New Approach to Geometric Series for Computational Applications

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Abstract: Computational science is a rapidly growing multi-and inter-disciplinary area where science, mathematics, computation, management and its collaboration use advance computing capabilities to understand and solve the most complex real-life problems. In this article, a new kind of geometric series and theorems are introduced for mathematical and computational applications.

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1. Introduction

Geometric series played a vital role in differential and integral calculus at the earlier stage of development and still continues as an important part of the study in science, engineering, management and its applications. In this article, a new kind of geometric series [1-6] is formulated for application [7] of computational science.

2. Theorem on Geometric Series

Theorem 2.1: Let R be a set of real numbers and $p \in R$. Then $\sum_{i=0}^{n-1} x^{pi} = \frac{x^{np}-1}{x^p-1}, p \neq 0$.

Proof. Let us prove this theorem using the following equality:

$$y^{n+1} = y^{n+1} \Rightarrow y^{n+1} = (y-1)y^n + y^n \Rightarrow y^{n+1} = (y-1)y^n + (y-1)y^{n-1} + y^{n-1}.$$

If we continue this expansion again and again, we can obtain the following series.

$$y^{n+1} = (y-1)y^n + (y-1)y^{n-1} + (y-1)y^{n-2} + (y-1)y^{n-3} + \cdots + (y-1)y^k + y^k.$$

$$i.e., \quad y^{n+1} - y^k = (y-1)(y^n + y^{n-1} + y^{n-2} + y^{n-3} + \cdots + y^k). \quad (1)$$

From the expansion (1), we obtain the series $\sum_{i=k}^{n} y^i = \frac{y^{n+1}-y^k}{y-1}, y \neq 1.$ (2)

Let y be x^p and substitute it in the series (2).

$$\text{Then, } \sum_{i=k}^{n} (x^p)^i = \frac{(x^p)^{n+1} - (x^p)^k}{x^p - 1} \Rightarrow \sum_{i=k}^{n} x^{pj} = \frac{x^{p(n+1)} - x^{pk}}{x^p - 1}, p \neq 0. \quad (3)$$

From the Equation (3), we conclude that

$$\sum_{i=0}^{n} x^{pi} = \frac{x^{p(n+1)}-1}{x^p-1}, p \neq 0 \quad \text{(OR)} \quad \sum_{i=0}^{n-1} x^{pi} = \frac{x^{np}-1}{x^p-1}, p \neq 0.$$

Hence, theorem is proved.

Lemma 2.1: if p is a nonnegative integer, $\left(\sum_{i=0}^{p-1} x^i\right)\left(\sum_{j=0}^{n-1} x^{pj}\right) = \sum_{k=0}^{np-1} x^k = \frac{x^{np}-1}{x-1}, x \neq 1.$

Proof.

$$\left(\sum_{i=0}^{p-1} x^i\right)\left(\sum_{j=0}^{n-1} x^{pj}\right) = \left(\frac{x^p-1}{x-1}\right)\left(\frac{x^{np}-1}{x^p-1}\right) = \frac{x^{np}-1}{x-1}, x \neq 1 = \sum_{k=0}^{np-1} x^k. \quad (4)$$

Hence, the lemma is proved.

Corollarry 2.1: $\left(\sum_{i=0}^{p-1} x^i\right)\left(\sum_{j=0}^{n} x^{pj}\right) - \sum_{i=0}^{np} x^i = \frac{x^{np}(x^p - x)}{x-1}, x \neq 1 = x^{np}\sum_{k=1}^{p-1} x^k.$

3. Conclusion

In this article, a new kind of geometric series and theorems are introduced for mathematical and computational applications. Also, this idea can enable the scientific researchers for further involvement in research and development.

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