$, where $\rho = \frac{M}{\frac{4}{3}\pi r^3}$ is the relativistic mass density of the point mass.

Outside the point mass: $$\boxed{\nabla^2\phi = -2\pi G\rho\left[\left(1-\frac{\phi}{c^2}\right)^5-\left(1-\frac{\phi}{c^2}\right)^3\right]\approx 4\pi G\rho\frac{\phi}{c^2}} \quad \textbf{(32)}$$

The relativistic Poisson equation at the point mass can be generated by considering only the positive terms and ignoring the negative terms in the Laplacian ($\nabla^2\phi$).

$\nabla^2\phi = \frac{3GM(1+z)^3}{(x^2+y^2+z^2)^{\frac{3}{2}}} = 4\pi G\rho(1+z)^3$, where ρ is the relativistic mass density of the point mass.

At the point mass: $$\boxed{\nabla^2\phi = 4\pi G\rho\left(1-\frac{\phi}{c^2}\right)^3} \quad \textbf{(33)}$$

For weak gravitational fields where the gravitational redshift z is negligible, the above equations (32) and (33) reduce to Laplace and nonrelativistic Poisson equations, respectively.

Outside the point mass: $\nabla^2\phi = -2\pi G\rho\left[(1+z)^5-(1+z)^3\right]\approx -2\pi G\rho[1-1]=0$

At the point mass: $\nabla^2\phi = 4\pi G\rho(1+z)^3 \approx 4\pi G\rho[1] = 4\pi G\rho$

8.5.2. Relativistic GEM equations

Relativistic Maxwell-like GEM equations can be generated using (33) and (32) at the point mass/current and outside the point mass/current, respectively. where E_G is the gravitoelectric field, B_G is the gravitomagnetic field, ϕ is the relativistic gravitational potential due to the gravitoelectric effect, and J is the relativistic mass current density.

At the point mass/current:	Outside the point mass/current:
$\nabla.\mathrm{E_G} = -4\pi G\rho\left(1-\frac{\phi}{c^2}\right)^3$	$\nabla.\mathrm{E_G} = 2\pi G\rho\left[\left(1-\frac{\phi}{c^2}\right)^5-\left(1-\frac{\phi}{c^2}\right)^3\right]$
$\nabla\times\mathrm{B_G} = -\frac{4\pi G\left(1-\frac{\phi}{c^2}\right)^3}{c^2}\mathrm{J}+\frac{1}{c^2}\frac{\partial E_G}{\partial t}$	$\nabla\times\mathrm{B_G} = \frac{2\pi G\left[\left(1-\frac{\phi}{c^2}\right)^5-\left(1-\frac{\phi}{c^2}\right)^3\right]}{c^2}\mathrm{J}+\frac{1}{c^2}\frac{\partial E_G}{\partial t}$
$\nabla.\mathrm{B_G} = 0$	$\nabla.\mathrm{B_G} = 0$
$\nabla\times\mathrm{E_G} = -\frac{\partial B_G}{\partial t}$	$\nabla\times\mathrm{E_G} = -\frac{\partial B_G}{\partial t}$

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8.6. Space-time interval (ds^2)

The invariant line element (ds^2) in the Minkowski space-time in spherical coordinates is given by $ds^2 = -c^2dt^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$. Applying the gravitational length contraction and time dilation (24) (16) as observed from the coordinate system at infinity or from the absolute reference frame, we obtain the equivalent of the Schwarzschild metric for this theory.

$$ds^2 = -\frac{c^2dt^2}{(1+z)^2} + \frac{dr^2}{(1+z)^2} + \frac{r^2}{(1+z)^2}(d\theta^2 + \sin^2\theta d\phi^2) \quad \textbf{(34)}$$

Schwarzschild metric equivalent:

$$\boxed{ds^2 = -c^2dt^2\left(1-\frac{2GM}{rc^2}\right) + dr^2\left(1-\frac{2GM}{rc^2}\right) + r^2\left(1-\frac{2GM}{rc^2}\right)(d\theta^2 + \sin^2\theta d\phi^2)}$$

OR

$$\boxed{ds^2 = -c^2dt^2\left(1-\frac{\phi}{c^2}\right)^{-2} + dr^2\left(1-\frac{\phi}{c^2}\right)^{-2} + r^2\left(1-\frac{\phi}{c^2}\right)^{-2}(d\theta^2 + \sin^2\theta d\phi^2)}$$

Once the (1+z) factor $\left(1-\frac{\phi}{c^2}\right)$ is identified by solving the relativistic Poisson equation (33), it can be substituted in (34) to obtain the space-time interval equation for this theory, which is equivalent to the equation $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$ in the general theory of relativity (Einstein, 1922). For any other inertial/noninertial reference frame, the respective coordinate length, time, and gravitational redshift z up to the origin of the reference frame must be used in (34) to make the line element (ds^2) invariant in all reference frames.

In this theory, space-time ends at the event horizon of a black hole as the time and length become zero and hence eliminates singularity, unlike in the general theory of relativity, where the length dr becomes infinity and custom coordinate systems are used to avoid infinities and to extend the space-time up to the singularity at the center of a black hole. Additionally, the slope of the light ray $\left(\frac{dt}{dr}\right)$ of the light cone is always equal to 1 (45 degrees) in this theory, even near the event horizon, unlike in the general theory of relativity, where the light cone closes up and the slope becomes infinity. Setting the line element $ds^2=0$, which is lightlike, and c=1 in (34), ignoring dθ and dϕ, we obtain $\frac{dt}{dr} = 1$.

Applying the cosmological length contraction and time dilation (4), we obtain the below equivalent of the FLRW metric of the flat space-time for this theory.

FLRW metric equivalent:

$$\boxed{ds^2 = -c^2dt^2\left(1-\frac{H_0r}{c}\right)^2 + dr^2\left(1-\frac{H_0r}{c}\right)^2 + r^2\left(1-\frac{H_0r}{c}\right)^2(d\theta^2 + \sin^2\theta d\phi^2)}$$

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9. MOND (Modified Newtonian dynamics)

We can generate the relativistic MOND formula and the respective Poisson equation using the (1+z) factor similar to the relativistic Newtonian theory of gravity. However, the relativistic effect in the deep-MOND regime is not significant due to lower accelerations. This theory proposes that the MOND regime can only be present around a black hole but not around ordinary masses such as gas and stars. In the absence of black holes, the gravitational field around ordinary masses will always be Newtonian at any acceleration. Therefore, the cutoff acceleration for the MOND regime, which is a_0 ($\sim 1.2 \times 10^{-10}$ m/s^2) (Bekenstein, 2009), is only applicable for the area around a central black hole but not for ordinary masses. Each cell in Figure 6 of the curved space around the black hole represents the Planck volume with Planck charge. The radial length becomes zero at the event horizon but will continue to expand to follow the relativistic Newton's law of gravitation as we move away from the black hole. Additionally, the number of Planck volumes on the circumference remains constant at any radius from the event horizon.

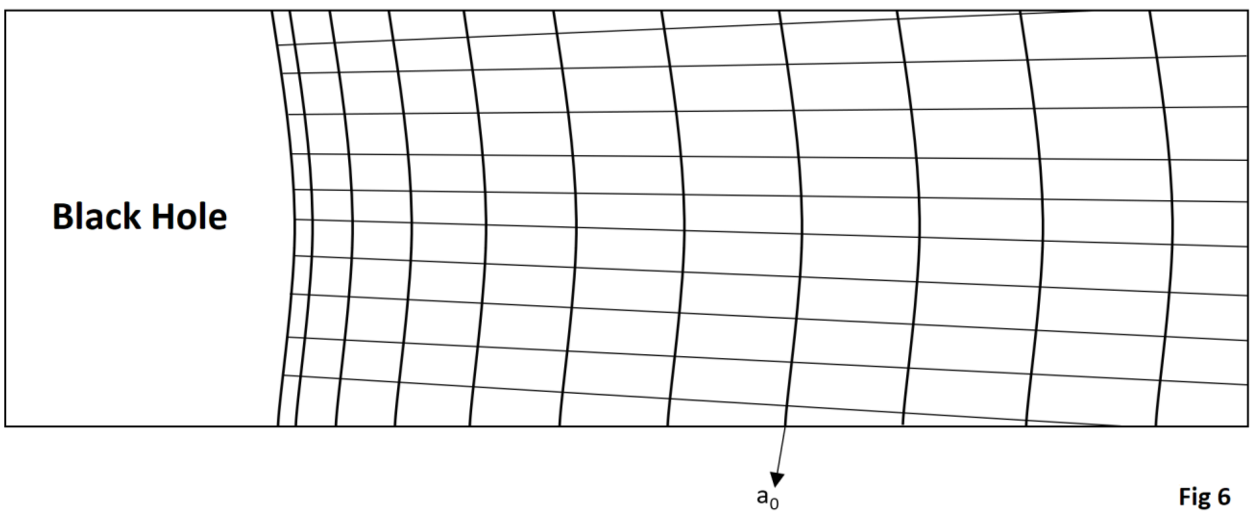

Fig 6

To understand the reason for the transition of the Newtonian regime to the MOND regime at a_0, we can flatten the above-curved space, which is depicted in Figure 7. As the relative Planck volume cannot increase more than the maximum allowed by the expansion of the universe, the relative size of the Planck volume should remain constant after a_0, which is the case in Figure 7 below.

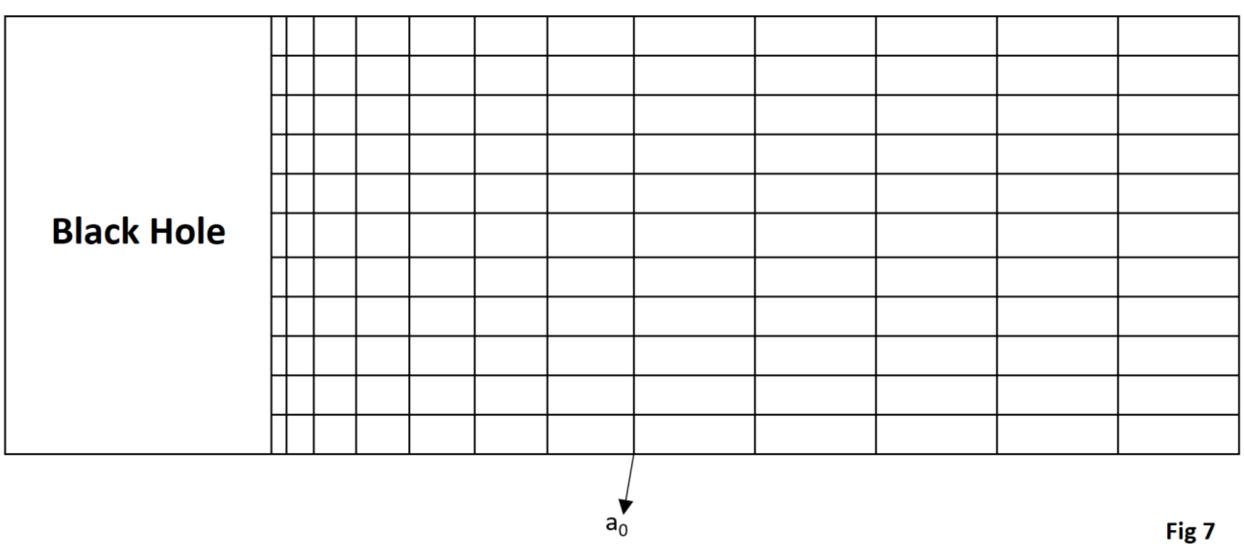

Fig 7

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Once the space becomes curved, as is the case around the black hole, the Planck volumes beyond a_0 also become curved, and hence, the relative Planck volumes also gradually increase as the stretching of space-time continues to increase beyond a_0 due to the curvature, as the circumference farther from a_0 should stretch more than the one closer to a0. However, the rate of change in the Planck volume for $a < a_0$ is less than the rate of change for $a > a_0$. Therefore, the Newtonian law of gravitation switches to the MOND law of gravitation at a_0.

Let us consider an elastic rubber band stretched around a cylinder; the radial force exerted by the rubber band on the cylinder would be $\frac{1}{2\pi}$ times the tension in the rubber band owing to the circumference of the cylinder given by 2πr. As the current acceleration of the universe is cH_0 (2) per this theory, the radial acceleration at a_0 should be $\frac{cH_0}{2\pi}$, which is $\sim 1.2 \times 10^{-10}$ m/s^2 per the above analogy. Here, the whole universe acts like a rubber band wrapped around the black hole at a_0. Additionally, we can see that the space-time grid ends at the event horizon of the black hole and does not extend into it as in the general theory of relativity.

Below, Figure 8 represents the distortion of the space-time grid around the ordinary matter. As the matter is present only in the four-dimensional space-time grid, it is depicted in red in the middle of Figure 8. Each cube in this figure is of one Planck volume with a Planck charge. As the number of circumferential Planck volumes and the respective area increase radially as we move away from the matter, cutoff acceleration such as a_0 is not applicable, and Newton's law of gravitation can be applied at any acceleration without using any modification such as MOND in the absence of black holes. This explains why the expected gravitational lensing is not observed around the gaseous part of the Bullet cluster (1E 0657-56) (Paraficz et al., 2016) but around the galactic matter as it is subjected to MONDian gravitational lensing due to the presence of black holes at the centers of the galaxies.

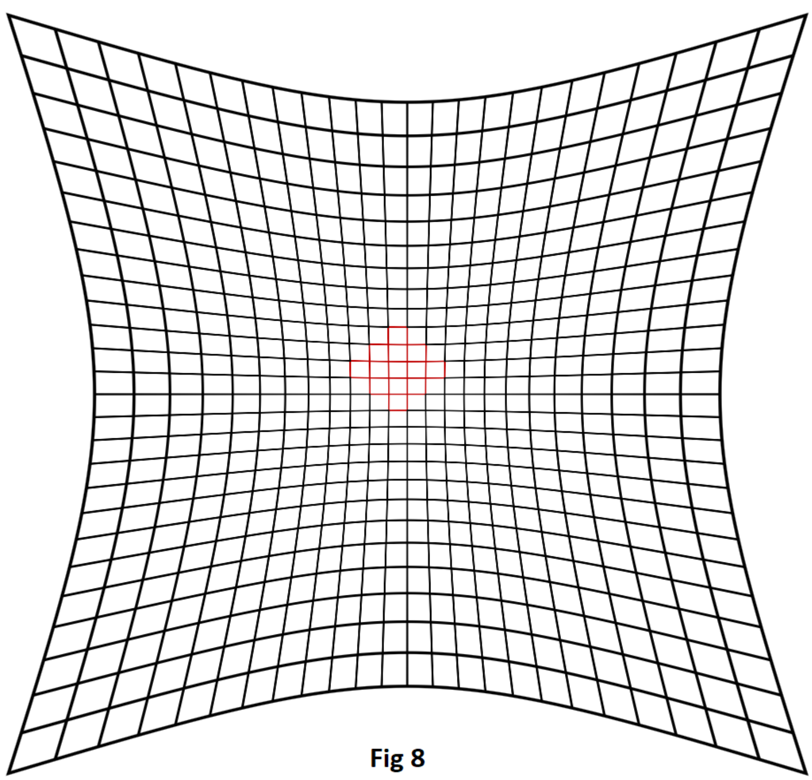

Fig 8

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As dark matter has not been detected in galaxy NGC 1052-DF2 (Van Dokkum et al., 2018), this theory predicts that galaxies that do not need dark matter to explain the rotation curves beyond the a_0 orbit should not have black holes at their centers or anywhere in the galaxies. Further research is needed to determine whether the relativistic MONDian gravity proposed in this theory can solve the missing mass problem in galaxy clusters.

9.1. The relativistic MONDian gravity

The MONDian gravitational law is given below.

$F = \frac{GMm}{r^2} f\left(\frac{r}{r_o}\right)$ (Sanders & Mcgaugh, 2002)

$f(x) \to 1$ for $x \ll 1$, $f(x) \to x$ for $x \gg 1$ and r_o is the radius at which a = a_0.

We can take the above MOND formula and introduce variable G and gravitational length contraction to develop a relativistic law similar to the relativistic Newtonian theory of gravity around a stationary black hole.

$F = \frac{G(1+z)Mm}{\frac{R_o}{(1+z_o)}\frac{R}{(1+z)}} = (1+z_o)(1+z)^2 \frac{GMm}{R_o R}$. where z_o is the gravitational redshift (24) at R_o.

$\frac{dU}{dR} = F = (1+z_o)(1+z)^2 \frac{GMm}{R_o R} = (1+z_o)\left(1-\frac{U}{mc^2}\right)^2 \frac{GMm}{R_o R}$ using (15)

$$\int_U^{U_o} \left(\frac{1}{mc^2 - U}\right)^2 dU = (1+z_o)\frac{GMm}{R_0 (mc^2)^2} \int_R^{R_o} \frac{1}{R} dR$$

$\frac{1}{(mc^2 - U_o)} - \frac{1}{(mc^2 - U)} = (1+z_o)\frac{GMm}{R_0 (mc^2)^2} \ln\left(\frac{R_o}{R}\right)$, divide the denominators by (mc^2).

$$\frac{1}{\left(1-\frac{U_o}{mc^2}\right)} - \frac{1}{\left(1-\frac{U}{mc^2}\right)} = (1+z_o)\frac{GM}{R_0 c^2} \ln\left(\frac{R_o}{R}\right)$$

$\frac{1}{(1+z_o)} - \frac{1}{(1+z)} = (1+z_o)\frac{GM}{R_0 c^2} \ln\left(\frac{R_o}{R}\right)$ using (15)

$$\boxed{1+z = \frac{(1+z_o)}{1-(1+z_o)^2 \frac{GM}{R_0 c^2} \ln\left(\frac{R_o}{R}\right)}} \quad \textbf{(35)}$$

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Relativistic MONDian gravitational force F for M ≥ m:

$$F = \left[1 + z_o \mu\left(\frac{z}{z_o}\right)\right](1+z)^2 \frac{GMm}{R^2} \mu\left(\frac{R}{R_o}\right) \qquad \textbf{(36)}$$

where $\mu(x) \to 1$ for $x \ll 1$, $\mu(x) \to$ x for $x \gg 1$

The above relativistic MONDian gravitational formula reduces to the relativistic Newtonian formula for gravity (25) for $R \le R_o$ and $z \ge z_o$.

Relativistic MONDian gravitational potential energy U for M ≥ m:

$$U = -mzc^2$$

Relativistic MONDian gravitational potential ϕ for M ≥ m:

$$\phi = \frac{U}{m} = -zc^2 \qquad \textbf{(37)}$$

9.2. MONDian gravitational lensing

In the deep-MOND regime, the fictitious gravitational force experienced by a photon can be equated to the fictitious centripetal acceleration of the photon to find the lensing formula around a stationary black hole similar to the formula (31) in the relativistic Newtonian regime.

$$a = (1+z_o)(1+z)^2 \frac{GM}{R_o R} = \frac{c^2}{\left(\frac{R}{1+z}\right)}$$

$(1+z_o)(1+z)\dfrac{GM}{R_o c^2} = 1$, which is the ratio of the gravitational acceleration to the centripetal acceleration of the photon at its closest approach to mass M. Replacing ratio 1 with $\mathrm{Tan}\left(\frac{\pi}{4}\right)$, we obtain the gravitational lensing equation in the deep-MOND regime. As this ratio should be proportional to the angle of deflection θ, we can generalize this equation by replacing $\mathrm{Tan}\left(\frac{\pi}{4}\right)$ with $\mathrm{Tan}\left(\frac{\theta}{4}\right)$ to find the angle of deflection for any radius R.

$$\theta = 4\,\mathrm{Tan}^{-1}\left((1+z_o)(1+z)\frac{GM}{R_o c^2}\right) \approx \frac{4GM}{R_o c^2} = \frac{4\sqrt{GMa_0}}{c^2} \qquad \textbf{(38)}$$

Therefore, in the absence of EFE (external field effect), the gravitational deflection angle θ remains almost constant in the deep-MOND regime.

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9.3. Relativistic Poisson equation for MOND

The relativistic MONDian gravitational potential in Cartesian coordinates based on (37) and (35) is given below.

$$\phi = -\left(\frac{(1+z_o)}{1-(1+z_o)^2 \frac{GM}{R_0 c^2} \ln\left(\frac{R_o}{\sqrt{x^2+y^2+z^2}} \right)} - 1 \right) c^2$$

The relativistic Poisson equation outside the point mass can be generated by applying the Laplace operator to the above gravitational potential.

$$\nabla^2 \phi = -\frac{2G^2 M^2 x^2 (1+z_o)^2 (1+z)^3}{(x^2+y^2+z^2)^2 R_o{}^2 c^2} - \frac{2G^2 M^2 y^2 (1+z_o)^2 (1+z)^3}{(x^2+y^2+z^2)^2 R_o{}^2 c^2} - \frac{2G^2 M^2 z^2 (1+z_o)^2 (1+z)^3}{(x^2+y^2+z^2)^2 R_o{}^2 c^2}$$
$$-\frac{2GMx^2 (1+z_o)(1+z)^2}{(x^2+y^2+z^2)^2 R_o} - \frac{2GMy^2 (1+z_o)(1+z)^2}{(x^2+y^2+z^2)^2 R_o} - \frac{2GMz^2 (1+z_o)(1+z)^2}{(x^2+y^2+z^2)^2 R_o} + \frac{3GM(1+z_o)(1+z)^2}{(x^2+y^2+z^2) R_o}$$

The relativistic Poisson equation at the point mass can be generated by considering only the positive terms and ignoring the negative terms in the Laplacian ($\nabla^2 \phi$).

$\nabla^2 \phi = \frac{3GM(1+z_o)(1+z)^2}{(x^2+y^2+z^2) R_o} = 4\pi G \rho (1+z_o)(1+z)^2 \frac{R}{R_o}$, where $\rho = \frac{M}{\frac{4}{3}\pi R^3}$ is the relativistic mass density of the point mass.

$$\frac{a_0}{|\nabla \phi|} = \frac{(1+z_o)^3 \frac{GM}{R_o{}^2}}{(1+z_o)(1+z)^2 \frac{GM}{R_o R}} = \frac{(1+z_o)^2 R}{(1+z)^2 R_o} \text{ based on (25) and (36).}$$

$$\nabla^2 \phi = 4\pi G \rho (1+z)^4 (1+z_o)^{-1} \frac{a_0}{|\nabla \phi|}$$

$$\nabla . \left[\mu\left(\frac{|\nabla \phi|}{a_0} \right) \nabla \phi \right] = 4\pi G \rho (1+z)^4 \left[1 + z_o \xi\left(\frac{z}{z_o} \right) \right]^{-1}$$

Relativistic Poisson equation for MOND:

$$\boxed{\nabla . \left[\mu\left(\frac{|\nabla \phi|}{a_0} \right) \nabla \phi \right] = 4\pi G \rho \left(1 - \frac{\phi}{c^2} \right)^4 \left[1 - \frac{\phi_o}{c^2} \xi\left(\frac{\phi}{\phi_o} \right) \right]^{-1}} \quad \textbf{(39)}$$

where $\mu(x) \to 1$ for $x \gg 1$, $\mu(x) \to x$ for $x \ll 1$ and $\xi(x) \to 1$ for $x \ll 1$, $\xi(x) \to x$ for $x \gg 1$

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The above relativistic Poisson equation for MOND (39) reduces to the relativistic Poisson equation in the Newtonian regime (33) for $\phi \geq \phi_o$ and $|\nabla\phi| \geq a_0$, and to the nonrelativistic equation when the gravitational redshift z is considered negligible. The nonrelativistic Poisson equation based on AQUAL ("A QUAdratic Lagrangian") is given below.

$\nabla.\left[\mu\left(\frac{|\nabla\phi|}{a_0}\right)\nabla\phi\right] = 4\pi G\rho$ (Mamon et al., 2005), where $\mu(x) \to 1$ for $x \gg 1$, $\mu(x) \to x$ for $x \ll 1$.

9.4. Relativistic AQUAL for MOND

Relativistic AQUAL $\mathcal{L}$ and the respective action $\mathcal{S}$ based on (39) are below.

$$\mathcal{L} = -\frac{1}{8\pi G} a_0{}^2 F\left(\frac{|\nabla\phi|^2}{a_0{}^2}\right) - \int \rho\left[\left(1-\frac{\phi}{c^2}\right)^4\left(1-\frac{\phi_o}{c^2}\xi\left(\frac{\phi}{\phi_o}\right)\right)^{-1}\right]d\phi \qquad \textbf{(40)}$$

Where $F(x) \to \frac{2}{3}x^{\frac{3}{2}}$ for $x \ll 1$, $F(x) \to x$ for $x \gg 1$ and $\xi(x) \to 1$ for $x \ll 1$, $\xi(x) \to x$ for $x \gg 1$

$$\mathcal{S} = \iint \mathcal{L}\, d^3r dt$$

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Methods:
The computations in this paper were performed by using Maple™. Maple 2020.2. Maplesoft, a division of Waterloo Maple Inc., Waterloo, Ontario. Maple is a trademark of Waterloo Maple Inc.

Data availability statement:
Data sharing is not applicable to this article, as no datasets were generated or analyzed during the current study.

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